From centralized to decentralized power systems:

Some stochastic optimization problems in energy management.

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Geilo, February 2018
EDF: A WORLD LEADER IN ENERGY IN A CHANGING WORLD

- **38 millions** customers in the world
- **117,1 g** of CO₂ per kWh produced
- **170,000** employees in the world
- **618,5 TWh** produced in the world
- **66.3 Billions €** turnover (49 % out of France)

### Electricity: generation, transmission, distribution, trading

- **1,1** Other energies
- **8,1** Hydro
- **4,3** Gas
- **11,2** Thermal (fossil)

### Natural Gas

Main activities in Europe:
- France, GB, Germany, Italy...

Industrial Operator in Asia and in the USA

#### Changing world:

- **Big cities**: 50 % of population live in towns, 70 % in 2050
- **Emerging Countries**: China, Brazil, India...
- **Finite resources and need to « de-carbon » energy**
- **New ways to generate, to manage, to use electricity**: Local setting with raise of:
  - new and numerous producers (regions, towns, industries, individuals) operating decentralized means of renewable production (wind/solar)
  - new uses of electricity: electrical vehicles, green transportation, data centers, …
  - new management techniques for an optimal use of electricity (demand response, …) based on technologies of information and communication (smart grids) and on mathematics

1/3 of energetical needs are covered by electricity
40 % of electricity in the world comes from **coal**, 20 % from natural **gas**, 16 % from hydro, 15 % from nuclear, 7 % from oil and 2 % from renewable energy

Some Stochastic Optimization Problems in Energy Management
MAIN OBJECTIVES OF THE TALK

1) To provide a brief introduction to energy management and its evolution

2) To present (not to solve) some stochastic optimization problems in energy management
   a) In the historical centralized framework
   b) In the decentralized framework induced by the evolution of electrical systems

3) To highlight the needs in stochastic optimization from the utility point of view

4) To open the discussion on the different paths for developments in this field
OUTLINE

Part I  The framework: electrical systems and energy management

Part II  Some energy management Problems in a centralized setting

Part III Which energy management problems in a decentralized setting?

Conclusions, perspectives
Part I

The framework:

electrical systems and energy management
AN ELECTRICAL SYSTEM

A set of three components electrically connected to each other on a given territory

**Generation (the offer, the supply)**
Electricity generated by power plants with different characteristics

**Load (the demand)**
Electricity consumed by
- Industrial/transport customers (23 – 138 KV)
- Commercial customers (4.16 – 34.5 KV)
- Residential customers (120 – 240 V)

**Transmission / distribution networks** (110-765 KV)
- Transmission network (in France: from 63 kV to 400 kV)
- Distribution network (in France: 230 V to 50 kV)

- Frequency
  - electrical expression of the rotating speed of the alternator (common parameter to all the components of a system: 50-60 Hz)
  - indicator of the gap between supply and demand:
  - If demand is higher than supply, the frequency decreases.
  - If supply is higher than demand, the frequency increases

- Voltage
  - local parameter of the electrical network: on a given point of the network, the voltage is a function of
  - injection (production): voltage increases
  - withdrawal (consumption): voltages decreases

Some Stochastic Optimization Problems in Energy Management
1. THE LOAD : USES AND USERS OF ELECTRICITY

Electricity consumption depends on customers profiles (demography, economic activity, sectors), networks losses, sale contracts

**Specific uses** (only by electricity).
- freezer, dryer, dishwasher, microwave
- audiovisual TV, DVD player, computer
- Lighting

**Competitive uses** (substitutions possible). The electricity demand depends on the market shares
- Mainly thermal : heating, domestic hot water, cooking
- Transportation : train vs coach / tram vs bus / electric vs gasoline vehicle

### Residential : Some power profiles

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Down</td>
<td>an american refrigerator</td>
</tr>
<tr>
<td>Up</td>
<td>three washing machines</td>
</tr>
<tr>
<td>Down</td>
<td>an elevator</td>
</tr>
</tbody>
</table>

### Demand (MW)

**Multi-year cycle**

<table>
<thead>
<tr>
<th>Year</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power (MW)</td>
<td>35000</td>
<td>40000</td>
<td>45000</td>
<td>50000</td>
</tr>
</tbody>
</table>

### Time series with cycles and yearly trend

**Annual cycle**

**Weekly cycle**

**Daily cycle**

Some Stochastic Optimization Problems in Energy Management
## 2. THE SUPPLY : THREE CHARACTERISTICS

<table>
<thead>
<tr>
<th>Capacity Factor</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>% of production hours expected per year on average over the life of the installation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coal</td>
<td>0.5</td>
<td>0.85</td>
</tr>
<tr>
<td>IGCC</td>
<td>0.5</td>
<td>0.85</td>
</tr>
<tr>
<td>Combustion Turbine</td>
<td>0.88</td>
<td>0.92</td>
</tr>
<tr>
<td>Combined Cycle</td>
<td>0.8</td>
<td>0.87</td>
</tr>
<tr>
<td>Nuclear</td>
<td>0.70</td>
<td>0.9</td>
</tr>
<tr>
<td>Biomass</td>
<td>0.8</td>
<td>0.85</td>
</tr>
<tr>
<td>Geothermal (hydrothermal)</td>
<td>0.87</td>
<td>0.95</td>
</tr>
<tr>
<td>Wind (onshore)</td>
<td>0.15</td>
<td>0.44</td>
</tr>
<tr>
<td>Wind (offshore)</td>
<td>0.30</td>
<td>0.45</td>
</tr>
<tr>
<td>Solar Thermal</td>
<td>0.22</td>
<td>0.42</td>
</tr>
<tr>
<td>PV</td>
<td>0.10</td>
<td>0.26</td>
</tr>
<tr>
<td>Hydro (lake)</td>
<td>0.30</td>
<td>0.50</td>
</tr>
<tr>
<td>Hydro (fil de l’eau)</td>
<td>0.80</td>
<td>1.00</td>
</tr>
</tbody>
</table>

### Total emissions of CO2

<table>
<thead>
<tr>
<th>Total emissions of CO2</th>
<th>Nuclear</th>
<th>10 gCO₂/kWh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Renewables</td>
<td>Hydro</td>
<td>6 gCO₂/kWh</td>
</tr>
<tr>
<td></td>
<td>Wind</td>
<td>7 gCO₂/kWh</td>
</tr>
<tr>
<td></td>
<td>Photovoltaic</td>
<td>55 gCO₂/kWh</td>
</tr>
<tr>
<td>Fossil fuel based</td>
<td>Coal</td>
<td>1038 gCO₂/kWh</td>
</tr>
<tr>
<td></td>
<td>Fuel</td>
<td>704 gCO₂/kWh</td>
</tr>
<tr>
<td></td>
<td>Gas</td>
<td>406 gCO₂/kWh</td>
</tr>
<tr>
<td></td>
<td>Cogeneration</td>
<td>400 gCO₂/kWh</td>
</tr>
</tbody>
</table>

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Electricity is very difficult to store

Storage : possible for small amounts of electricity and at very high costs. Three techniques:

- Hydro storage : pumping power station
- Economical incentive «storage» : shedding contracts
- Battery
2. THE SUPPLY : OPERATING A PLANT

- **Starts the plant**
  - start curves: vary depending on the previous stop time
  - Minimal duration of operation
  - Maximum number of start-up per day

- **Produces**
  - Levels of production: $P_{\text{min}}$, $P_1$, $P_2$, …, $P_{\text{max}}$
  - Minimum duration in a given level of production
  - Maximum number of modulations per day
  - Increases the production (Gradient of power variation upward)
  - Decreases the production (Gradient of power variation downward)

- **Stops the plant**
  - stop curves: vary depending on the previous stop time
  - Minimal duration of stop
4. UNCERTAINTIES ON ELECTRICAL SYSTEMS

- Loss of customers (competition)
- Exchanges
- Economic situation
- Customers behavior
- Fall of a pylon
- Load curve
- Networks

Weather Climat
- Extreme Phenomenons (drought, flood, ...)

Influence/Correlation

- Random outages
- Shutdown duration
- Temperature of rivers (cooling)
- Hydro inflows
- Volumes in reservoirs
- Speed wind
- Sunshine
- State of the atmosphere
- Season

Variability of Wind production
Variability of Hydro Inflows
Load (In winter: -1°C → + 1500 MW)

Some Stochastic Optimization Problems in Energy Management
5. ENERGY MANAGEMENT: AT A GLANCE

Energy Management = optimal management of the three components of the electrical system
objective: Efficiency & Cost effectiveness

**ENERGY MANAGEMENT**
- Demand-Side management
- Network management
- Power Systems management

**DEcision process**
- Organization of teams / Timing of decisions
- Decision-making tools & decision-makers

**HISTORICAL LEGAL ISSUE**
- Monopoly - Integrated Utility
  - Generation
  - Transmission
  - Distribution

**PRESENT LEGAL ISSUE**
- Deregulation, markets
  - Generation Companies
  - Transmission company
  - Distribution companies
  - ISO (system reliability)
  - Electricity markets
  - Prosumers

Management = Software (physical management is excluded)
5. ENERGY MANAGEMENT: OPTIMIZE DECISIONS

Find the best decisions in view to operate the system at short/mid/long terms and at minimal cost

Long-Term Prices (uranium, coal, gas, oil)

Supply-side: Management of power plants

MT Issue: Power Plants Stops for Maintenance (& Refueling)

Fuel Supplies & Stocks Management

Efficient nuclear management saves 100 Millions € / year

Efficient hydro management saves 200-300 Millions € / year on fossil fuel cost

ST Issue: Daily Generation Planning & adjustment (Unit Commitment)

LT Issue: Investments (new power plants)

Non-availability of a nuclear power plant during winter week: 1.2 to 1.5 Millions €

Electricity Markets

Risk management: sales, purchases, hedging

Long-term (20-50 years)

Mid-term (1-5 years)

Short-term (1 hour – 1 day)

Network Side

Investments (networks)

Dispatching

Design customers contracts

Strategy for the use of load-sheds

Load forecast

Long-term demand

Demand-Side: Forecasting and Contracts for customers

Fuel costs 3-4 Billions € / year (nuclear 2.4 Billions €)

Some Stochastic Optimization Problems in Energy Management
A 2-step process

Is the demand too high regarding capacity of production?

Yes

PB 0: Assessment on network losses, exchanges with others utilities + Decision to start shedding contracts (main tool of flexibility on demand): managed upstream of the optimization process and independently → Demand is a data of an optimization problem

No

PB 1: Optimization of production for this demand

Goal: Minimize the global cost of production

Constraints I: Satisfy the demand

Constraints II: Respect the
- technical constraints of power plants
- technical constraints related to the network

Representation of uncertainty
- Laws, Uncertainty sets
- Scenarios

Modeling constraints on production units
- Detailed
- CT

Mixture of 3 challenging features: Uncertainty + Big size + Mixed variables
Some stochastic optimization problems
In energy management

Centralized setting
Some Stochastic Optimization Problems in Energy Management

THE CENTRALIZED SETTING

Legislator (regulatory rules, constraints)

Utility Company (Monopoly)

- Minimize its costs
- Satisfy the demand
- Respect operating constraints

Manages Power Systems
Thermal / hydro power plants and some commands on flexibilities on demand (Sheddings)

Production Planning

Manages Networks
Transmission, distribution network: dispatching, security

Adjustment

Customers

Sheddings signals

Demand
The data of the problem
You have a set of power plants in order to satisfy a given demand on the next 24 hours. Each power plant is characterized by:
- Costs: generation, starting, …
- Operation constraints

The problem to solve
ST1 - Forecast at D-1 the demand for the next 24h of day D

ST2 - Determine at D-1 the best generation planning for the next 24h of day D:
- minimal global cost
- satisfaction of demand
- feasible planning: respects all the operation constraints

ST3 - Adjust at each half an hour of D-Day the initial generation schedule estimated at D-1 with the specific constraint: changes in the planning must only concern a limited number of power plants (typically 30 over 250 power plants)
A 2-step process

Is the demand too high regarding capacity of production?

Yes

PB 0: Decision to start shedding contracts (main tool of flexibility on demand): managed upstream of the optimization process and independently.

No

Demand is a data of the optimization problem

PB 1: Optimization of production for this demand

Goal: Minimize the global cost of production

Constraints I: Satisfy the demand

Constraints II: Respect the
- technical constraints of production units (Pmin/Pmax / Starts/stops / Minimum Up/Down time / Gradients, …)

Representing uncertainty: integrated in reserves

Modeling constraints on power plants: at short term, a precise description of operation constraints is needed

Mixture of 2 challenging features: Big size + Mixed variables
THE UNIT COMMITMENT PROBLEM : RESOLUTION SCHEME

Detailed description of constraints on power plants – Deterministic setting

\[
\min_u \sum_{l \in L} \left( \sum_{t \in T} C_{l,t} \left( u_{l,t}, x_{l,t} \right) \right)
\]

\[
\forall t \in T, \sum_{l \in L} P_{l,t} \left( u_{l,t}, x_{l,t} \right) = d_t, \ \forall t \in T, \sum_{l \in L} R_{l,t} \left( u_{l,t}, x_{l,t} \right) \geq r_t
\]

\[
\forall l \in L, \left( u_{l,t}, x_{l,t} \right) \in D_l
\]

Nonconvex, nonlinear, mixed problem of big size

Coordination : bundle method

\[
\sum_{j=1}^{n} p_{ij} = d_i
\]

Price \(\downarrow\) Production

59 nuclear power plants
50 thermal plants (coal, fuel, gas)

Price \(\downarrow\) Production

50 hydro valley

Simple Lagrangian
(compute : marginal costs)

Augmented Lagrangian
(compute : production plannings)

Simplified formulation | Linear Programme (LP) | Quadratic Programme (QP)
Exact Formulation    | Mixed Linear Programme (MLP) | Mixed Quadratic Programme (MQP)

Big valley (La Durance) : 13000 constraints, 18000 variables (5000 binaries)
Medium valley (l’Arc) : 12000 constraints, 15000 variables (4000 binaries)
Small valley (la Romanche) : 10000 constraints, 13000 variables (3000 binaries)

Some Stochastic Optimization Problems in Energy Management
HOW THE ELECTRICAL SYSTEM CAN FACE UNCERTAINTIES?

The Primary Reserve (Frequency-Power)
- Automated system
- 3000 MW at European level (simultaneous loss of 2 biggest power plants 2x1500 MW)
- Reaction time: few seconds
- In France: from 600 to 800 MW
- 50% available in 15 s, 100% in 30 s

The Secondary Reserve (Frequency-Power)
- Automated system
- In France: de 500 à 1000 MW
- Released from 100 to 200 s
- End of mobilization < 15 mn

The Tertiary Reserve
- Human decision
- Fast reserve (hydro) <13 mn: 1000 MW
- Additional reserve (hydro, turbine) <30 mn: 500 MW

Thermal plants do not operate at maximal capacity!

The setpoint power of a thermal group is between its technical minimum (TM) and its maximum available power (MAP)
THE UCP : COMPUTING THE RESERVES

Probabilistic Approach

\[ \text{IP} \left[ \sum_{j=1}^{N} a_{ij} \left( \xi_{\theta} \right) x_{ij} \geq d_i \left( \xi_{\delta} \right) \right] \geq 1 - p, \quad i = 1, \ldots, m \]

\[ a_{ij} \quad \text{Availability coefficient} \]

\[ d \quad \text{Demand} \]

Gaussian setting:

\[ \sum_{j=1}^{N} \text{IE} \left[ a_{ij} x_{ij} \right] + \Phi^{-1} \left( 1 - p \right) \sqrt{\sum_{j=1}^{N} \sigma_{ij}^2 x_{ij}^2 + \sigma_{d_i}^2} \geq \text{IE} \left[ d_{ij} \right] \]

Mean production \( R_{1\%} \) Reserve for the risk 1%

Hoeffding setting (no hypothesis on laws):

**Lemma 1 : Any Individual Chance Constraint**

\[ \mathbb{P} \left[ \langle A_i(\xi), x \rangle \geq b_i(\xi_{\delta}) \right] \geq \alpha_i, \quad \forall i \in I \]

is approximated by the 2 (convex) conic quadratic inequalities:

\[ \langle \mathbb{E} [A_i(\xi)], x \rangle - \sqrt{\left( 1/2 \right) \ln \left( 1 - \alpha_i \right)} \| \Delta_i x + \delta_{b_i} \|_2 \geq \mathbb{E} (b_i) \]

\[ \langle \mathbb{E} [A_i(\xi)], x \rangle \geq \mathbb{E} (b_i) \]

\[ \delta_{b_i} = (b_i^{\text{max}} - b_i^{\text{min}}) \]

\[ \Delta_j = \begin{pmatrix} a_{1j}^{\text{max}} - a_{1j}^{\text{min}} \\ \vdots \\ a_{mj}^{\text{max}} - a_{mj}^{\text{min}} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \]
DEALING WITH UNCERTAINTY

Which methods for energy management?

Investigations: probabilistic models

\[
\begin{align*}
\min_{x \in X} & \quad f[c(x, \xi)] \\
\text{s.t.} & \quad \mathbb{P}[A(\xi)x \geq b(\xi)] \geq 1 - p \\
& \quad Px \leq h
\end{align*}
\]

\[
A(\xi) = \begin{pmatrix}
A^0(\xi_\theta) & A^\eta(\xi_\eta) & A^\mu(\xi_\mu) & A^\sigma(\xi_\sigma) & A^c(\xi_c)
\end{pmatrix}
\]

\[
A^\alpha(\xi_\alpha) = \begin{pmatrix}
a^{\alpha}_{11} & a^{\alpha}_{1N\alpha} & 0 & \ldots & 0 & \ldots & 0 \\
0 & \ldots & 0 & a^{\alpha}_{i1} & a^{\alpha}_{iN\alpha} & \ldots & 0 \\
0 & \ldots & 0 & 0 & \ldots & 0 & a^{\alpha}_{m1} \\
0 & \ldots & 0 & 0 & \ldots & a^{\alpha}_{mN\alpha}
\end{pmatrix}
\]

\[
\max_{x \in X} \ IP \left[ a \leq Ax \leq b \right]
\]

Applications: integration of renewables, statistics, portfolio, inverse problems, …
EXPERIMENT - THE HYDRO SUB-PROBLEM OF THE UCP

To determine the production of a valley associated to a price signal

- Cost function: water values (pre-computed)

- Constraints (simplified situation):
  - Flow constraints
  - Production level bounds
  - Reservoir bounds (exploitation policy, security)

- Random Inflows: we assume that they follow some causal time series model with Gaussian innovations (correlated between reservoirs)

Some Valleys

Some Stochastic Optimization Problems in Energy Management
F1 : Deterministic

F2 : Joint Chance Constraint

\[ \min_{x \geq 0} \quad c^T x \]
\[ \text{s.t.} \quad Ax \leq b \]
\[ p \leq \mathbb{P}[a^r + A^r x \leq \eta \leq b^r + B^r x]. \]

F3 : Individual Chance Constraints

F4 : Robust

F5 : Max-p

\[ \max_{x \geq 0} \quad \varphi(x) := \mathbb{P}[a^r + A^r x \leq \eta \leq b^r + A^r x] \]
\[ \text{s.t.} \quad Ax \leq b \]
Numerical results

Water Trajectories for reservoir « Saut Mortier ».

- **F1 : Det**
- **F2 : JCCP**
- **F3 : ICCP**
- **F4 : Robust Conic**
- **F5 : Max-p**

<table>
<thead>
<tr>
<th>Inst.</th>
<th>Item / Problem</th>
<th>Det</th>
<th>JCCP</th>
<th>ICCP</th>
<th>Conic</th>
<th>Max-p</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>nbViolation</td>
<td>100</td>
<td>20</td>
<td>35</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Cost (€)</td>
<td>$-1.0478e^5$</td>
<td>$-1.0340e^5$</td>
<td>$-1.0422e^5$</td>
<td>$-1.0282e^5$</td>
<td>$-9.9176e^4$</td>
</tr>
</tbody>
</table>

- The deterministic average-solution leads to almost sure violation of bounds
- JCCP gives a good trade-off between cost and robustness
- The Max-p solution is very robust, but very expensive
Which energy management problems in a decentralized setting?
THE DECENTRALIZED SETTING

Legislator (regulatory rules, constraints)

Network Manager: Responsible for the global Supply-Demand balance

Wholesale Producer
(Thermal and hydro power plants, some commands of flexibilities on demand)

Minimize its global cost
Supply-demand balance at its perimeter
Risk Management

Manages big flexibilities (EVC, HDW, Sheddings)

WP’s Customers

Electricity Market

Buy Sell

Other Producers
Minimize its global cost
Supply-demand balance at its perimeter
Risk Management

Providers/Traders

Local Actors
(PV, wind and battery)

Maximize their profit
Own Supply-demand balance
Risk management

Sell

Demand

Buy Sell

WP’s Offers Business

Competitors’ Offers Business

Other Customers

Demand

Other Producers

Some Stochastic Optimization Problems in Energy Management
RELATIONSHIPS BETWEEN WHOLESALE PRODUCER AND ACTORS

**Wholesale Producer:**
(Thermal and hydro power plants, some levers of flexibilities on demand)

- Minimize its costs
- Supply-demand balance at its perimeter
- Manages big flexibilities (EVC, HDW, Sheddings)

**Local Actor:**
(PV, wind and battery)

- Maximize its profit

**Commands on demand to facilitate sheddings via attractive pricing schemes**

**Optimization problem with uncertainty**

Wholesale producer: « Leader »
\[
\min_{\tilde{x}} \tilde{c}_1 \tilde{x} + \tilde{d}_1 \tilde{y}
\]
\[
\text{s.t. } \tilde{A}_1 \tilde{x} + \tilde{B}_1 \tilde{y} \leq \tilde{b}_1
\]
\[
\text{max } \tilde{d}_2 \tilde{y}
\]
\[
\text{s.t. } \tilde{A}_2 \tilde{x} + \tilde{B}_2 \tilde{y} \leq \tilde{b}_2
\]

Local Actor: « Follower »
\[
\tilde{y} : \text{commands on its production and its own demand}
\]

Buy / sell at wholesale producer

Some Stochastic Optimization Problems in Energy Management
Joint optimization of production portfolio and flexibilities on demand

Supply
- Classical production portfolio (nuclear, thermal, hydro)
- Renewables Production manageable by using batteries

Demand
- Three main flexibilities
  - Shedding Contracts
  - Electrical Vehicles Charging (EVC)
  - Hot Domestic Water (HDW)

Shedding Profile
- Shedding on 24 h

Electrical Vehicles Charging
- Shift of EVC and HDW consumptions while respecting some constraints:
  1) Respect of the maximal peak and 2) Energy conservation
  2 slots: 09-17h et 18-06h

Some Stochastic Optimization Problems in Energy Management
### Results (obtained with fictitious data unrelated with EDF)

#### Flexibilities

<table>
<thead>
<tr>
<th>Mix</th>
<th>Nuclear</th>
<th>Coal</th>
<th>Fuel</th>
<th>Gas</th>
<th>Hydro</th>
<th>Wind</th>
<th>PV</th>
</tr>
</thead>
<tbody>
<tr>
<td>2015</td>
<td>49.1 %</td>
<td>21.4 %</td>
<td>12.7 %</td>
<td>4.2 %</td>
<td>8.4 %</td>
<td>3.2 %</td>
<td>1.1 %</td>
</tr>
<tr>
<td>2030</td>
<td>28.2 %</td>
<td>15.7 %</td>
<td>11.2 %</td>
<td>5.6 %</td>
<td>12.8 %</td>
<td>16  %</td>
<td>10.6 %</td>
</tr>
</tbody>
</table>

#### Battery management

We compare the 2 following situations:
- Situation 1: The big producer manages the battery
- Situation 2: The local player manages the battery

<table>
<thead>
<tr>
<th>Flexibility optimized</th>
<th>Saving on global cost (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2015</td>
</tr>
<tr>
<td>Electrical Vehicles</td>
<td>0.26 %</td>
</tr>
<tr>
<td>Hot Domestic Water</td>
<td>0.62 %</td>
</tr>
<tr>
<td>Sheddings</td>
<td>0.04 %</td>
</tr>
<tr>
<td>The three</td>
<td>0.84 %</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Earning Situation 1</th>
<th>Earning Situation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>For the big producer</td>
<td>1.45 %</td>
<td>0.72 %</td>
</tr>
<tr>
<td>For the Local Actor</td>
<td>0 %</td>
<td>4.09 %</td>
</tr>
<tr>
<td>For the both</td>
<td>1.41 %</td>
<td>0.82 %</td>
</tr>
</tbody>
</table>

It is generally more cost-effective to manage the battery by the big producer and give financial compensation to the local player… but … the local player has personal interest in managing himself his battery.
A new French 2016 law allows self-consumption.

1 business building A

- Load curves
- Flexibilities: EVC, HDW
- 1 PV panel on the roof + battery
  - Sharing the PV production
  - Sell the excedent to building B

1 trading building B

- Load curves
- No PV panel

Problems

- Building A: each company who invested in the PV + battery aims at maximizing its profit
- Building B: each shop have to decide: buy electricity to the building A or not?
- DSO: tarification for using the network
- Wholesale producer: estimating the residual demand induced by the self-consumption
TYPES OF PROBLEMS

Classical problems in a decentralized setting

→ stochastic optimization

Competition :

→ game theory, equilibriums

Relationships with customers :

→ Tarification
→ Residual demands of prosumers
→ stochastic bi-level optimization
CONCLUSIONS AND PERSPECTIVES

1) Even in "historical" situation of a centralized management, energy management optimization poses problems of extreme difficulty (large, uncertainty, continuous / discrete variables, big size)

2) The need to adapt to the new context of energy leads to new challenging optimization problems in interaction with economy and game theory. → Significant strengthening of the research effort.

EDF has a broad research program with the support of many academic institutions. Cooperation on these problems is possible via the two associated labs:

**FIME : A research Lab in Finance for Energy markets**

- A strong network in economics & applied mathematics
- 6 EDF research engineers
- 12 academic researchers
- 15 PhD students and postdocs in 2016
- 20 publications in peer-reviewed journals/year
- 4 High quality training sessions/year
- 6 Published books since 2009

www.fime-lab.org

**PGMO : the Gaspard Monge Program for Optimization**

- A strong network in optimisation, operational research & data science (+ education)
- 2 EDF research engineers
- 150 academic researchers
- 110 Laboratories contributing to projects
- 5 publications in peer-reviewed journals/year
- 35 publications in peer-reviewed journals/year

www.fondation-hadamard.fr/en/pgmo

Some Stochastic Optimization Problems in Energy Management
THANK YOU
APPENDIX
General Probabilistic Model

Theoretical Extension to Closed Loop Setting:

Based on exploiting the link between Joint and Individual Chance Constraints

Denote $\mathcal{E}_i = \{\xi : \langle A_i(\xi), x \rangle \geq b_i \}$

$$\mathbb{P}[A(\xi)x \geq b] = \mathbb{P}[\mathcal{E}_i] \prod_{k=1, k \neq i}^m \mathbb{P}[\mathcal{E}_k | \mathcal{E}_i \cap \bigcap_{j=1}^{k-1} \mathcal{E}_j]$$

Start with time step $i = 1$, sequential decision using conditional probabilities.
Probabilistic Model : Individual Chance Constraint

\[
\begin{align*}
\min_x & \quad c^t x \\
\text{s.t.} & \quad \mathbb{P}[\langle A_i(\xi^\theta), x \rangle \geq b_i(\xi^\delta)] \geq \alpha_i, \forall i \\
& \quad \mathbb{P}[\langle H_i, x^\eta \rangle \leq h_i^{\text{min}}(\xi^\eta)] \geq \beta_i, \forall i \\
& \quad \mathbb{P}[\langle H_i, x^\eta \rangle \geq h_i^{\text{max}}(\xi^\eta)] \geq \beta_i, \forall i \\
& \quad \langle P_i, x \rangle \leq h_i, \quad i = 1, \ldots, m \\
& \quad x \geq 0
\end{align*}
\]

\[
\begin{align*}
\begin{array}{l}
\quad h_i^{\text{min}}(\xi^\eta) = v_0 + \sum_{i0=1}^i f_{i0}(\xi^\eta) - v_i^{\text{min}} \\
\quad h_i^{\text{max}}(\xi^\eta) = v_0 + \sum_{i0=1}^i f_{i0}(\xi^\eta) - v_i^{\text{max}}
\end{array}
\end{align*}
\]

\[
\begin{align*}
\quad H_{ij} = \mathbb{I}_{j \leq i}
\end{align*}
\]
Available Information

historical data

\[ a_{ij}(\xi_\alpha) \in [a_{ij}(\xi_\alpha)_{min}, a_{ij}(\xi_\alpha)_{max}] \]
\[ b_i(\xi_\delta) \in [b_i(\xi_\delta)_{min}, b_i(\xi_\delta)_{max}] \]

Exploiting minimal information on random processes:

\[ a_{ij}(\xi)_{min}, a_{ij}(\xi)_{mean}, a_{ij}(\xi)_{max} \]

Hypothesis: Independence of \[ a_{ij}(\xi_\alpha), b_i(\xi_\delta) \]
Convex Approximation

**Proof**: Based on Hoeffding’s Theorem: for independent and bounded random variables $X_1, \ldots, X_m$, noting $S = \sum_{i=1}^{m} X_i$, the individual chance constraint is bounded as follows:

$$\forall \tau \geq 0, \, \mathbb{P}[S \geq \mathbb{E}[S] + \tau] \leq \exp \left( \frac{-2\tau^2}{\sum_{i=1}^{m} (X_{ij} - \mathbb{E}[X_{ij}])^2} \right).$$

We define a matrix $A' = (A \quad -b)$ of dimension $(m, n+1)$ and the vector $x' = (x \quad x_{n+1})$ with $x_{n+1} = 1$. We pose $X_{ij} = a'_{ij}x'_j$. □

$$\mathbb{P}\left[ \sum_{j=1}^{n} a_{ij}x_j \geq b_i \right] \iff \mathbb{P}\left[ \sum_{j=1}^{n+1} a'_{ij}x'_j \geq 0 \right]$$
Computing the solution: solve a SOCP

**ICCP: Individual Chance Constraint Program**

\[
\begin{align*}
\min & \quad c^t x \\
\text{s.t.} & \quad \mathbb{P}\left[ \langle A_i(x), x \rangle \geq b_i(x) \right] \geq \alpha_i \quad i = 1, \ldots, m \\
& \quad Px \leq h \\
& \quad x \geq 0
\end{align*}
\]

**SOCP: Second Order Conic Program**

\[
\begin{align*}
\min_x & \quad c^t x \\
\text{s.t.} & \quad \| \tilde{A}_l x + \tilde{b}_l \|_2 \leq \tilde{f}_l x + \tilde{d}_l, \quad l \in (1, L)
\end{align*}
\]

Resolution (polynomial complexity): Interior Points Method

**Equivalent SDP representation**

\[
\begin{align*}
\min_x & \quad c^t x \\
\text{s.t.} & \quad \begin{bmatrix}
(\tilde{f}_l^t x + \tilde{d}_l)I & \tilde{A}_l x + \tilde{b}_l \\
(\tilde{A}_l x + \tilde{b}_l)^t & \tilde{f}_l^t x + \tilde{d}_l
\end{bmatrix} \succeq 0, \quad l \in (1, L)
\end{align*}
\]
Resolution of Joint CCP Problem

The joint CCP can be shown to be convex.

Step 1: Gradient can be computed by repeated use of Genz’ code.

Corollary 1

Let \( \xi \) be a Gaussian Random variable of dimension \( n \). Let \( x, A, B, a, b \) be vectors and matrices of appropriate dimension. Now consider the mapping \( \varphi : x \mapsto \mathbb{P}[a + Ax \leq \xi \leq Bx + b] \). We have:

\[
\nabla \varphi = \nabla_a F_\xi(a, b)^T A + \nabla_b F_\xi(a, b)^T B \\
\Delta \varphi = A^T \Delta_a F_\xi(a, b) A + A^T \Delta_{ab} F_\xi(a, b) B + B^T \Delta_{ba} F_\xi(a, b) A + B^T \Delta_{bb} F_\xi(a, b) B.
\]

Step 2: We can apply a cutting planes algorithm to solve the problem.
The Hydro Sub-Problem: Solving the Joint CCP Problem

Step 1: gradient of a joint chance constraint

**Theorem 1** Assume that $\xi \sim \mathcal{N}(\mu, \Sigma)$ with some positive definite covariance matrix $\Sigma$. Then, for $i = 1, \ldots, n$,

$$\frac{\partial}{\partial b_i} F_\xi(a, b) = f_{\xi_i}(b_i) F_{\xi(b_i)}(\tilde{\alpha}, \tilde{b}) \quad (2)$$

$$\frac{\partial}{\partial a_i} F_\xi(a, b) = -f_{\xi_i}(a_i) F_{\xi(a_i)}(\tilde{a}, \tilde{b}). \quad (3)$$

Here, $f_{\xi_i}$ is as in Lemma 1, $\tilde{\xi}(b_i), \tilde{\xi}(a_i)$, are $n - 1$-dimensional random vectors distributed according to $\tilde{\xi}(b_i), \tilde{\xi}(a_i) \sim \mathcal{N}(\hat{\mu}, \hat{\Sigma})$ such that $\hat{\mu}$ results from the vector $\mu + \sigma_{ii}^{-1} (b_i - \mu_i) \sigma_i$ (in case of $b_i$) or from the vector $\mu + \sigma_{ii}^{-1} (a_i - \mu_i) \sigma_i$ (in case of $a_i$) by deleting component $i$ and $\hat{\Sigma}$ is defined as in Lemma 1. Moreover $\tilde{a}$ and $\tilde{b}$ result from $a$ and $b$ by deleting the respective component $i$. 
Resolution of Joint CCP Problem

◊ Step 2: cutting planes algorithm

1. Let $x_0$ be the solution of the "average" problem, $x_s$ a Slater point. Set $A_0 = A$, $b_0 = b$ and $k = 0$.

2. Find $\lambda^*$ such that $x_k^* = (1 - \lambda^*)x_k + \lambda^*x_s$ and $|\varphi(x_k^*) - p| < \varepsilon$.

3. Add constraint $-\nabla \varphi(x_k^*)^T x \leq \varphi(x_k^*) - \nabla \varphi(x_k^*)^T x_k^* - p$ to the matrix.

4. Solve $\min_x c^T x; s.t. A_k x \leq b_k$ to find $x_{k+1}$.

5. If $\varphi(x_{k+1}) > p - \delta$ then stop, else move $k = k + 1$ and go in step 2.
F4 : Robust Conic

Uncertainty set : find some set $\mathcal{E}_p$ such that $\mathbb{P}[\eta \in \mathcal{E}_p] \approx p$ such as

$$\mathcal{E}_p = \left\{ x : x^\top \Sigma^{-1} x \leq n + \Phi^{-1}(p) \sigma_C \right\}$$

with

$$\sigma_C = \sqrt{\sum_{i=1}^{n} \mathbb{E} \left( y_i^4 \right) - n}$$

$$y = L^{-1} \eta$$

$$\Sigma = LL^\top.$$ 

Solve : $\quad \min_{x \geq 0} \quad c^\top x$

s.t. $\quad Ax \leq b$

$$a^r + A^r x \leq \inf \mathcal{E}_p$$

$$b^r + A^r x \geq \sup \mathcal{E}_p$$

$\inf \mathcal{E}_p :$ largest vector $x^i \in \mathbb{R}^n$ having $x^i \leq y \ \forall y \in \mathcal{E}_p$