Electricity Auctions in the Presence of Transmission Constraints and Transmission Costs*

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Abstract

Electricity markets are moving through integration around the world. However, our understanding of those markets is still limited. I characterize the Bertrand equilibrium in a discriminatory-price electricity auction when suppliers submit a single offer price for their entire production capacity and they face transmission constraints and linear tariffs for the injection of electricity into the grid. With *point of connection* tariffs, which are used in the majority of the European countries, suppliers pay a tariff for the total electricity injected into the grid. In contrast, with *transmission tariffs*, suppliers only pay a tariff for the electricity sold in the other market. Transmission tariffs outperform point of connection tariffs by maximizing consumers welfare and transmission efficiency. The consequences of an increase in transmission capacity differ considerably depending on the tariff. If the transmission tariffs are *zero*, an increase in transmission capacity is pro-competitive. In contrast, if the transmission tariffs are *positive*, an increase in transmission capacity is pro-competitive only when the transmission capacity is low.

KEYWORDS: electricity auctions, wholesale electricity markets, transmission capacity constraints, network tariffs, energy economics.
JEL codes: D43, D44, L13, L94

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1 Introduction

The integration of electricity markets around the world has increased the importance of congestion between countries/states and has initiated a discussion of how to harmonize network tariffs. In general, transmissions from regions with low prices to regions with high prices benefit social welfare. In deregulated electricity markets, more transmission would, in addition, normally improve the market competitiveness. However, it is very costly to expand the transmission capacity. In order to focus the investments to points in the grid where the gains in terms of enhanced market performance will be the largest, one needs a better understanding of how transmission capacity influences the competition between spatially distributed producers. The contribution of this paper is to characterize the outcome of a discriminatory-price electricity auction, and how it depends on transmission constraints and the tariffs to access the grid.

The analysis employs a simple duopoly model similar to that in Fabra et al. (2006). In the basic set up, the two suppliers have symmetric production capacities and nil production costs that are located in two different markets ("North" and "South") connected through a transmission line with a limited transmission capacity.[1] When the transmission line is capacity constrained, the equilibrium prices differ across markets. Those differences in prices generate a congestion rent which, as in Borenstein et al. (2000) and in the majority of the European countries (ENTSO-E, 2015; European Commission, 2015, Nord Pool, 2007; Price Coupling Regions 2016a; Price Coupling Regions 2016b), I assume to be captured by the transmission system operator. When the competition in the spot electricity market is perfect, the introduction of financial rights to capture the congestion rents can re-stablish perfect competition by avoiding the externalities caused by loop flows in the transmission network (Hogan, 1992; Chao et al., 1996).[2]. However, in the presence of lumpy investments it is necessary to introduce tariffs to finance the investments in transmission capacity.[3]

To finance the electricity grid, suppliers pay a monetary charge (tariff) to the network owner when using the grid. The charge is linear and it depends on how much power the suppliers inject into the grid (*point of connection tariff*) or transmit through the grid (*transmission tariff*). The majority of European countries (ENTSO-E, 2013; ENTSO-E, 2016) have point of connection tariffs. With the point of connection tariffs scheme, suppliers pay a linear tariff for the electricity injected into the grid, i.e., the electricity sold in their own market and the electricity sold in the other market. In contrast, for transmission tariffs, electricity suppliers would only pay a linear tariff for the electricity sold to the other market.

Each supplier faces a perfectly inelastic demand in each market that is known with certainty when suppliers submit their offer prices. Each supplier submits a single price

[1] The term "transmission capacity constraint" is used throughout this article in the electrical engineering sense: a transmission line is constrained when the flow of power is equal to the capacity of the line, as determined by engineering standards.

[2]When the competition in the spot electricity market is imperfect the allocation of transmission rights plays a crucial role determining the equilibrium (Joskow and Tirole, 2000; Gilbert et al., 2004)

[3]For a complete study of the expansion of electricity grid and the tariff system in Europe review Energinet (2015), ENTSO-E (2014), ENTSO-E (2016), European Commission (2013), Nord Pool (2010), Svenska Kräfnat (2015).

offer for its entire capacity[4] in a discriminatory price auction such as those used in the UK wholesale electricity market. The assumption of price-inelastic demand can be justified by the fact that the vast majority of consumers purchase electricity under regulated tariffs that are independent of the prices set in the wholesale market, at least in the short run. The assumption that suppliers have perfect information concerning market demand is reasonable when applied to markets where offers are "short lived", such as in Spain, where there are 24 hourly day-ahead markets each day.

When *transmission tariffs* are *zero*, the supplier located in the high-demand market faces a high residual demand and it submits higher bids than the supplier located in the low-demand market (size effect). When the transmission tariffs are *positive* and high enough, the supplier located in the high-demand market faces lower costs and it submits lower bids than the supplier located in the low-demand market to extract the efficiency rents (cost effect); given that the majority of consumers are located in that market, consumers aggregate welfare could be larger than when *transmission tariffs* are *zero*.[5] In contrast, when suppliers are charged by the power that they inject into the grid (*point of connection* tariffs), given that they pay the same tariff independent of the market in which they are selling electricity, the strategic component of being located in the high-demand market disappears and the equilibrium is only determined by the size effect and the supplier located in the high-demand market submits higher bids than the supplier located in the low-demand market. Moreover, due to demand being inelastic, the tariff is passed through to consumers that are worse off than in the *zero* transmission tariffs scenario.[6] Therefore, in terms of consumers welfare maximization, *positive* transmission tariffs outperforms *zero* transmission tariffs that outperforms *point of connection tariffs.*

With *positive* transmission tariffs, an increase in transmission capacity is pro-competitive only when the transmission capacity is low. In that case, an increase in transmission capacity substantially increases the competition between suppliers that move from an isolated market scenario to a connected market scenario; simultaneously, and due to the low capacity of the line, the suppliers sell a small part of their production capacity into the other market and an increase in transmission capacity slightly increases suppliers' costs. Hence, an increase in the transmission capacity is pro-competitive. If the transmission capacity is high, an increase in transmission capacity slightly increases the competition between suppliers, but substantially increases their costs since they sell a large part of their production capacity into the other market; therefore, the suppliers raise their bids to cover the increase in costs, and an increase in transmission capacity is anti-competitive. If the transmission tariffs are *zero*, or with *point of connection tariffs*, an increase in transmission always increase competition between suppliers, and it doesn't modify their costs; therefore, and increase in transmission capacity is always pro-competitive.

[4] Fabra et al. (2006) show that the equilibrium outcome allocation does not change when firms submit single price offers for their entire capacity and when they submit a set of price-quantity offers.

[5] If the transmission tariffs are very high, the supplier located in the high-demand market submits lower bids than the supplier located in the low-demand market. However, due to the high tariffs, the suppliers have to submit high bids to cover the costs, the equilibrium price in both markets is high and consumers' aggregate welfare decreases. Therefore, positive transmission tariffs only increase consumers' aggregate welfare when the tariff is high, but not when it is very high.

[6] This is in line with the pass-through literature (Marion and Muehlegger, 2011; Fabra and Reguant, 2014).

My model contrasts with the previous models of price competition with production capacity constraints (Kreps and Scheinkman, 1983; Osborne and Pitchik, 1986; Deneckere and Kovenock, 1996; Fabra et al., 2006) where the results are exclusively driven by production capacity constraints. In the presence of transmission constraints, there are two relevant constraints that explain the results. If the production capacity is binding, the equilibrium is symmetric even when the realization of demands across markets is asymmetric. If the transmission capacity is binding, the equilibrium is asymmetric even when the suppliers are symmetric in production capacity and production costs. Therefore, this model provides a complete and novel analysis of the role played by the structural variables of the model (demand realization, production capacity and transmission capacity) determining equilibrium outcome allocations.[7]

In the presence of transmission constraints and *positive* transmission tariffs the literature of truncated Pareto distributions available nowadays cannot be applied to work out the equilibrium outcome allocations (Zaninetti et al., 2008; Aban, 2007). Therefore, this paper also expand the literature of truncated Pareto distributions.

This paper also contributes to the literature that analyzes electricity markets. Borenstein et al. (2000) characterize the equilibrium in an electricity network where suppliers compete in quantities as in a Cournot game. Holmberg and Philpott (2012) solve for symmetric supply function equilibria in electricity networks when demand is uncertain ex-ante, but they do not consider any transmission costs. Escobar and Jofré (2010) analyze the effect of transmission losses and transmission costs on equilibrium outcome allocations, but they neglect transmission constraints. Downward et al. (2014) found that the introduction of a tax on suppliers' profit sometimes increases consumer welfare. However, in their analysis, all suppliers produce in the same market and therefore, the results are not driven by any type of size effect similar to the one described in this paper. Hence, this paper is the first to characterize equilibrium outcomes in networks with both transmission constraints and transmission costs. The paper also shows that the interaction between transmission costs and transmission constraints is non-straightforward.

Hogan (1992) introduces the concept of a contract network that maintains short-run efficiency through an optimal spot-price calculation of transmission prices and provides the correct long-term signals to invest in capacity. Chao and Peck (1996) propose a market mechanism for electricity power transmission that consists of tradable transmission capacity rights and a trading rule that also induces short-run efficiency and long-term correct signals. Joskow and Tirole (2000) work out the equilibrium in an electricity market when financial and physical rights are introduced and they find that in a perfect competition scenario, as in Chao and Peck (1996), the financial and physical transmission rights generate the same equilibrium outcome allocations; in contrast, in an imperfect competition scenario, the financial transmission rights outperform the physical transmis-

[7]Fabra et al. (2006) show that in electricity auctions, the equilibrium depends on the type of auction implemented. In particular, they find that equilibrium outcome allocations differ substantially when uniform and discriminatory price auction are implemented. In Blazquez (2015), I work out the equilibrium in a zonal price electricity market when uniform and discriminatory price auctions are implemented. I find that when the demand is low, the discriminatory price auction outperforms the uniform price auction by maximizing consumers welfare and transmission efficiency. When the demand is high, the discriminatory price auction outperforms the uniform price auction by maximizing consumers welfare, but no rank between auctions can be established in terms of transmission efficiency.

sion rights by generating lower equilibrium prices and increasing efficiency in production. This paper depart from that literature in two directions. First, as in Joskow and Tirole (2000), I assume imperfect competition in the spot electricity market, but in contrast with their approach which assume that production capacity is installed in only one market, I assume that production capacity is installed in both markets. Second, I assume lumpy costs which requires the introduction of tariffs to finance the transmission investments.

The results of this paper could also be of relevance for the trade literature. For instance, Krugman (1980), Flam and Helpman (1987), Brezis et al. (1993) and Motta et al. (1997) explain differences in prices and profits in international trade models based on product differentiation or product cost advantages. By introducing transport costs and transport constraints, this paper finds related results, even if the product is homogeneous and suppliers have identical production technologies.

The article proceeds as follows. Section 2 describes the set up of model and the timing of the game. Section 3 characterizes the equilibrium in the presence of transmission capacity constraints and zero transmission tariffs. Section 4 characterizes the equilibrium when transmission tariffs are positive. Section 5 compares equilibrium outcomes and consumer welfare when transmission tariffs and point of connection tariffs are implemented. Section 6 concludes the paper. The analysis of point of connection tariffs and all proofs are found in the Appendix.

2 The model

Set up of the model. There exist two electricity markets, market North and market South, that are connected by a transmission line with capacity T. When suppliers transmit electricity through the grid from one market to the other, they face a symmetric[8] and linear[9] transmission tariff t.

There exist two duopolists with capacities k_n and k_s, where subscript n means that the supplier is located in market North and subscript s means that the supplier is located in market South. The suppliers' marginal costs of production are c_n and c_s for production levels less than the capacity, while production above the capacity is impossible (i.e., infinitely costly). Suppliers are symmetric in capacity $k_n = k_s = k > 0$ and symmetric in production costs $c_n = c_s = c = 0$.[10] The level of demand in any period, θ_n in market

[8]In order to reduce the transmission losses, transmission tariffs can include a locational and a seasonal component similar to those added to point of connection tariffs. The locational component of the tariff penalizes the injection of electricity into points of the grid that generate high flows of electricity. The seasonal/period-of-day component of the tariff penalizes the transmission of electricity when the losses are larger. Due to the locational and seasonal elements, suppliers face asymmetric linear tariffs. I characterize the equilibrium for symmetric transmission tariffs; however, the model can easily be modified to introduce this type of asymmetries. For a complete analysis of losses in Europe and a complete description of the algorithm implemented to work out power losses, consult the document "ENTSO-E ITC Transit Losses Data Report 2013". For a comparison of European tariff systems, check out the document "ENTSO-E Overview of transmission tariffs in Europe: Synthesis 2016."

[9]The transmission tariffs are linear in electricity markets. However, the model can be modified to assume convex costs. When the transmission costs are convex, the existence of the equilibrium is guaranteed by Dixon (1984).

[10]In this paper, I analyze the effect that transmission capacity constraints and tariffs to access the grid have on equilibrium outcome allocations. In order to focus on that effect, I assume that suppliers

North and θ_s in market South, is independent across markets[11] and independent of market price, i.e., perfectly inelastic. Moreover, $\theta_i \in [\underline{\theta}_i, \overline{\theta}_i] \subseteq [0, k+T]$, $i = n, s$.

The capacity of the transmission line can be lower than the installed capacity in each market $T \leq k$, i.e., the transmission line could be congested for some realization of demands (θ_s, θ_n). When $T > k$, the transmission line is not congested and the equilibrium is as in Fabra et al. (2006).

Timing of the game. Having observed the realization of demands $\theta \equiv (\theta_s, \theta_n)$, each supplier simultaneously and independently submits a bid specifying the minimum price at which it is willing to supply up to its capacity, $b_i \leq P$, $i = n, s$, where P denotes the "market reserve price", possibly determined by regulation. P can be interpreted as the price at which all consumers are indifferent between consuming and not consuming, or a price cap imposed by the regulatory authorities (von der Fehr and Harbord, 1993). Moreover, as I show in the next section, the equilibrium in this model could be in mixed strategies. In that case, when the demand is inelastic, the introduction of a price cap guarantees the existence of the upper bound of the support in a mixed strategies equilibrium (Baye et al., 1992; Fabra et al., 2006).

Let $b \equiv (b_s, b_n)$ denote a bid profile. On basis of this profile, the auctioneer calls suppliers into operation. If suppliers submit different bids, the capacity of the lower-bidding supplier is dispatched first. If the capacity of the lower-bidding supplier is not sufficient to satisfy total demand, the higher-bidding supplier's capacity, supplier s, is then dispatched to serve residual demand. If the two suppliers submit equal bids, then supplier i is ranked first with probability ρ_i, where $\rho_n + \rho_s = 1$, $\rho_i = 1$ if $\theta_i > \theta_j$, and $\rho_i = \frac{1}{2}$ if $\theta_i = \theta_j$, $i = n, s$, $i \neq j$.[12]

The output allocated to supplier $i, i = n, s$, denoted by $q_i(\theta, b)$, is given by

$$q_i(b; \theta, T) = \begin{cases} \min\{\theta_i + \theta_j, \theta_i + T, k_i\} & \text{if } b_i < b_j \\ \rho_i \min\{\theta_i + \theta_j, \theta_i + T, k_i\} + & \\ \qquad [1 - \rho_i] \max\{0, \theta_i - T, \theta_i + \theta_j - k_j\} & \text{if } b_i = b_j \\ \max\{0, \theta_i - T, \theta_i + \theta_j - k_j\} & \text{if } b_i > b_j \end{cases} \tag{1}$$

are symmetric in capacity and production costs. The generalization of the equilibrium to introduce asymmetries in capacity and costs complicates the theoretical analysis and the interpretation of the results and it is outside the scope of this paper.

[11] In the majority of electricity markets, demand in one market is higher than demand in the other market. Moreover, there exists the possibility of some type of correlation between demands across markets. In this paper, I assume uniform distribution and independence of demand. However, the model can be modified to introduce different distributions of demand and a correlation between demands across markets.

[12] The implemented tie breaking rule is such that if the bids of both suppliers are equal and demand in market i is larger than demand in market j, the auctioneer first dispatches the supplier located in market i. This tie breaking rule minimizes the transmission costs and given that in this model, those costs are the unique ones, it also minimizes the total costs. This tie breaking rule is in line with those used in the literature where the tie breaking rule minimizes the total costs. Moreover, as I show in the next section, the equilibrium in this model could be in mixed strategies. In that case, the tie breaking rule ensures the existence of a mixed strategies equilibrium in the Bertrand game with transmission constraints and transmission costs (Dasgupta and Maskin, 1986).

Figure 1: Supplier n's output function ($k_n = k_s = 60, T = 40$)

total_residual-eps-converted-to.pdf

The output function plays an important role in determining the equilibrium and thus, it is explained in detail. Below, I describe the construction of supplier n's output function; the output for supplier s is symmetric.

The total demand that can be satisfied by supplier n when it submits the lower bid ($b_n < b_s$) is defined by $\min\{\theta_n + \theta_s, \theta_n + T, k\}$. The realization of (θ_s, θ_n) determines three different areas (left-hand panel, figure **??**).

$$\min\{\theta_n + \theta_s, \theta_n + T, k\} = \begin{cases} \theta_s + \theta_n & \text{if } \theta_s < T \text{ and } \theta_n + \theta_s < k \\ \theta_n + T & \text{if } \theta_s > T \text{ and } \theta_n + T < k \\ k & \text{if } \theta_n + T > k \text{ and } \theta_n + \theta_s > k \end{cases}$$

When demand in both markets is low and the transmission line is not congested, supplier n can satisfy total demand ($\theta_s + \theta_n$). If the demand in market South is larger than the transmission capacity $\theta_s > T$, supplier n cannot satisfy the demand in market South, even when it has enough production capacity for this; therefore, the total demand that supplier n can satisfy is ($\theta_n + T$). Finally, if the demand is large enough, the total demand that supplier n can satisfy is its own production capacity (k).

The residual demand that supplier n satisfies when it submits the higher bid ($b_n > b_s$) is defined by $\max\{0, \theta_n - T, \theta_s + \theta_n - k\}$. The realization of (θ_s, θ_n) determines three different cases (right-hand panel, figure **??**).

$$\max\{0, \theta_n - T, \theta_s + \theta_n - k\} = \begin{cases} 0 & \text{if } \theta_n < T \text{ and } \theta_s + \theta_n < k \\ \theta_n - T & \text{if } \theta_n > T \text{ and } \theta_s < k - T \\ \theta_s + \theta_n - k & \text{if } \theta_s + \theta_n > k \text{ and } \theta_s > k - T \end{cases}$$

When demand in both markets is low and the transmission line is not congested, supplier s satisfies total demand and therefore, the residual demand that remains for supplier n is zero. The total demand that supplier s can satisfy diminishes due to the transmission constraint. As soon as the demand in market North is larger than the transmission capacity ($\theta_n > T$), it cannot be satisfied by supplier s and thus, some residual demand ($\theta_n - T$) remains for supplier n. When total demand is large enough, supplier s cannot satisfy total demand and some residual demand ($\theta_s + \theta_n - k$) remains for supplier n.

When both suppliers submit the same bid ($b_n = b_s$). Supplier n's demand change discontinuously around the diagonal (central panel, figure **??**). When the demand in market North is larger than the demand in market South, supplier n satisfies the total demand. When the demand in market North is lower than the demand in market South, supplier n satisfies the residual demand. Finally, when the demand in both markets is the same, supplier n satisfies the demand in its own market.

Finally, the payments are worked out by the auctioneer. When the auctioneer runs

Figure 2: Supplier n's profit function ($k_n = k_s = 60, T = 40, t > 0$)

profit_total_residual-eps-converted-to.pdf

a discriminatory price auction, the price received by a supplier for any positive quantity dispatched by the auctioneer is equal to its own offer price, whenever a bid is wholly or partly accepted. Hence, for a given realization of demands $\theta \equiv (\theta_s, \theta_n)$ and a bid profile $b \equiv (b_s, b_n)$, supplier i's profits, $i = n, s$, can be expressed as

$$\pi_i^d(b; \theta, T, t) = \begin{cases} b_i \min\{\theta_i + \theta_j, \theta_i + T, k\} - & \\ \quad t \max\{0, \min\{\theta_j, T, k - \theta_i\}\} & \text{if } b_i \leq b_j \text{ and } \theta_i > \theta_j \\ b_i \theta_i & \text{if } b_i = b_j \text{ and } \theta_i = \theta_j \\ b_i \max\{0, \theta_i - T, \theta_i + \theta_j - k\} - & \\ \quad t \max\{0, \theta_j - k\} & \text{otherwise} \end{cases}$$

Given the relevance of the payoff function determining the equilibrium, I explain it in detail. As for the outcome function, I focus on supplier n's payoff function; the one for supplier s is symmetric. If $b_n \leq b_s$ and $\theta_n > \theta_s$, supplier n is dispatched first and satisfies total demand. Supplier n's payoff function is $\pi_n^d(b; \theta, T) = b_n \min\{\theta_n + \theta_s, \theta_n + T, k\}$. In addition to this expression, due to the transmission tariff, supplier n is charged a transmission tariff t for the power sold in market South. [13] The transmission costs have four different possible values: $t\theta_s$ when the realization of demand in market North is low and the transmission line is not congested; tT when the realization of demand in market North is low and the transmission line is congested; when the realization of demand in market North is high but lower than its production capacity, the transmission costs are $t(k - \theta_n)$; finally, when demand in market North is larger than the production capacity k, supplier n cannot sell any electricity in market South and the transmission costs are zero. Hence, after adding the transmission costs, supplier n's payoff is equal to $\pi_n^d(b; \theta, T, t) = b_n \min\{\theta_n + \theta_s, \theta_n + T, k\} - t \ max\{0, min\{\theta_s, T, k - \theta_n\}\}$ (left-hand panel, figure **??**).

If $b_n = b_s$ and $\theta_n = \theta_s$, each supplier satisfies the demand in its own market and no electricity flows through the grid. Supplier n's payoff function is $\pi_n^d(b; \theta, T) = b_n \theta_n$.

In the rest of the cases, supplier n is dispatched last and satisfies the residual demand. Supplier n's payoff function is $\pi_n^d(b; \theta, T, t) = b_n \min\{\theta_s + \theta_n, \theta_n + T, k\}$. In addition to this expression, due to the transmission tariff, supplier n is charged a transmission tariff t for the residual demand satisfied in market South. Therefore, after adding the transmission costs, supplier n's payoff is equal to $\pi_n^d(b; \theta, T) = b_n \max\{0, \theta_n - T, \theta_s + \theta_n - k\} - t \max\{0, \theta_s - k\}$ (right-hand panel, figure **??**).

[13] Given that the production costs are nil, the marginal costs in this model come from the transmission tariff. In this paper, I use the standard definition of marginal costs as the cost of producing one more unit of electricity.

Figure 3: *Zero* transmission tariffs. Equilibrium areas ($k_n = k_s = k = 60, T = 40, c = 0$)

area_I-eps-converted-to.pdf

3 Effect of transmission capacity constraints

I characterize the equilibrium in the presence of transmission capacity constraints and *zero* transmission tariffs and then I analyze the effect of an increase in transmission capacity.

Lemma 1. In the presence of transmission constraints and *zero* transmission tariffs, when the demand is low (area A), the equilibrium is in pure strategies, when the demand is intermediate (areas $A1, B1$) or high (area $B2$), a pure strategies equilibrium does not exist (figure **??**).[14]

Proof. When the demand is low (area A), both suppliers have enough capacity to satisfy total demand in both markets and the transmission line is not congested. Therefore, they compete fiercely to be dispatched first in the auction. Hence, the equilibrium is the typical Bertrand equilibrium where both suppliers submit bids equal to their marginal cost.

When the demand is intermediate (areas $A1$, $B1$) or high (area $B2$), at least one of the suppliers faces a positive residual demand. Therefore, a pure strategies equilibrium does not exist. First, an equilibrium such that $b_i = b_j = c$ does not exist because at least one supplier has the incentive to increase its bid and satisfy the residual demand. Second, an equilibrium such that $b_i = b_j > c$ does not exist because at least one supplier has the incentive to undercut the other to be dispatched first. Finally, an equilibrium such that $b_j > b_i > c$ does not exist because supplier i has the incentive to shade the bid submitted by supplier j. $\square$

A pure strategies equilibrium does not exist when the demand is intermediate or high. However, the model satisfies the properties established by Dasgupta and Maskin (1986) which guarantee that a mixed strategies equilibrium exists.

Lemma 2. In the presence of transmission constraints and *zero* transmission tariffs, in a mixed strategies equilibrium, no supplier submits a bid lower than bid ($\underline{b}_i$) such that $\underline{b}_i \min\{\theta_i + \theta_j, \theta_i + T, k\} = P \max\{0, \theta_i - T, \theta_i + \theta_j - k\}$. Moreover, the support of the mixed strategies equilibrium for both suppliers is $S = \left[\max\left\{\underline{b}_i, \underline{b}_j\right\}, P\right]$.

Proof. Given that the demand is inelastic, the supplier's profit is maximized when it sets the reservation price. Therefore, the reservation price is the upper-bound of the support.

Each supplier can guarantee for itself the payoff $P \max\{0, \theta_i - T, \theta_i + \theta_j - k\}$, since each supplier can always submit the highest bid and satisfy the residual demand. Therefore, in a mixed strategy equilibrium, no supplier submits a bid that generates a payoff

[14] Equation **??** defines three different areas (figure **??**). The intersection of the areas in the left and the right-hand panels in that figure generates the equilibrium areas in figure **??**.

Figure 4: *Zero* transmission tariffs. Mixed strategy equilibrium

CDF_I-eps-converted-to.pdf

equilibrium lower than $P \max\{0, \theta_i - T, \theta_i + \theta_j - k\}$. Hence, no supplier submits a bid lower than $\underline{b}_i$, where $\underline{b}_i$ solves $\underline{b}_i \min\{\theta_i + \theta_j, \theta_i + T, k\} = P \max\{0, \theta_i - T, \theta_i + \theta_j - k\}$.

No supplier can rationalize submitting a bid lower than $\underline{b}_i, i = n, s$. In the case when $\underline{b}_i = \underline{b}_j$, the support is symmetric. In the case when $\underline{b}_i < \underline{b}_j$, supplier i knows that supplier j never submits a bid lower than $\underline{b}_j$. Therefore, in a mixed strategy equilibrium, supplier i never submits a bid b_i such that $b_i \in (\underline{b}_i, \underline{b}_j)$, because supplier i can increase its expected payoff choosing a bid b_i such that $b_i \in [\underline{b}_j, P]$. Hence, the equilibrium strategy support for both suppliers is $S = [\max\{\underline{b}_i, \underline{b}_j\}, P]$ $\square$

Using Lemmas one and two, I characterize the equilibrium.

Proposition 1. In the presence of transmission constraints and *zero* transmission tariffs, the characterization of the equilibrium falls into one of the next two categories.

i Low demand (area A). The equilibrium strategies pair is in pure strategies.

ii Intermediate demand (areas $A1$, $B1$) and high demand (area $B2$). The equilibrium strategies pair is in mixed strategies.

When the demand is low (area A), suppliers compete fiercely to be dispatched first in the auction and the equilibrium is the typical Bertrand equilibrium in which both suppliers submit bids equal to their marginal cost. Suppliers' marginal costs are nil since the production and transmission tariffs are nil. Therefore, equilibrium prices and profits are also nil. Moreover, given that suppliers' bids are equal, no electricity flows through the grid.

As soon as the transmission line becomes congested (intermediate demand, areas $A1$ and $B1$), the supplier located in the high-demand market (supplier n) faces a high residual demand and the supplier located in the low-demand market (supplier s) cannot sell its entire production capacity. Equation **??** in annex 1 defines the asymmetric mixed strategies equilibrium and the shape of the cumulative distribution functions that are represented in the left-hand panel in figure **??**. As can be observed, the slope of supplier s's cumulative distribution function in the lower bound of the support is steeper than the slope of supplier n's cumulative distribution function. Given that the derivative of the cumulative distribution function is the probability distribution function, supplier s submits lower bids with higher probability than supplier n. In the upper bound of the support, supplier s's cumulative distribution function is continuous, i.e., supplier s assigns zero probability to the maximum bid allowed by the auctioneer; in contrast, supplier n's cumulative distribution function is discontinuous, i.e., supplier n assigns a positive probability to the maximum bid allowed by the auctioneer. Therefore, when the demand is intermediate, the supplier located in the high-demand market randomizes submitting higher bids with a higher probability, i.e., its cumulative distribution function stochastically dominates the

Figure 5: *Zero* transmission tariffs. Increase in transmission capacity $\triangle T$, $\theta_s = 5, \theta_n = 55, k = 60, c = 0, t = 0, P = 7$

increase_T_t-eps-converted-to.pdf

cumulative distribution function of the supplier located in the low-demand market.

Finally, when the demand is high (area $B2$), the transmission capacity is not binding, but the production capacity is. Therefore, both suppliers face the same residual and total demand and the equilibrium is symmetric, i.e., both suppliers randomize using the same cumulative distribution function (right-hand panel, figure **??**). When the transmission capacity is high enough $T \geq k$, the transmission line is not congested and the equilibrium when the demand is intermediate (areas $A1$ and $B1$) coincides with the equilibrium in area $B2$, i.e., the equilibrium is as in Fabra et al. (2006). It is important to notice that when the equilibrium is in mixed strategies (symmetric or asymmetric), the probability that both suppliers submit the same bid is zero.

These results are in contrast to the models of price competition with production capacity constraints (Kreps and Scheinkman, 1983; Osborne and Pitchik, 1986; Deneckere and Kovenock, 1996; Fabra et al., 2006) where the results are exclusively driven by production capacity constraints. In the presence of transmission constraints, there are two relevant constraints that explain the results. If the production capacity is binding, the equilibrium is symmetric even when the realization of demands is asymmetric. If the transmission capacity is binding, the equilibrium is asymmetric even when the suppliers are symmetric in production capacity. Therefore, this model provides a complete and novel analysis of the role played by the structural variables of the model (demand realization, production capacity and transmission capacity) determining equilibrium outcome allocations.

To conclude this section, I analyze the effect of an increase in transmission capacity on the main variables of the model.

Proposition 2. In the presence of transmission constraints and *zero* transmission tariffs, an increase in transmission capacity ($\triangle T$) reduces the lower bound of support $\underline{b}$ and reduces the expected bids for both suppliers (an increase in transmission capacity is pro-competitive). Moreover, an increase in transmission capacity reduces the profit of the supplier located in the high-demand market. However, an increase in transmission capacity modifies the profit of the supplier located in the low-demand market in a non monotonic pattern (table **??** and figure **??**).

An increase in transmission capacity reduces the residual demand and, according to lemma two, the lower bound of the support decreases (left-hand panel, figure **??**; column two, table **??**; equation **??** in annex 1). A decrease in the lower bound of the support implies that both suppliers randomize submitting lower bids and therefore, the expected bid decreases for both suppliers (right-hand panel, figure **??**; columns five and six, table **??**; equations **??** and **??**, annex 1). Finally, an increase in transmission capacity reduces the expected bid and the residual demand of the supplier located in the high-demand market

Table 1: *Zero* transmission tariffs. Increase in transmission capacity $\triangle T$ ($\theta_s = 5, \theta_n = 55, k = 60, c = 0, t = 0, P = 7$). Main variables.

T	$\underline{b}$	π_n	π_s	$E_n(b)$	$E_s(b)$
0	7	385.07	35	7	7
5	5.835	350.1	58.35	6.8963	6.3795
15	4.668	280.08	93.36	6.5587	5.6770
25	3.501	210.06	105.03	5.9261	4.8530
35	2.335	140.1	93.4	4.8981	3.8464
45	1.168	70.08	58.4	3.2589	2.5102
55	0	0	0	0	0

as does its expected profit (central panel, figure **??**; column three, table **??**; equation **??**, annex 1). In contrast, an increase in transmission capacity reduces the expected bid and increases the total demand of the supplier located in the low-demand market. When the transmission capacity is low, the increase in demand dominates the decrease in the expected bid and its expected profit increases. However, when the transmission capacity is large enough, the decrease in bids dominates and its expected profit decreases (central panel, figure **??**; column four, table **??**; equation **??**, annex 1).

4 Effect of transmission capacity constraints and transmission tariffs

I characterize the equilibrium in the presence of transmission capacity constraints and *positive* transmission tariffs and then I analyze the effect of an increase in transmission capacity.

Lemma 3. In the presence of transmission constraints and *positive* transmission tariffs, in the low demand area (area A), the equilibrium is in pure strategies. In the intermediate demand area (area $A1$) and when the transmission tariffs are high, the equilibrium is in pure strategies; otherwise, a pure strategies equilibrium does not exist. Moreover, due to the presence of transmission tariffs, the pure strategies equilibrium is asymmetric. In the intermediate demand areas (areas $B1a, B1b$) or in the high-demand areas (area $B2a, B2b$), a pure strategies equilibrium does not exist (figure **??**).

Proof. In the low demand area (area A), both suppliers have enough capacity to satisfy total demand and the transmission line is not congested. Therefore, the competition to be dispatched first is fierce. Moreover, the supplier located in the high-demand market (supplier i) faces lower total marginal costs. Hence, the equilibrium is the typical Bertrand equilibrium with asymmetries in costs where the supplier located in the high-demand market extracts the efficiency rents. The *pure strategies equilibrium* is $b_i = b_j = \dfrac{t\theta_i}{\theta_i + \theta_j}$.

The *equilibrium profits* are:

Figure 6: *Positive* transmission tariffs. Equilibrium areas ($k_n = k_s = k = 60, T = 40, c = 0, t > 0$)

area_II-eps-converted-to.pdf

$$\overline{\pi}_j = (\theta_i + \theta_j)\frac{t\theta_i}{\theta_i + \theta_j} - t\theta_i = 0; \;\; \overline{\pi}_i = (\theta_i + \theta_j)\frac{t\theta_i}{\theta_i + \theta_j} - t\theta_j = t(\theta_i - \theta_j) > 0$$

The *equilibrium price* is $\dfrac{t\theta_i}{\theta_i + \theta_j}$

Electricity flows from the high-demand market to the low-demand market, i.e., the electricity losses are minimized.

When the demand belongs to area $A1$, the transmission constraint binds for the supplier located in the low-demand market (supplier j); therefore, only the supplier located in the high-demand market can satisfy total demand. The supplier located in the high-demand market prefers to submit a low bid and extract the efficiency rent instead of submitting a high bid and satisfying the residual demand if $(\theta_i+\theta_j)\dfrac{tT}{\theta_j + T} - t\theta_j \geq P(\theta_i - T)$. In such a case, the pure strategies equilibrium is $b_i = b_j = \dfrac{tT}{\theta_j + T}$, i.e., supplier i prefers to extract the efficiency rents when the transmission tariffs are high enough.

The *equilibrium profits* are:

$$\overline{\pi}_j = (\theta_j + T)\frac{tT}{\theta_j + T} - tT = 0; \;\; \overline{\pi}_i = (\theta_i + \theta_j)\frac{tT}{\theta_j + T} - t\theta_j > 0$$

The *equilibrium price* is $\dfrac{tT}{\theta_i + T}$

The electricity flows from the high-demand market to the low-demand market, i.e., the electricity losses are minimized.

In the rest of the cases, a pure strategies equilibrium does not exist and the proof proceeds as in lemma one. $\square$

A pure strategy equilibrium does not exist in the intermediate or high-demand areas. However, the implemented tie breaking rule guarantees that the model satisfies the properties established by Dasgupta and Maskin (1986) which ensure that a mixed strategy equilibrium exists.

Lemma 4. In a mixed strategies equilibrium, in the presence of transmission tariffs and *positive* transmission costs, no supplier submits a bid lower than bid ($\underline{b}_i$) such that

$$\underline{b}_i \min\{\theta_i+\theta_j, \theta_i+T, k\} - t\max\{0, min\{\theta_j, T, k-\theta_i\}\} =$$
$$P\max\{0, \theta_i - T, \theta_i+\theta_j-k\} - t\max\{0, \theta_j - k\}.$$

Moreover, the support for the mixed strategies equilibrium for both suppliers is $S = \left[\max\left\{\underline{b}_i, \underline{b}_j\right\}, P\right]$.

Proof. The proof proceeds as in lemma two. □

Using lemmas three and four, I characterize the equilibrium.

Proposition 3. In the presence of transmission constraints and *positive* transmission tariffs, the characterization of the equilibrium falls into one of the next three categories.

i Low demand (area A). The equilibrium strategies pair is in pure strategies.

ii Intermediate demand (area $A1$). When the transmission tariffs are high, the equilibrium strategies pair is in pure strategies. In contrast, when the transmission tariffs are low, the equilibrium strategies pair is in mixed strategies.

iii Intermediate demand (areas $B1a$, $B1b$) and high demand (areas $B2a$, $B2b$). The equilibrium strategies pair is in mixed strategies.

In the low demand area (area A), suppliers compete fiercely to be dispatched first in the auction and the equilibrium is the typical Bertrand equilibrium with asymmetries in costs where the supplier located in the high-demand market extracts the efficiency rents. Therefore, the profits of the supplier located in the high-demand market are positive; the profits of the supplier located in the low-demand market are nil; the equilibrium price in both markets is the same; and the electricity flows from the low-demand market to the high demand market.

In the intermediate demand area (area $A1$), the transmission capacity binds for the supplier located in the low-demand market; therefore, only the supplier located in the high-demand market can satisfy total demand and the equilibrium crucially depends on the value of the transmission tariffs (low, intermediate or high).[15] If the transmission tariffs are high enough, the supplier located in the high-demand market prefers to satisfy the total demand by submitting a low bid and extracting the efficiency rents. If the transmission tariffs are intermediate, the supplier located in the high-demand market faces lower marginal costs and thus, it has incentives to submit low bids to extract the efficiency rents. Simultaneously, it faces a high residual demand and thus, it has incentives to submit high bids. The two economic forces work in opposite directions and non cumulative distribution function stochastically dominates the other (left-hand panel, figure **??**; equation **??**, annex 2).[16] This is in contrast to the *zero* transmission tariffs case where the cumulative distribution function of the supplier located in the low-demand market stochastically dominates the one of the supplier located in the high-demand market (left-hand panel, figure **??**). Finally, if the transmission tariffs are low, the supplier located in

[15]Those tariffs are defined in equations **??** and **??** and are summarized in figure **??** in annex 2.

[16]It is important to remind that in the presence of transmission constraints and when the transmission tariffs are intermediate or high, the equilibrium is in mixed strategies (lemma 3).

Figure 7: *Positive* transmission tariffs. Mixed strategy equilibrium

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the high-demand market prefers to submit a high bid and satisfy the residual demand; therefore, the cumulative distribution function of the supplier located in the high-demand market stochastically dominates the one of the supplier located in the low-demand market.

When the demand is intermediate, but larger than in Area $A1$ (areas $B1a$ and $B1b$), the same logic applies and the stochastic or non-stochastic dominance crucially depends on transmission tariffs. Moreover, since both suppliers face a positive residual demand, a pure strategies equilibrium does not exist even when the transmission tariffs are high.

In the high demand areas (areas $B2a$ and $B2b$), the transmission capacity is not binding, but the production capacity is. Therefore, both suppliers face the same demand. However, due to the transmission tariffs, the supplier located in the high-demand market faces lower total marginal costs and it submits lower bids to extract the efficiency rents. Hence, the cumulative distribution function of the supplier located in the low-demand market stochastically dominates the cumulative distribution function of the supplier located in the high-demand market (right-hand panel, figure **??**). This is in contrast to the *zero* transmission tariffs case where both suppliers randomize using the same cumulative distribution function (right-hand panel, figure **??**).

Finally, when the demand belongs in the diagonal, both suppliers face the same demand and marginal costs. In the low demand area (area A), the equilibrium is a symmetric pure strategies equilibrium; otherwise (areas $B1a$ and $B2b$), the equilibrium is a symmetric mixed strategies equilibrium.

The results presented in proposition 3 not only complement the models of price competition with capacity constraints introducing transmission constraints and transmission tariffs, but also contribute to the literature of truncated Pareto distributions. In particular, in annex 2, I show that the standard approach followed by the truncated Pareto distribution literature to characterize the equilibrium and to stablish the stochastic dominance relation between cumulative distribution functions cannot be applied when the suppliers face *positive* transmission tariffs. In annex 2, I compare the characterization of the equilibrium when the transmission tariffs are *zero* and when they are *positive*, and I explain the reasons that made necessary an extension of the theory of truncated Pareto distributions.

To conclude this section, I analyze the effect of an increase in transmission capacity on equilibrium outcome allocations.

Proposition 4. In the presence of transmission constraints and transmission tariffs, an increase in transmission capacity ($\triangle T$) has different effects depending on the transmission capacity (figure **??**, table **??**).

i. When the transmission capacity is low, an increase in transmission capacity de-

Figure 8: *Positive* transmission tariffs. Increase in transmission capacity $\triangle T$, ($\theta_s = 5, \theta_n = 55, k = 60, c = 0, t = 1.5, P = 7$)

increase_T_tpositive-eps-converted-to.pdf

creases the lower bound of the support of the supplier located in the high-demand market and increases the lower bound of the support of the supplier located in the low-demand market; reduces the expected bids of both suppliers (an increase in transmission capacity is pro-competitive); reduces the profit of the supplier located in the high-demand market and modifies the profit of the supplier located in the low-demand market in a non-monotonic pattern.

ii. When the transmission capacity is low, an increase in transmission capacity decreases the lower bound of the support of the supplier located in the high-demand market and increases the lower bound of the support of the supplier located in the low-demand market; increases the expected bids of both suppliers (an increase in transmission capacity is anti-competitive); increases the expected profit of the supplier located in the high-demand market and does not modify the expected profit of the supplier located in the low-demand market.

When the transmission capacity is low ($T \leq 44$ for the numerical examples in table **??** and figure **??**), an increase in transmission capacity substantially increases the competition between suppliers that move from an isolated market scenario to a connected market scenario; simultaneously, and due to the low capacity of the line, an increase in transmission capacity slightly increases the marginal costs since now, the suppliers export more electricity to the other market and that electricity is charged by the transmission tariff. Hence, the competitive effect dominates, and an increase in transmission capacity has the same effects on equilibrium outcome variables as when the transmission tariff are nil (proposition 2): The lower bound of the support decreases and so do suppliers' expected bids (right-hand panel, figure **??**; columns five and six, table **??**); reduces the expected profit of the supplier located in the high-demand market and modifies the profit of the supplier located in the low-demand market in a non-monotonic pattern (central panel, figure **??**; columns three and four, table **??**).

If the transmission capacity is high enough ($T > 44$), an increase in transmission capacity slightly increases the competition between suppliers, but substantially increases their marginal cost. Therefore, the lower bound of the support increases since the suppliers have to submit higher bids to compensate the increase in costs (left-hand panel, figure **??**). An increase in the lower bound of the support entailed that both suppliers randomize submitting higher bids and therefore, the expected bids increase for both suppliers (right-hand panel, figure **??**; columns five and six, table **??**). Finally, an increase in transmission capacity increases the expected profit of the supplier located in the high-demand market since the increase in costs allows it to extract the efficiency rents; in contrast, the expected profit of the supplier located in the low-demand market does not change because the increase in profits derived from an increase in the expected bid is compensated by the increase in marginal costs (central panel, figure **??**; columns three and four, table **??**).

Table 2: *Positive* transmission tariffs. Increase in transmission capacity $\triangle T$ ($\theta_s = 5, \theta_n = 55, k = 60, t = 1.5, P = 7$) . Main variables.

T	$\underline{b}$	$\overline{\pi}_n$	$\overline{\pi}_s$	$E_n(b)$	$E_s(b)$
0	7	385.07	35	7	7
5	5.959	350.05	52.09	6.9079	6.4483
15	4.793	280.09	73.36	6.5206	5.7490
25	3.626	210.07	71.28	5.7253	4.9301
35	2.459	140.05	45.86	4.2942	3.9307
45	1.351	73.575	0	1.3569	2.7304
55	1.376	75	0	1.3821	3.5075

5 Model comparison and consumer welfare

In this section, I compare equilibrium outcome allocations and their effects on consumer welfare in the presence of transmission constraints and when the transmission tariffs are *nil* and *positive*. I also compare these results with the equilibrium when suppliers face a *point of connection tariff*.[17]

In the presence of a congested transmission lines and zero transmission tariffs (Model I), the supplier located in the high-demand market faces a high residual demand, while the supplier located in the low-demand market cannot sell its entire production capacity. Therefore, the supplier located in the high-demand market has incentives to submit higher bids than the supplier located in the low-demand market. Given that the majority of consumers are located in the high-demand market, the aggregate payment that consumers face to acquire electricity is large (column eight, table **??**).

When positive transmission tariffs are implemented (Model II), the supplier located in the low-demand market faces high marginal costs and thus, its expected bid is high. In contrast, the supplier located in the high-demand market faces low marginal costs and for high enough transmission tariffs, it finds more profitable to extract the efficiency rents undercutting the supplier located in the low-demand market.[18] These changes in equilibrium prices induce a drastic decrease on consumers electricity expenses (column eight, table **??**). Moreover, the introduction of positive transmission tariffs makes the electricity to flow from the high to the low-demand market reducing the flow of electricity and thus, increasing transmission efficiency.

Finally, if suppliers face a point of connection tariff (Model III), they pay the same transmission tariff for the electricity sold in their own market and the electricity sold in the other market. Therefore, the competitive advantage derived of being located in the high-demand market disappears. Moreover, given that electricity demand is very inelastic, an increase in suppliers costs is passed through to consumers that face an increase

[17] As I explain in the introduction, with point of connection tariffs, the suppliers are charged by the total power injected into the grid. In annex 4, I have characterized the equilibrium when suppliers face a point of connection tariff.

[18] As I indicate in the previous section, this result is only valid when the tariff is intermediate, but not when it is high. Intermediate and high tariffs are defined in equations **??** and **??** and are summarized in figure **??** in annex 2.

Table 3: Effect of the transmission constraint and different tariffs to access the grid on the equilibrium outcome ($\theta_s = 5$, $\theta_n = 55$, $k = 60$, $c = 0$, $P = 7$)

	T	$\underline{b}$	$\overline{\pi}_n$	$\overline{\pi}_s$	$\overline{\pi} = \overline{\pi}_n + \overline{\pi}_s$	$E_n(b)$	$E_s(b)$	$\theta_n E_n(b) + \theta_s E_s(b)$
Model I	40	1.75	105	184	289	4.2	3.2	247
Model II	40	1.87	105	24	129	3.1	3.3	187
Model III	40	2.87	82.5	62	144.5	4.8	4	284

Model I: zero transmission costs. Model II: transmission tariff ($t = 1.5$). Model III: point of connection tariff ($t = 1.5$)

in equilibrium prices in both markets. Hence, consumers welfare decrease (column eight, table ??).

The comparison between the three models suggests that the introduction of transmission tariffs could increase aggregate consumers welfare. In contrast, point of connection tariffs always reduce aggregate consumers welfare. However, it is important to emphasize that these results are only valid when the tariffs are symmetric.

The symmetric transmission tariffs scenario is relevant as a benchmark model. In that case, the regulator does not interfere in the market and equilibrium market allocations are only determined by the structural parameters of the model. Moreover, in that scenario, the knowledge of the market and the information required by the regulator to implement those tariffs is minimal. However, as described in the model section, in the majority of countries, point of connection tariffs present some type of asymmetry (seasonal or locational components) to reduce the flow of electricity. Therefore, it is important to characterize the equilibrium when asymmetric transmission and point of connection tariffs are implemented. Moreover, given that symmetric transmission tariffs outperform point of connection tariffs by maximizing consumers welfare and transmission efficiency, it is a noteworthy economic policy issue to determine whether introducing the "correct" asymmetry in point of connection tariffs could lead to the same outcome as when symmetric transmission tariffs are implemented, i.e., under particular tariff designs, asymmetric point of connection tariffs and symmetric transmission tariffs could be equivalent.

6 Conclusion

Electricity markets are moving through integration processes around the world. In such a context, there exists an intense debate to analyze the effect of transmission constraints and tariffs to access the grid on suppliers' strategies. The contribution of this paper is to characterize the outcome of an electricity market auction in the presence of transmission constraints and different tariffs to access the grid.

In the presence of transmission constraints and *zero* transmission tariffs, the supplier located in the high-demand market faces a high residual demand and it submits higher bids than the supplier located in the low-demand market. When the transmission tariffs are *positive*, the supplier located in the high-demand market faces lower marginal costs. If the transmission tariffs are high enough, the supplier located in the high-demand market submits lower bids than the supplier located in the low-demand market to extract the efficiency rents; given that the majority of consumers are located in that market, consumers aggregate welfare could increase. Moreover, the electricity flows from the high-demand market to the low-demand market and transmission losses are minimized; therefore, the introduction of positive transmission tariffs increases transmission efficiency.

When suppliers are charged according to the power that they inject into the grid (*point of connection* tariffs), given that they pay the same tariff independent of the market where they are selling electricity, the strategic component of being located in the high-demand market disappears. Moreover, due to the fact that demand is inelastic, the tariff is passed through to consumers that are worse off than in the *zero* transmission tariffs scenario and thereby also worse off than in the *positive* transmission tariffs scenario.

The consequences of an increase in transmission capacity differ considerably due to the transmission tariffs. If the transmission tariffs are *zero*, an increase in transmission capacity is pro-competitive. In contrast, if the transmission tariffs are *positive*, an increase in transmission capacity could be anti-competitive. When *point of connection tariffs* are implemented, an increase in transmission capacity is always pro-competitive.

The results that I present in this paper complement the models of price competition with capacity constraints (Kreps and Scheinkman, 1983; Osborne and Pitchik, 1986; Deneckere and Kovenock, 1996; and Fabra et al., 2006). Moreover, I also provide novel results that expand the literature of truncated Pareto distributions.

In the next future, I would like to extend the model to introduce more suppliers in both markets to study other relevant problems as mergers, investment in production and transmission capacity, entry decisions, etcetera.[19]

The symmetric transmission model presented in the paper is useful to compare equilibrium outcome allocations when different tariffs are implemented. However, the model can easily be modified to analyze models that include some type of seasonal and geographical component in the tariffs. In the next future, I would like to explore the design of novel tariffs that include a seasonal and a geographical component.

[19]It is important to emphasize that the introduction of more suppliers into the model complicates the characterization of the equilibrium (Baye et al., 1992). Moreover, the presence of transmission constraints introduce asymmetries in the residual demand and therefore, it is not possible to follow Janssen et al. (2003) to work out a symmetric equilibrium.

Annex 1. Effect of transmission capacity constraints

Proposition 1. Characterization of the equilibrium in the presence of transmission constraints and *zero* transmission tariffs.

When demand is low (area A, figure **??**): $b_n = b_s = c = 0$, the *equilibrium profit* is zero for both suppliers. No electricity flows through the grid.

When demand is intermediate (areas $A1$ and $B1$, figure **??**) or high (area $B2$, figure **??**), a pure strategies equilibrium does not exist, as it is proved in lemma one; however, the model presented in section two satisfies the properties established by Dasgupta and Maskin (1986) which guarantee that a mixed strategies equilibrium exists. In particular, the discontinuities of $\pi_i, \forall i, j$ are restricted to the strategies such that $b_i = b_j$. Furthermore, it is simple to confirm that by reducing its price from a position where $b_i = b_j$, a supplier discontinuously increases its profit. Therefore, $\pi_i(b_i, b_j)$ is everywhere left lower semi-continuous in b_i and hence, weakly lower semi-continuous. Obviously, $\pi_i(b_i, b_j)$ is bounded. Finally, $\pi_i(b_i, b_j) + \pi_j(b_i, b_j)$ is continuous because discontinuous shifts in the clientele from one supplier to another only occur where both suppliers derive the same profit per customer. Therefore, theorem five in Dasgupta and Maskin (1986) applies and hence, a mixed strategies equilibrium exists.

The existence of the equilibrium is guaranteed by Dasgupta and Maskin (1986). However, they did not provide an algorithm to work out the equilibrium. Nevertheless, using the approach proposed by Karlin (1959), Shapley (1957), Shilony (1977), Varian (1980), Kreps and Scheinkman (1984), Osborne and Pitchik (1986), Deneckere and Kovenock (1996) and Fabra et al. (2006), the equilibrium characterization is guaranteed by construction. I use the approach proposed by this branch of the literature to work out the mixed strategies equilibrium. In particular, I work out the *lower bound of the support*, the *cumulative distribution function*, the *probability distribution function*, the *expected equilibrium price* and the *expected profit.*

Lower Bound of the Support. The lower bound of the support is defined according to lemma two.

Cumulative Distribution Function (CDF). To work out the CDF, I follow Varian (1980) and Kreps and Scheinkman (1984).

For further reference,
$L_i(\theta, k, T) = min\,\{\theta_i + \theta_j, \theta_i + T, k\}$,
$H_i(\theta, k, T) = max\,\{0, \theta_i - T, \theta_i + \theta_j - k\}$ and
$C_i(\theta, k, T) = \dfrac{min\,\{\theta_i + \theta_j, \theta_i + T, k\}}{min\,\{\theta_i + \theta_j, \theta_i + T, k\} - max\,\{0, \theta_i - T, \theta_i + \theta_j - k\}}$.

In the first step, the payoff function for any supplier is:

$$\begin{aligned} \pi_i(b) &= b\,[F_j(b)H_i(\theta, k, T) + (1 - F_j(b))L_i(\theta, k, T)] = \\ &= -b\ F_j(b)\,[L_i(\theta, k, T) - H_i(\theta, k, T)] + \\ &\quad b\ L_i(\theta, k, T) \end{aligned} \tag{2}$$

In the second step, $\pi_i(b) = \overline{\pi}_i \forall b \in S_i, i = n, s$, where S_i is the support of the mixed strategies. Then,

$$\begin{aligned} \overline{\pi}_i &= -b\ F_j(b)\left[L_i(\theta,k,T) - H_i(\theta,k,T)\right] + \\ &\quad b\ L_i(\theta,k,T) \Rightarrow \\ F_j(b) &= \frac{b\ L_i(\theta,k,T) - \overline{\pi}_i}{b\left[L_i(\theta,k,T) - H_i(\theta,k,T)\right]} \end{aligned} \tag{3}$$

The third step, at $\underline{b}$, $F_i(\underline{b}) = 0 \forall i = n, s$. Then,

$$\overline{\pi}_i = \underline{b} L_i(\theta,k,T) \tag{4}$$

In the fourth step, plugging **??** into **??**, I obtain the mixed strategies for both suppliers.

$$\begin{aligned} F_j(b) &= \frac{b\ L_i(\theta,k,T) - \underline{b}\ L_i(\theta,k,T)}{b\left[L_i(\theta,k,T) - H_i(\theta,k,T)\right]} = \\ &= \frac{L_i(\theta,k,T)}{L_i(\theta,k,T) - H_i(\theta,k,T)} \frac{b - \underline{b}}{b} = C_i(\theta,k,T)\frac{b - \underline{b}}{b}\ \ \forall i = n, s \end{aligned} \tag{5}$$

For further reference,
$L_j(b) = b\ min\left\{\theta_i + \theta_j, \theta_i + T, k\right\}$,
$H_j(b) = b\ max\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\}$

It is easy to verify that equation $F_j(b)$ is indeed a cumulative distribution function with the following properties. First, in the third step, I have established that $F_j(\underline{b}) = 0$. Second, using partial derivatives it can be show that the cumulative distribution function is increasing $\frac{\partial F_j(b)}{\partial b} = C_i \frac{\underline{b}}{b^2} > 0$. Third, the cumulative distribution function is concave, $\frac{\partial F_j(b)}{\partial b^2} = -C_i \frac{\underline{b}}{b^3} < 0$. Fourth, $F_j(b) \leq 1 \forall b \in S_j$. Fifth, if $C_i(\theta,k,T) > C_j(\theta,k,T)$, then $F_j(b) > F_i(b) \forall b \in S$, i.e., $F_i(b)$ stochastic dominates $F_j(b)$. If $C_i(\theta,k,T) = C_j(\theta,k,T)$, then $F_j(b) = F_i(b) \forall b \in S$, i.e., $F_j(b)$ and $F_i(b)$ are symmetric. Finally, $F_j(b)$ is continuous in the support because $L_i(b) - L_i(\underline{b})$ and $L_i(b) - H_i(b)$ are continuous functions in the support.

Probability Distribution Function.

$$\begin{aligned} f_j(b) &= \frac{\partial F_j(b)}{\partial b} \\ &= \frac{L_i(\theta,k,T)\underline{b}\left(L_i(\theta,k,T) - H_i(\theta,k,T)\right)}{b^2\left(L_i(\theta,k,T) - H_i(\theta,k,T)\right)^2} \\ &= \frac{L_i(\theta,k,T)}{L_i(\theta,k,T) - H_i(\theta,k,T)} \frac{\underline{b}}{b^2} = C_i(\theta,k,T)\frac{\underline{b}}{b^2}\ \ \forall i = n, s \end{aligned} \tag{6}$$

Expected Equilibrium Bid.

$$
\begin{aligned}
E_j(b) &= \int_{\underline{b}}^{P} b f_j(b) \partial b \\
&= \int_{\underline{b}}^{P} \frac{b\ L_i(\theta,k,T)\underline{b}}{b^2\left(L_i(\theta,k,T)-H_i(\theta,k,T)\right)} \partial b + P\left(1-F_j(P)\right) \\
&= \frac{L_i(\theta,k,T)\underline{b}}{L_i(\theta,k,T)-H_i(\theta,k,T)} \left[ln(b)\right]_{\underline{b}}^{P} + P\left(1-F_j(P)\right) \\
&= \underline{b}\ C_i(\theta,k,T)\left[ln(b)\right]_{\underline{b}}^{P} + P\left(1-F_j(P)\right)\ \ \forall i=n,s \qquad (7)
\end{aligned}
$$

where $(1 - F_j(P))$ in equation **??** is the probability assigned by firm j to the maximum price allowed by the auctioneer.[20]

Expected Profit. The expected profit is defined by equation **??** and it is equal to $\overline{\pi}_j = \underline{b}\ min\left\{\theta_i + \theta_j, \theta_j + T, k\right\} = \underline{b}L_i(\theta, k, T)$.

Lemmas 1 and 2, and equations **??**, **??**, **??** and **??** fully characterize the equilibrium. To facilitate the understanding of the equilibrium, in area $A1$, I work out the lower bound of the support, the cumulative distribution function, the probability distribution function, the expected bid, and the expected profit. I also establish the stochastic dominance rank between suppliers' cumulative distribution functions. Moreover, the characterization of the equilibrium in area $A1$ will be useful to compare the equilibrium when $t = 0$ and when $t > 0$.

First, I work out the *lower bound of the support* on the border between areas $B1$ and $B2$, $\theta_s = k - T$. On the border, $\underline{b}_n$ solves

$$\underline{b}_n min\left\{\theta_n + \theta_s, \theta_n + T, k\right\} = P\ max\left\{0, \theta_n - T, \theta_s + \theta_n - k\right\},$$

therefore $\underline{b}_n = \dfrac{P(\theta_n - T)}{k}$ and $\underline{b}_s$ solves

$$\underline{b}_s min\left\{\theta_n + \theta_s, \theta_s + T, k\right\} = P\ max\left\{0, \theta_s - T, \theta_s + \theta_n - k\right\},$$

therefore $\underline{b}_s = \dfrac{P(\theta_n + \theta_s - k)}{\theta_s + T}$. Plugging the value of θ_s on the border between these areas into $\underline{b}_s$ formula, I obtain $\underline{b}_s = \dfrac{P(\theta_n + k - T - k)}{k - T + T} = \dfrac{P(\theta_n - T)}{k} = \underline{b}_n$. Therefore, on the border between areas $A1$ and $B1$, $\underline{b}_s = \underline{b}_n = \dfrac{P(\theta_n - T)}{k}$.

Taking partial derivatives $\dfrac{\partial \underline{b}_n}{\partial \theta_s} = \dfrac{-P(\theta_n - T)}{(\theta_n + \theta_s)^2} < 0$ and $\dfrac{\partial \underline{b}_s}{\partial \theta_s} = \dfrac{P(k + T - \theta_n)}{(\theta_s + T)^2} > 0$. Therefore, $\underline{b}_n > \underline{b}_s$. Hence, the support of the mixed strategies equilibrium is defined by

[20] When the transmission line is congested, the mixed strategy equilibrium is asymmetric. In such an equilibrium, the cumulative distribution function for the supplier located in the low-demand market is continuous in the upper bound of the support. In contrast, the cumulative distribution function of the supplier located in the high-demand market has a mass point in the upper-bound of the support, which means that the supplier located in the high-demand market submits the maximum bid allowed by the auctioneer with a positive probability $(1 - F_j(P))$. Hence, in order to work out the expected value, in addition to the integral, it is necessary to add the term $P\left(1 - F_j(P)\right)$. Figure **??** illustrates these characteristics.

$S = [max\{\underline{b}_n, \underline{b}_s\}, P] = [\underline{b}_n, P]$. In particular, $S = \left[\frac{P(\theta_n - T)}{(\theta_n + \theta_s)}, P\right]$.

Second, I work out the *cumulative distribution function.*

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \frac{\theta_n + \theta_s}{\theta_s + T}\frac{b - \underline{b}}{b} = C_n(\theta, k, T)\frac{b - \underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \frac{\theta_s + T}{\theta_s + T}\frac{b - \underline{b}}{b} = C_s(\theta, k, T)\frac{b - \underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases} \tag{8}$$

Given that $\underline{b}_n > \underline{b}_s$, it is easy to show that $F_s(P)$ is continuous in the upper bound of the support, and that $F_n(P)$ is discontinuous in the upper bound of the support:

$$\begin{aligned} F_s(P) &= \frac{\theta_n + \theta_s}{\theta_s + T}\frac{P - \frac{P(\theta_n - T)}{\theta_n + \theta_s}}{P} = C_n(\theta, k, T)\frac{P - \frac{P(\theta_n - T)}{\theta_n + \theta_s}}{P} = 1 \\ F_n(P) &= \frac{\theta_s + T}{\theta_s + T}\frac{P - \frac{P(\theta_n - T)}{\theta_n + \theta_s}}{P} = C_s(\theta_n, k, T)\frac{P - \frac{P(\theta_n - T)}{\theta_n + \theta_s}}{P} < 1 \end{aligned}$$

Third, the *probability distribution function* is equal to:

$$\begin{aligned} f_s(b) &= \frac{\partial F_s(b)}{\partial b} = \frac{\theta_n + \theta_s}{\theta_s + T}\frac{\underline{b}}{b^2} = C_n(\theta, k, T)\frac{\underline{b}}{b^2} \\ f_n(b) &= \frac{\partial F_n(b)}{\partial b} = \frac{\theta_s + T}{\theta_s + T}\frac{\underline{b}}{b^2} = C_s(\theta, k, T)\frac{\underline{b}}{b^2} \end{aligned} \tag{9}$$

Fourth, the *expected bid* is determined by:

$$\begin{aligned} E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} \frac{\theta_n + \theta_s}{\theta_s + T}\frac{\underline{b}}{b}\partial b = \frac{\theta_n + \theta_s}{\theta_s + T}\underline{b}\,[ln(b)]_{\underline{b}}^{P} \\ &= C_s(\theta, k, T)\underline{b}\,[ln(b)]_{\underline{b}}^{P} \\ E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_n)\partial b = \int_{\underline{b}}^{P} \frac{\underline{b}}{b^2}\partial b = \frac{\theta_s + T}{\theta_s + T}\underline{b}\,[ln(b)]_{\underline{b}}^{P} + (1 - F_n(P))\,P \\ &= C_n(\theta, k, T)\underline{b}\,[ln(b)]_{\underline{b}}^{P} + (1 - F_n(P))\,P \end{aligned} \tag{10}$$

Given that $F_n(b)$ is discontinuous in the upper bound of the support, to work out supplier n's expected bid is necessary to multiply the maximum bid allowed by the auctioneer by the probability that supplier n assigns to that bid $(1 - F_n(P))\,P$.

Fifth, the expected profit is defined by equation **??** and is equal to $\overline{\pi}_n = \underline{b}(\theta_s + \theta_n)$ and $\overline{\pi}_s = \underline{b}(\theta_s + T)$.

Finally, I show that $F_n(b)$ stochastically dominates $F_s(b)$. Following Kreps and Scheinkman (1984) and the theory of truncated Pareto distributions (Aban et al., 2007; Zaninetti et al., 2008), it is enough to show that $C_s(\theta,k,T) < C_n(\theta,k,T)$. Given than $\theta_n + \theta_s \geq T + \theta_s$, it is straightforward to check that $\frac{\theta_s + T}{\theta_s + T}\frac{b-\underline{b}}{b} \leq \frac{\theta_n+\theta_s}{\theta_s+T}\frac{b-\underline{b}}{b} \forall b \in (\underline{b}, P)$. Therefore, $F_n(b) \leq F_s(b) \forall b \in [\underline{b}, P]$, i.e., $F_n(b)$ stochastically dominates $F_s(b)$.

Following a similar approach and applying equations **??**, **??**, **??** and **??**, it is straight forward to characterize the equilibrium in the areas $B1$ and $B2$.

Proposition 2. effect of an increase in transmission capacity in the presence of transmission constraints and *zero* transmission tariffs.

*Area A*1.

$$\frac{\partial \underline{b}}{\partial T} = \frac{-P}{(\theta_s+\theta_n)} < 0 \tag{11}$$

$$\frac{\partial F_n(P)}{\partial T} = \frac{1}{(\theta_s+\theta_n)} > 0 \tag{12}$$

$$\begin{aligned}\frac{\partial E_n(b)}{\partial T} &= \frac{\partial \underline{b}}{\partial T}\left[ln\left(\frac{P}{\underline{b}}\right)\right] + \underline{b}\left[\frac{\underline{b}}{P}\frac{-\frac{\partial \underline{b}}{\partial T}P}{\underline{b}^2}\right] - \frac{\partial F_n(P)}{\partial T} \\ &= \frac{\partial \underline{b}}{\partial T}\left[ln\left(\frac{P}{\underline{b}}\right) - 1\right] - \frac{\partial F_n(P)}{\partial T} < 0 \Leftrightarrow ln\left(\frac{P}{\underline{b}}\right) > 1\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\partial E_s(b)}{\partial T} &= \frac{\partial \underline{b}}{\partial T}\frac{\theta_s+\theta_n}{\theta_s+T}\left[ln\left(\frac{P}{\underline{b}}\right)\right] - \underline{b}\frac{\theta_s+\theta_n}{(\theta_s+T)^2}\left[ln\left(\frac{P}{\underline{b}}\right)\right] + \underline{b}\frac{\theta_s+\theta_n}{\theta_s+T}\left[\frac{\underline{b}}{P}\frac{-\frac{\partial \underline{b}}{\partial T}P}{\underline{b}^2}\right] \\ &= \frac{\partial \underline{b}}{\partial T}\frac{\theta_s+\theta_n}{\theta_s+T}\left[ln\left(\frac{P}{\underline{b}}\right) - 1\right] - \underline{b}\frac{\theta_s+\theta_n}{(\theta_s+T)^2}\left[ln\left(\frac{P}{\underline{b}}\right)\right] < 0 \Leftrightarrow ln\left(\frac{P}{\underline{b}}\right) > 1\end{aligned} \tag{14}$$

$$\frac{\partial \pi_n}{\partial T} = -P < 0 \tag{15}$$

$$\begin{aligned}\frac{\partial \pi_s}{\partial T} &= \frac{-P}{(\theta_s+\theta_n)}(\theta_s+T) + \frac{P(\theta_n - T)}{(\theta_s+\theta_n)} = \frac{P(\theta_n - 2T - \theta_s)}{(\theta_s+\theta_n)} > 0 \\ &\Leftrightarrow \theta_n > 2T + \theta_s\end{aligned} \tag{16}$$

*Area B*1. The proof follows the same approach than in area $A1$.

Annex 2. Effect of transmission capacity constraints and transmission tariffs

Proposition 3. Characterization of the equilibrium in the presence of transmission constraints and *positive* transmission tariffs.

In areas A and $A1$ (figure **??**), the equilibrium is in pure strategies, and the equilibrium strategies, price, and profits are worked out in lemma 3. In the rest of the areas, a pure strategies equilibrium doesn't exist. However, the tie breaking rule implemented in the model determines that in case of a tie, the supplier located in the high demand market is dispatched first, i.e., the transmission costs are minimized. Therefore, the model satisfies the properties established by Dasgupta and Maskin (1986) which guaranteed that a mixed strategies equilibrium exists.

As in proposition one, I work out the general formulas of the *lower bound of the support*, the *cumulative distribution function*, the *probability distribution function*, the *expected equilibrium price* and the *expected profit*.

Lower Bound of the Support. The lower bound of the support is defined according to lemma four.

Cumulative Distribution Function.

For further reference:

$$\begin{aligned} H_i(\theta,k,T) &= max\left\{0,\theta_i - T, \theta_j + \theta_i - k\right\} \\ Ht_i(\theta,k,T) &= max\left\{0,\theta_j - k\right\} \\ L_i(\theta,k,T) &= min\left\{\theta_i + \theta_j, \theta_i + T, k\right\} \\ Lt_i(\theta,k,T) &= max\left\{0, min\left\{\theta_i, T, k-\theta_i\right\}\right\} \end{aligned}$$

In the first step, the payoff function for any firm is:

$$\begin{aligned} \pi_i(b) &= F_j(b)\left[b\left(H_i(\theta,k,T)\right) - t\left(Ht_i(\theta,k,T)\right)\right] + \\ &\quad (1-F_j(b))\left[b\left(L_i(\theta,k,T)\right) - t\left(Lt_i(\theta,k,T)\right)\right] = \\ &= -F_j(b)\left[b\left(L_i(\theta,k,T)\right) - t\left(Lt_i(\theta,k,T)\right) - b\left(H_i(\theta,k,T)\right) + t\left(Ht_i(\theta,k,T)\right)\right] \\ &\quad b\left(L_i(\theta,k,T)\right) - t\left(Lt_i(\theta,k,T)\right) \end{aligned} \tag{17}$$

In the second step, $\pi_i(b) = \overline{\pi}_i \forall b \in S_i, i = n, s$, where S_i is the support of the mixed strategy. Then,

$$\begin{aligned} \overline{\pi}_i &= -F_j(b)\left[b\left(L_i(\theta,k,T)\right) - t\left(Lt_i(\theta,k,T)\right) - b\left(H_i(\theta,k,T)\right) + t\left(Ht_i(\theta,k,T)\right)\right] \\ &\quad b\left(L_i(\theta,k,T)\right) - t\left(Lt_i(\theta,k,T)\right) \Rightarrow \\ F_j(b) &= \frac{b\left(L_i(\theta,k,T)\right) - t\left(Lt_i(\theta,k,T)\right) - \overline{\pi}_i}{b\left[L_i(\theta,k,T) - H_i(\theta,k,T)\right] - t\left[Lt_i(\theta,k,T) - Ht_i(\theta,k,T)\right]} \end{aligned} \tag{18}$$

In the third step, at $\underline{b}$, $F_i(\underline{b}) = 0 \forall i = n, s$. Then,

$$\overline{\pi}_i = b\left(L_i(\theta,k,T)\right) - t\left(Lt_i(\theta,k,T)\right) \tag{19}$$

Fourth step, plugging **??** into **??**, I obtain the mixed strategies for both firms.

$$F_j(b) = \frac{(b-\underline{b})L_i(\theta,k,T)}{b\left[L_i(\theta,k,T)-H_i(\theta,k,T)\right]-t\left[Lt_i(\theta,k,T)-Ht_i(\theta,k,T)\right]} = \quad \forall i=n,s \tag{20}$$

As can be observed in equation **??**, when $t=0$, the cumulative distribution function can be written as a constant $C_i(\theta,k,T)$ multiplied by $\frac{b-\underline{b}}{b}$, which simplified the computation of the probability distribution function, the expected value, and it facilitates the analysis of the stochastic dominance relation between cumulative distribution functions. In contrast, when $t>0$, the denominator in equation **??** cannot be factorized, and the cumulative distribution function cannot be simplified. Therefore, when $t>0$, the approach followed by the truncated Pareto distributions literature to work out the cumulative distribution function, the probability distribution function, the expected equilibrium price and the expected profit cannot be applied and I have developed new methods to fully characterize the equilibrium.

Probability Distribution Function.

$$\begin{aligned} f_j(b) &= \frac{\partial F_j(b)}{\partial b} \\ &= \frac{L_i(\cdot)\left[\underline{b}\left[L_i(\theta,k,T)-H_i(\theta,k,T)\right]-t\left[Lt_i(\theta,k,T)-Ht_i(\theta,k,T)\right]\right]}{\left[b\left[L_i(\theta,k,T)-H_i(\theta,k,T)\right]-t\left[Lt_i(\theta,k,T)-Ht_i(\theta,k,T)\right]\right]^2} \\ & \quad \forall i=n,s \end{aligned} \tag{21}$$

For further reference:

$$\begin{aligned} n(\cdot) &= L_i(\cdot)\left[\underline{b}\left[L_i(\theta,k,T)-H_i(\theta,k,T)\right]-t\left[Lt_i(\theta,k,T)-Ht_i(\theta,k,T)\right]\right] \\ d_1(\cdot) &= \left[L_i(\theta,k,T)-H_i(\theta,k,T)\right] \\ d_2(\cdot) &= \left[Lt_i(\theta,k,T)-Ht_i(\theta,k,T)\right] \end{aligned}$$

As can be observed in equation **??**, when $t=0$, the cumulative distribution function can be written as a constant $C_i(\theta,k,T)$ multiplied by $\frac{\underline{b}}{b^2}$. In contrast, when $t>0$, equation **??** cannot be simplified.

Expected Equilibrium Bid.

$$\begin{aligned} E_j(b) &= \int_{\underline{b}}^{P} b f_j(b)\partial b \\ &= \int_{\underline{b}}^{P} \frac{b\left(n(\cdot)\right)}{\left[b\left(d_1(\cdot)\right)-t\left(d_2(\cdot)\right)\right]^2}\partial b + P(1-F_j(P)) \;\; \forall i=n,s \end{aligned}$$

I work out the expected equilibrium bid by substitution of variables. In particular:

$$\begin{aligned} U &= \left[b\left(d_1(\cdot)\right) - t\left(d_2(\cdot)\right)\right] \Rightarrow b = \frac{U + t\left(d_2(\cdot)\right)}{d_1(\cdot)} \\ \frac{\partial U}{\partial b} &= d_1 \Rightarrow \partial b = \frac{\partial U}{\partial d_1} \end{aligned}$$

Therefore:

$$\begin{aligned} E_j(b) &= \int_{\underline{b}}^{P} \frac{\left(\frac{U + t\left(d_2(\cdot)\right)}{d_1(\cdot)}\right) n(\cdot)}{U^2} \frac{\partial U}{d_1(\cdot)} + P(1 - F_j(P)) \\ &= \frac{n(\cdot)}{d_1(\cdot)} \left[\int_{\underline{b}}^{P} \frac{U \partial U}{U^2} + \int_{\underline{b}}^{P} \frac{t\left(d_2(\cdot)\right) \partial U}{U^2}\right] + P(1 - F_j(P)) \\ &= \frac{n(\cdot)}{d_1(\cdot)^2} \left[ln(U) - \frac{t\left(d_2(\cdot)\right)}{U}\right]_{\underline{b}}^{P} + P(1 - F_j(P)) \end{aligned}$$

Substituting again:

$$\begin{aligned} E_j(b) = & \frac{n(\cdot)}{d_1(\cdot)^2} \\ & \left[ln\left(\frac{P\left(d_1(\cdot)\right) - t\left(d_2(\cdot)\right)}{\underline{b}\left(d_1(\cdot)\right) - t\left(d_2(\cdot)\right)}\right) - \frac{t\left(d_2(\cdot)\right)}{P\left(d_1(\cdot)\right) - t\left(d_2(\cdot)\right)} + \frac{t\left(d_2(\cdot)\right)}{\underline{b}\left(d_1(\cdot)\right) - t\left(d_2(\cdot)\right)}\right] \\ & + P(1 - F_j(P)) \end{aligned} \tag{22}$$

As can be observed in equation **??**, when $t = 0$, the expected equilibrium bid can be worked out easily, and can be written as a constant $C_i(\theta, k, T)$ multiplied by $\underline{b}\left[ln(b)\right]_{\underline{b}}^{P}$. In contrast, when $t > 0$, to work out the expected equilibrium bid is necessary to integrate by substitution of variables.

Lemmas 3 and 4, and equations **??**, **??**, **??** and **??** fully characterize the equilibrium. To facilitate the understanding of the equilibrium and to establish comparisons with the equilibrium when $t = 0$, in area $A1$, I work out the lower bound of the support, the cumulative distribution function, the probability distribution function, the expected bid, and the expected profit. I also establish the stochastic dominance rank between suppliers' cumulative distribution functions.

First, the *lower bound of the support* is:

$$\begin{aligned} \underline{b}_n \theta_n + \underline{b}_n \theta_s - t\theta_s = P(\theta_n - T) \Rightarrow \underline{b}_n &= \frac{P(\theta_n - T) + t\theta_s}{\theta_n + \theta_s} \\ \underline{b}_s \theta_s + \underline{b}_s T - tT = 0 \Rightarrow \underline{b}_s &= \frac{tT}{\theta_s + T} \end{aligned} \tag{23}$$

Second, I work out the *cumulative distribution function.*

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})(\theta_n+\theta_s)}{b\left[(\theta_s+\theta_n)-(\theta_n-T)\right]-t\ min\left\{\theta_s, k-\theta_n\right\}} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})(\theta_s+T)}{b(\theta_s+T)-tT} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases} \tag{24}$$

Third, the *probability distribution function* is equal to:

$$\begin{aligned} f_s(b) = \frac{\partial F_s(b)}{\partial b} &= \frac{(\theta_n+\theta_s)(\underline{b}(\theta_s+T)-t\theta_s)}{(b(\theta_s+T)-t\theta_s)^2} \\ f_n(b) = \frac{\partial F_n(b)}{\partial b} &= \frac{(\theta_s+T)(\underline{b}(\theta_s+T)-tT)}{(b(\theta_s+T)-tT)^2} \end{aligned} \tag{25}$$

Fourth, the *expected bid* is determined by:

$$\begin{aligned} E_s(b) &= \int_{\underline{b}}^{P} bf_s(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{(\theta_n+\theta_s)(\underline{b}(\theta_s+T)-t\theta_s)}{(b(\theta_s+T)-t\theta_s)^2} + (1-F_s(P))P \\ &= \frac{(\theta_n+\theta_s)(\underline{b}(\theta_s+T)-t\theta_s)}{(\theta_s+T)^2} \\ &\quad \left[ln\left(\frac{P(\theta_s+T)-t\theta_s}{\underline{b}(\theta_s+T)-t\theta_s}\right) - \frac{t\theta_s}{P(\theta_s+T)-t\theta_s} + \frac{t\theta_s}{\underline{b}(\theta_s+T)-t\theta_s}\right] \\ &\quad +(1-F_s(P))P \\ E_n(b) &= \int_{\underline{b}}^{P} bf_n(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{(\theta_s+T)(\underline{b}(\theta_s+T)-tT)}{(b(\theta_s+T)-tT)^2} + (1-F_n(P))P \\ &= \frac{(\underline{b}(\theta_s+T)-tT)}{(\theta_s+T)} \\ &\quad \left[ln\left(\frac{P(\theta_s+T)-tT}{\underline{b}(\theta_s+T)-tT}\right) - \frac{tT}{P(\theta_s+T)-tT} + \frac{tT}{\underline{b}(\theta_s+T)-tT}\right] \\ &\quad +(1-F_n(P))P \end{aligned} \tag{26}$$

I have solved equation **??** by substituting the variables:

$$\begin{aligned} U &= b(\theta_s+T)-t\theta_s \Rightarrow b = \frac{U+t\theta_s}{\theta_s+T} \\ \frac{\partial U}{\partial b} &= \theta_s+T \Rightarrow \partial b = \frac{\partial U}{\theta_s+T} \\ \text{and} & \\ U &= b(\theta_s+T)-tT \Rightarrow b = \frac{U+tT}{\theta_s+T} \\ \frac{\partial U}{\partial b} &= \theta_s+T \Rightarrow \partial b = \frac{\partial U}{\theta_s+T} \end{aligned}$$

Fifth, the *expected profit* is defined by equation ?? and is equal to $\overline{\pi}_n = \underline{b}(\theta_s + \theta_n) - t\theta_s$ and $\overline{\pi}_s = \underline{b}(\theta_s + T) - tT$.

When $t = 0$, the cumulative distribution function can be written as $C_i(\theta, k, T)$ multiplied by $\frac{\underline{b}}{b^2}$. Therefore, following the theory of truncated Pareto distributions, the stochastic dominance rank between $F_n(b)$ and $F_s(b)$ can be established by comparing $C_n(\theta, k, T)$ with $C_s(\theta, k, T)$. In contrast, when $t > 0$, to establish the stochastic dominance rank, it is necessary to follow a different approach. First, I work out the transmission tariff that equalizes $\underline{b}_s$ and $\underline{b}_n$, and the transmission tariff that equalizes $\left.\frac{\partial F_s(b)}{\partial b}\right|_{b=\underline{b}}$ and $\left.\frac{\partial F_n(b)}{\partial b}\right|_{b=\underline{b}}$.

I work out the transmission tariff that equalizes $\underline{b}_s$ and $\underline{b}_n$.

$$\begin{aligned} \hat{t} \mid \underline{b}_n &= \frac{P(\theta_n - T) + \hat{t}\theta_s}{\theta_n + \theta_s} = \frac{\hat{t}T}{\theta_s + T} = \underline{b}_s \Leftrightarrow \\ \hat{t} &= \frac{P(\theta_n - T)(\theta_s + T)}{T(\theta_n + \theta_s) - \theta_s(\theta_s + T)} \end{aligned} \tag{27}$$

Moreover, if $\underline{b}_n \geq \underline{b}_s$, then $F_s(b)$ is continuous in the upper bound of the support, and $F_n(b)$ is discontinuous in the upper bound of the support. If $\underline{b}_n < \underline{b}_s$, then $F_s(b)$ is discontinuous in the upper bound of the support, and $F_n(b)$ is continuous in the upper bound of the support.

$$\begin{aligned} \text{If } \underline{b}_n \geq \underline{b}_s \Rightarrow F_s(P) &= 1 \\ F_n(P) &= \frac{(P(\theta_s + T) - t\theta_s)(\theta_s + T)}{(P(\theta_s + T) - tT)(\theta_s + \theta_n)} \\ \text{If } \underline{b}_n < \underline{b}_s \Rightarrow F_s(P) &= \frac{(P(\theta_s + T) - tT)(\theta_s + \theta_n)}{(P(\theta_s + T) - t\theta_s)(\theta_s + T)} \\ F_n(P) &= 1 \end{aligned} \tag{28}$$

I work out the slope of the cumulative distribution function at the lower bound of the support, and I work out the transmission tariff that equalizes $\left.\frac{\partial F_s(b)}{\partial b}\right|_{b=\underline{b}}$ and $\left.\frac{\partial F_n(b)}{\partial b}\right|_{b=\underline{b}}$. To do that, first, I have to work out the slope of the cumulative distribution function,

$$\begin{aligned} \frac{\partial F_s(b)}{\partial b} &= \frac{(\theta_s + \theta_n)(\underline{b}(\theta_s + T) - t\theta_s)}{(b(\theta_s + \theta_n) - t\theta_s - b(\theta_n - T))^2} > 0 \\ \frac{\partial F_n(b)}{\partial b} &= \frac{(\theta_s + T)(\underline{b}(\theta_s + T) - tT)}{(b(\theta_s + T) - tT)^2} > 0. \end{aligned}$$

Therefore, the slope of the cumulative distribution function at the lower bound of the support is defined by:

$$\begin{aligned} \left.\frac{\partial F_s(b)}{\partial b}\right|_{b=\underline{b}} &= \frac{(\theta_s + \theta_n)}{(\underline{b}(\theta_s + \theta_n) - t\theta_s - \underline{b}(\theta_n - T))} \\ \left.\frac{\partial F_n(b)}{\partial b}\right|_{b=\underline{b}} &= \frac{(\theta_s + T)}{(\underline{b}(\theta_s + T) - tT)}. \end{aligned}$$

Figure 9: Relation between transmission tariffs and stochastic dominance

$F_n(b) < F_s(b)$ | $F_n(b) \lesseqgtr F_s(b)$ | $F_n(b) > F_s(b)$

$\underline{b}_n > \underline{b}_s$ | $\underline{b}_n < \underline{b}_s$

$t = 0$ | $\tilde{t} = \hat{t}\frac{\theta_n - T}{\theta_n + \theta_s}$ | $\hat{t} = \frac{P(\theta_n - T)(\theta_s + T)}{T(\theta_n + \theta_s) - \theta_s(\theta_s + T)}$

$\partial F_n(\underline{b})/\partial b < F_s(\underline{b})/\partial b$
$F_n(P) < F_s(P) = 1$

$\partial F_n(\underline{b})/\partial b = F_s(\underline{b})/\partial b$
$F_n(P) < F_s(P) = 1$

$\partial F_n(\underline{b})/\partial b > F_s(\underline{b})/\partial b$
$F_n(P) = F_s(P) = 1$

Hence, the transmission tariff that equalizes $\left.\frac{\partial F_s(b)}{\partial b}\right|_{b=\underline{b}}$ and $\left.\frac{\partial F_n(b)}{\partial b}\right|_{b=\underline{b}}$ is defined by:

$$\tilde{t} \mid \left.\frac{\partial F_n(b)}{\partial b}\right|_{b=\underline{b}} = \frac{(\theta_s + T)}{\underline{b}(\theta_s + T - tT)} = \frac{(\theta_s + \theta_n)}{(\underline{b}(\theta_s + \theta_n) - t\theta_s - \underline{b}(\theta_n - T))} = \left.\frac{\partial F_s(b)}{\partial b}\right|_{b=\underline{b}} \Leftrightarrow$$
$$\tilde{t} = \frac{\underline{b}(\theta_n - T)(\theta_s + T)}{T(\theta_n + \theta_s) - \theta_s(\theta_s + T)}. \tag{29}$$

Given that $\underline{b} < P$, it is straightforward to check that $\tilde{t} < \hat{t}$. Moreover, to work out a close form solution of $\tilde{t}$, it is enough to plug the value of $\underline{b}$ computed in **??** into **??**. For the particular values of demand (θ_s, θ_n) that belong to area $A1$, the formula is defined by $\tilde{t} = \hat{t}\frac{(\theta_n - T)}{(\theta_s + \theta_n)}$.

As can be observed in figure **??**, equations **??** and **??** define two thresholds. When $t \in [0, \tilde{t}]$, the cumulative distribution function of the supplier located in the high demand market $(F_n(b))$ stochastic dominates the one of the supplier located in the low demand market $(F_s(b))$. First, $F_n(b)$'s slope at the lower-bound of the support is lower than $F_s(b)$'s slope at the lower-bound of the support. Second, $F_n(P) < F_s(P)$. Therefore, given that the cumulative distribution function is continuous, increasing and concave, $F_n(b) < F_s(b) \forall b \in [\underline{b}, P]$, i.e., $F_n(b)$ stochastically dominates $F_s(b)$. When $t \in (\tilde{t}, \hat{t}]$, none cumulative distribution functions stochastically dominates the other. First, $F_n(b)$'s slope at the lower-bound of the support is higher than $F_s(b)$'s slope at the lower-bound of the support. Second, $F_n(P) < F_s(P)$. Therefore, given that the cumulative distribution function is continuous, increasing and concave, $F_n(b)$ and $F_s(b)$ cross each other for some $b \in [\underline{b}, P]$. Therefore, none cumulative distribution function stochastically dominates the other. Finally, when $t \geq \hat{t}$, using the same procedure as above, it can be shown that $F_n(b) > F_s(b) \forall b \in [\underline{b}, P]$, i.e., $F_s(b)$ stochastically dominates $F_n(b)$.

Proposition 4. Effect of an increase in transmission capacity in the presence of transmission constraints and *positive* transmission tariffs.

In the presence of transmission capacity constraints and transmission tariffs, the "size" and "cost" effects determine the equilibrium. These two mechanisms work in opposite directions modifying the relevant variables of the model (lower bound of the support, expected bids and expected profits) in a non-monotonic pattern. Therefore, no clear conclusions can be obtained through the analysis of the partial derivatives. However, the effect of an increase in transmission capacity can be studied by using equations ??, ??, ?? and ??.

In this section, I present the static comparative analysis in order to illustrate the difficulties to obtain a close form solution. I present the results for area $A1$. In the rest of the areas, the analysis follows a similar approach.

$$\begin{aligned}
\frac{\partial \underline{b}_n}{\partial T} &= \frac{-P}{(\theta_s+\theta_n)} < 0 \\
\frac{\partial \underline{b}_s}{\partial T} &= \frac{t(\theta_s+T)-tT}{(\theta_s+T)^2} = \frac{t\theta_s}{(\theta_s+T)^2} > 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial F_n(P)}{\partial T} = {} & \frac{(2P(\theta_s+T)-t\theta_s)\left((P(\theta_s+T)-tT)(\theta_n+\theta_s)\right))}{\left((P(\theta_s+T)-tT)(\theta_n+\theta_s)\right)^2} + \\
& \frac{t(\theta_n+\theta_s)(P(\theta_s+T)-t\theta_s)(\theta_s+T)}{\left((P(\theta_s+T)-tT)(\theta_n+\theta_s)\right)^2} > 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E_n(b)}{\partial T} = {} & \frac{\frac{\partial \underline{b}}{\partial T}(\theta_s+T)+(\underline{b}-t)(\theta_s+T)-\underline{b}(\theta_s+T)+tT}{(\theta_s+T)^2} \\
& \left[ln\left(\frac{P(\theta_s+T)-tT}{\underline{b}(\theta_s+T)-tT}\right) - \frac{tT}{P(\theta_s+T)-tT} + \frac{tT}{\underline{b}(\theta_s+T)-tT} \right] + \\
& \frac{\underline{b}(\theta_s+T)-tT}{\theta_s+T} \\
& \left[\frac{\underline{b}(\theta_s+T)-tT}{P(\theta_s+T)-tT} \right] \\
& \left[\frac{(P-t)(\underline{b}(\theta_s+T)-tT) - \left(\frac{\partial \underline{b}}{\partial T}(\theta_s+T)+\underline{b}-t\right)(P(\theta_s+T)-tT)}{(\underline{b}(\theta_s+T)-tT)^2} \right] + \\
& \frac{\underline{b}(\theta_s+T)-tT}{\theta_s+T} \left[-\frac{t(P(\theta_s+T)-tT)-(P-t)tT}{(P(\theta_s+T)-tT)^2} \right] + \\
& \frac{\underline{b}(\theta_s+T)-tT}{\theta_s+T} \left[\frac{t(\underline{b}(\theta_s+T)-tT) - \left(\frac{\partial \underline{b}}{\partial T}(\theta_s+T)+\underline{b}-t\right)tT}{(\underline{b}(\theta_s+T)-tT)^2} \right]
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial E_s(b)}{\partial T} &= \frac{\frac{\partial \underline{b}}{\partial T}(\theta_s+T)^3(\theta_s+\theta_n)+\underline{b}(\theta_n+\theta_s)(\theta_s+T)^2-2(\theta_s+T)\left[(\theta_s+\theta_n)(\underline{b}(\theta_s+T)-t\theta_s)\right]}{(\theta_s+T)^4} \\
& \left[ln\left(\frac{P(\theta_s+T)-t\theta_s}{\underline{b}(\theta_s+T)-t\theta_s}\right)-\frac{t\theta_s}{P(\theta_s+T)-t\theta_s}+\frac{t\theta_s}{\underline{b}(\theta_s+T)-t\theta_s}\right]+ \\
& \frac{(\theta_n+\theta_s)(\underline{b}(\theta_s+T)-tT)}{(\theta_s+T)^2} \\
& \left[\frac{(\underline{b}(\theta_s+T)-t\theta_s)}{P(\theta_s+T)-t\theta_s}\right] \\
& \left[\frac{P(\underline{b}(\theta_s+T)-t\theta_s)-\left(\frac{\partial \underline{b}}{\partial T}(\theta_s+T)+\underline{b}\right)(P(\theta_s+T)-t\theta_s)}{(\underline{b}(\theta_s+T)-t\theta_s)^2}\right]+ \\
& \frac{(\theta_n+\theta_s)(\underline{b}(\theta_s+T)-t\theta_s)}{(\theta_s+T)^2}\left[-\frac{Pt\theta_s}{(P(\theta_s+T)-t\theta_s)^2}\right]+ \\
& \frac{(\theta_n+\theta_s)(\underline{b}(\theta_s+T)-t\theta_s)}{\theta_s+T}\left[\frac{-\underline{b}t\theta_s-\left(\frac{\partial \underline{b}}{\partial T}(\theta_s+T)t\theta_s\right)}{(\underline{b}(\theta_s+T)-t\theta_s)^2}\right]
\end{aligned}
$$

$$\frac{\partial \pi_n}{\partial T} = -P < 0$$

$$
\begin{aligned}
\frac{\partial \pi_s}{\partial T} &= \frac{-P}{(\theta_s+\theta_n)}(\theta_s+T)+\frac{P(\theta_n-T)+t\theta_s}{(\theta_s+\theta_n)}-t \\
&= \frac{P(\theta_n-2T-\theta_s)-t\theta_n}{(\theta_s+\theta_n)}
\end{aligned}
$$

Annex 3. Equilibrium when suppliers pay a point of connection tariff

In this paper, I assume that suppliers face transmission constraints and that they are charged by a linear transmission tariff for the electricity sold in the other market. Under this assumption, I show that suppliers' strategies are affected by the "size" and the "cost" effects that work in the opposite direction and determine equilibrium outcome allocations. However, when suppliers face transmission constraints and they are charged on basis of the total electricity that they inject into the grid (point of connection tariff), the suppliers pay the same transmission tariff for the electricity sold in their own market and the electricity sold in the other market.

Therefore, with point of connection tariffs, the competitive advantage (cost effect) derived from the location in the high-demand market disappears and equilibrium market outcomes exclusively depend on the size effect. Moreover, given that electricity demand is very inelastic, an increase in production costs is passed through to consumers that face an increase in equilibrium prices in both markets. This result is in line with the pass through literature (Marion and Muehlegger 2011; Fabra and Reguant 2014). Hence, a change in the design of transmission tariffs from the design used in the majority of the countries to the one proposed in this article could induce a large improvement in consumer welfare.

The approach to work out the *lower bound of the support*, the *cumulative distribution function*, the *probability distribution function*, the *expected equilibrium price* and the *expected profit* is similar to the one used in propositions 1 and 3. To facilitate the understanding of the equilibrium and to establish comparisons with the transmission tariffs scenario, in area $A1$, I work out the *lower bound of the support*, the *cumulative distribution function*, the *probability distribution function*, the *expected equilibrium price* and the *expected profit*. I also analyze the effect that an increase in transmission capacity has on the main variables of the model. I conclude the annex by comparing the equilibrium outcome of the three model specifications.

First, I work out the *lower bound of the support*. Using the same approach as in annex one, it is straightforward to show that $\underline{b}_n > \underline{b}_s$. Hence, $S = [max\{\underline{b}_n, \underline{b}_s\}, P] = [\underline{b}_n, P]$, where $\underline{b}_n$ can be derived from $(\underline{b}_n - t)(\theta_n + \theta_s) = (P - t)(\theta_n - T)$, where t is the point of connection tariff that it is taken as a cost by the suppliers. Therefore, in area $A1$, $S = \left[t + \dfrac{(P-t)(\theta_n - T)}{(\theta_n + \theta_s)}, P\right]$.

Second, I work out the *cumulative distribution function*.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{\theta_n + \theta_s}{\theta_s + T}\dfrac{b - \underline{b}}{b - t} = C_n(\theta, k, T)\dfrac{b - \underline{b}}{b - t} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{\theta_s + T}{\theta_s + T}\dfrac{b - \underline{b}}{b - t} = C_s(\theta, k, T)\dfrac{b - \underline{b}}{b - t} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases} \tag{30}$$

As can be observed in equation **??** when the transmission tariffs are null ($t = 0$), and in equation **??** with point of connection tariffs, the cumulative distribution function can be written as a constant $C_i(\theta, k, T)$ multiplied by $\dfrac{b - \underline{b}}{b}$ or by $\dfrac{b - \underline{b}}{b - t}$. This contrast with the positive transmission tariffs scenario ($t > 0$), where the denominator in equation **??** cannot be factorized, and the cumulative distribution function cannot be simplified.

Given that $\underline{b}_n > \underline{b}_s$, it is easy to show that $F_S(P)$ is continuous in the upper bound of the support, and that $F_n(P)$ is discontinuous in the upper bound of the support.

$$
\begin{aligned}
F_s(P) &= \frac{\theta_n+\theta_s}{\theta_s+T}\frac{P-t-\dfrac{(P-t)(\theta_n-T)}{\theta_n+\theta_s}}{P-t} = C_n(\theta,k,T)\frac{P-t-\dfrac{(P-t)(\theta_n-T)}{\theta_n+\theta_s}}{P-t} = 1 \\
F_n(P) &= \frac{\theta_s+T}{\theta_s+T}\frac{P-t-\dfrac{P-t(\theta_n-T)}{\theta_n+\theta_s}}{P-t} = C_s(\theta,k,T)\frac{P-t-\dfrac{P-t(\theta_n-T)}{\theta_n+\theta_s}}{P-t} = \\
&= \frac{(\theta_s+T)}{(\theta_n+\theta_s)} < 1
\end{aligned}
$$

Third, the *probability distribution function* is equal to:

$$
\begin{aligned}
f_s(b) &= \frac{\partial F_s(b)}{\partial b} = \frac{\theta_n+\theta_s}{\theta_s+T}\frac{\underline{b}-t}{(b-t)^2} = C_n(\theta,k,T)\frac{\underline{b}-t}{(b-t)^2} \\
f_n(b) &= \frac{\partial F_n(b)}{\partial b} = \frac{\theta_s+T}{\theta_s+T}\frac{\underline{b}-t}{(b-t)^2} = C_s(\theta,k,T)\frac{\underline{b}-t}{(b-t)^2} \qquad (31)
\end{aligned}
$$

As can be observed in equation **??** when the transmission tariffs are null ($t=0$), and in equation **??** with point of connection tariffs, the probability distribution function can be written as a constant $C_i(\theta,k,T)$ multiplied by $\dfrac{\underline{b}}{b^2}$ or by $\dfrac{\underline{b}-t}{(b-t)^2}$. This contrast with the positive transmission tariffs scenario ($t>0$), where the denominator in equation **??** cannot be simplified.

Fourth, the expected bid is determined by:

$$
\begin{aligned}
E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{\theta_n+\theta_s}{\theta_s+T}\frac{(\underline{b}-t)}{(b-t)^2}\partial b = \\
&\quad \frac{\theta_n+\theta_s}{\theta_s+T}(\underline{b}-t)\left[ln\left(\frac{P-t}{\underline{b}-t}\right) - \frac{t}{P-t} + \frac{t}{\underline{b}-t}\right] \\
E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_n)\partial b = \int_{\underline{b}}^{P} b\frac{\underline{b}-t}{(b-t)^2}\partial b + (1-F_n(P))\,P = \\
&\quad (\underline{b}-t)\left[ln\left(\frac{P-t}{\underline{b}-t}\right) - \frac{t}{P-t} + \frac{t}{\underline{b}-t}\right] + (1-F_n(P))\,P \qquad (32)
\end{aligned}
$$

In equation **??**, I have solved by substituting variables:

$$
\begin{aligned}
U &= b-t \Rightarrow b = U+t \\
\frac{\partial U}{\partial b} &= 1 \Rightarrow \partial b = \partial U
\end{aligned}
$$

As can be observed in equation **??**, with point of connection tariffs, the expected bid formula cannot be simplified as $C_i(\theta,k,T)$ multiplied by $\underline{b}\ ln\left(\dfrac{P}{\underline{b}}\right)$ as when the transmission tariffs are null ($t=0$). In contrast, given that to work out the expected bid, it is necessary to integrate by parts, the expected bid with point of connection tariffs (equation

Table 4: The effect of transmission constraints, transmission tariffs and point of connection tariffs on the main variables of the model ($\theta_s = 5$, $\theta_n = 55$, $k = 60$, $c = 0$, $P = 7$)

	T	$\underline{b}$	$\overline{\pi}_n$	$\overline{\pi}_s$	$\overline{\pi}$	$E_n(b)$	$E_s(b)$	$\theta_n E_n(b) + \theta_s E_s(b)$
Model I	40	1.75	105	184	289	4.2	3.2	247
Model II	40	1.87	105	24	129	3.1	3.3	187
Model III	40	2.87	82.5	62	144.5	4.8	4	284

??), and the expected bid with positive transmission tariffs ($t > 0$) (equation **??**) are very similar.

Fifth, the expected profit is defined by $\overline{\pi}_n = (\underline{b} - t)(\theta_s + \theta_n)$ and $\overline{\pi}_s = (\underline{b} - t)(\theta_s + T)$.

Figure 10: Cumulative Distribution Functions of models I, II and III.

`model_I_II_III-eps-converted-to.pdf`

Model comparison

In the last part of the annex, I compare the equilibrium outcome of the three different model specifications: transmission constraints and *zero* transmission tariffs (model I); transmission constraints and *positive* transmission tariffs (model II) and, finally, transmission constraints and *positive* point of connection tariffs (model III).

As can be observed in figure **??**, the three different model specifications affect suppliers' strategies in different ways, and that induces important changes on the main variables of the model (table **??**).

I have discussed the three models in detail in section four (pages. 18-19). I refer the reader to those pages to follow the analysis.

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