Stochastic Dispatch Models for Electricity Markets

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Decisions under uncertainty

- Generators, retailers, consumers
  - Bidding
  - Hedging
  - Investing
- System operator / Exchange
  - Transfer capacities
  - Reliability and security constraints
- What about the dispatch problem itself?
Congestion management and balancing

Reasons for inadequate congestion handling:
- Congestion within areas not considered (in full)
- “Loop-flow” not included in market clearing
- Inadequate representation of capacity constraints
Flexibility costs and uncertainty

- High uncertainty
- Low uncertainty

Flexibility costs: extra adjustment costs
e.g. due to
- resetting plans,
- non-optimal operation,
- more expensive units
- rules of the auction

Day-ahead market
Regulation market
Delivery hour (e.g. 08:00-08:59)

Time
Alternative market clearing models

- Integrated stochastic model
  - Pritchard, Zakeri & Philpott (OR, 2010)
- Improved conventional (myopic) dispatch
  - Morales et al. (EJOR, 2014)
- Reserve market
  - Morales et al. (IEEE TPS, 2009 & 2012)
Stochastic market clearing

- Literature:
  - Bouffard et al. (2005a,b); Bouffard and Galiana (2008); Ruiz et al. (2009b,a); Papavasiliou et al. (2011); Papavasiliou and Oren (2012); Khazaei et al. (2014); Bjørndal et al. (2016a,b)

- Pricing issues:
  - Kaye et al. (1990); Wong and Fuller (2007); Pritchard et al. (2010); Morales et al. (2012, 2014); Zavala et al. (2017); Zakeri et al. (2016).
Our starting point is an energy-only stochastic market clearing as in Pritchard et al. (2010)
- Compare to a sequential market clearing model with myopic clearing of the day-ahead part of the market

Organization of bidding and information responsibilities (Bjørndal et al., 2016a)

How should the day-ahead part of the market be modeled? (Bjørndal et al., 2016b)
- Network flow and balance constraints

Pricing (Zakeri et al., 2016)
- Revenue adequacy for the system operator
- Cost recovery for generators
Model

Day-ahead and real-time generation ($\geq 0$) and load ($\leq 0$) quantities:

$$x_i \in C^1_i \quad \forall i \in I$$

$$X_{i\omega} \in C^2_i(\omega, x_i) \quad \forall i \in I, \omega \in \Omega$$

Upregulation $X^u_{i\omega} = \max\{X_{i\omega} - x_i, 0\}$ or downregulation $X^d_{i\omega} = \max\{x_i - X_{i\omega}, 0\}$ for flexible entities.
Generator cost functions

\[ a_i + b_i x_i \]

\[ a_i - a_i^d \]

\[ a_i^u - a_i \]

\[ X_{i \omega_1} \]

\[ x_i \]

\[ X_{i \omega_2} \]
Load benefit curves

\[ a_i \]

\[ b_i \]

\[ a_i + b_i x_i \]

\[ -X_i \omega_1 \]

\[ -x_i \]

\[ -X_i \omega_2 \]

\[ X_i^{u} \]

\[ X_i^{d} \]
Examples of bid curves

\[ b_i x_i + b_i^u X_i^{u} \]

\[ b_i X_i \omega \]

\[ b_i x_i \]

\[ x_i \]

\[ X_i \omega \]
Objective function

Cost of real-time quantity at day-ahead parameters:

\[ c_i(X_{i\omega}) = a_i X_{i\omega} + 0.5 b_i (X_{i\omega})^2 \]

Flexibility cost:

\[ \tilde{c}_i(x_i, X_{i\omega}) = (a_i^u - a_i) X_{i\omega}^u + 0.5 (b_i^u - b_i) (X_{i\omega}^u)^2 \\
+ (a_i - a_i^d) X_{i\omega}^d + 0.5 (b_i^d - b_i) (X_{i\omega}^d)^2 \]
Stochastic market clearing model

\[
\min_{x,f,X,F} \mathbb{E} \left[ \sum_{i \in I} \left( c_i(X_i) + \tilde{c}_i(x_i, X_i) \right) \right]
\]

\text{s.t.}

\begin{align*}
    x_i & \in C_i^1 & \forall i & \in I \\
    X_{i\omega} & \in C_i^2(\omega, x_i) & \forall i & \in I, \ \omega \in \Omega \\
    \tau_n(f) + \sum_{i \in l(n)} x_i & = 0 & \forall n & \in N \\
    \tau_n(F_\omega) + \sum_{i \in l(n)} X_{i\omega} & = 0 & \forall n & \in N, \ \omega \in \Omega \\
    f & \in U^1 \\
    F_\omega & \in U^2 & \forall \omega & \in \Omega
\end{align*}

\text{(1a)} \text{ (1b)} \text{ (1c)} \text{ (1d)} \text{ (1e)} \text{ (1f)} \text{ (1g)}
Myopic market clearing model - day-ahead part

\[
\min_{x,f} \sum_{i \in I} c_i(x_i) \tag{2a}
\]

s.t.

\[
x_i \in C^1_i \quad \forall i \in I \tag{2b}
\]

\[
\tau_n(f) + \sum_{i \in l(n)} x_i = 0 \quad \forall n \in N \tag{2c}
\]

\[
f \in U^1 \tag{2d}
\]
Myopic market clearing model - real-time part, scenario $\omega$ 

\[
\min_{X_\omega, F_\omega} \sum_{i \in I} \left( c_i(X_{i\omega}) + \bar{c}_i(x_i, X_{i\omega}) \right) 
\]

s.t.

\[
X_{i\omega} \in C_i^2(\omega, x_i) \quad \forall i \in I 
\]

\[
\tau_n(F_\omega) + \sum_{i \in I(n)} X_{i\omega} = 0 \quad \forall n \in N \quad [\lambda_{n\omega}] 
\]

\[
F_\omega \in U^2 
\]
Stochastic market clearing model

\[
\min_{x,f,X,F} \mathbb{E} \left[ \sum_{i \in I} \left( c_i(X_i) + \bar{c}_i(x_i, X_i) \right) \right]
\]

s.t.

\[
x_i \in C_i^1 \quad \forall i \in I \tag{1b}
\]

\[
X_{i\omega} \in C_i^2(\omega, x_i) \quad \forall i \in I, \ \omega \in \Omega \tag{1c}
\]

\[
\tau_n(f) + \sum_{i \in I(n)} x_i = 0 \quad \forall n \in N \tag{1d}
\]

\[
\tau_n(F_\omega) + \sum_{i \in I(n)} X_{i\omega} = 0 \quad \forall n \in N, \ \omega \in \Omega \tag{1e}
\]

\[
f \in U^1 \tag{1f}
\]

\[
F_\omega \in U^2 \quad \forall \omega \in \Omega \tag{1g}
\]
Day-ahead constraints

- We assume that $U^2$ represents the network constraints in a DC load flow model without losses.
- What should $U^1$ represent?
Alternative day-ahead network representations

1. Nodal model, i.e., $U^1 = U^2$
2. Zonal model
   - No loop flow
   - Aggregate flow capacities set by system operator(s)
3. Unconstrained flow, i.e., $U^1 = \mathbb{R}^{|L|}$
4. Unconstrained flow and balance

$$\min[v_{stoch}^{nodal}, v_{stoch}^{zonal}] \geq v_{stoch}^{bal} \geq v_{stoch}^{unc}$$
Example 1

- \(P(1) = P(2) = 0.5\)
- Real-time quantities \(X_\omega\) are given above
- All cost parameters equal zero, except \(a_1^u = a_2^u = 1\) and \(a_3^u = 0.25\)
- All lines have identical impedances
- Capacity of line (2,3) is 40
Example 1 - stochastic model

\[
\begin{align*}
\text{min } & \quad 0.5 \cdot 1 \cdot ([30 - x_1]^+ + [0 - x_1]^+ + [0 - x_2]^+ + [60 - x_2]^+) \\
& \quad + 0.5 \cdot 0.25 \cdot ([-30 - x_3]^+ + [-60 - x_3]^+) \\
\text{s.t. } & \quad x_1 + x_2 + x_3 = 0 \\
& \quad -40 \leq \frac{x_2 - x_3}{3} \leq 40
\end{align*}
\]
Example 1 - optimal day-ahead schedules

\[ v_{\text{stoch}} = 0 \]

\[ v_{\text{unc}} = 0 \]

\[ v_{\text{bal}} = 0.5 \cdot (-30 - (-90)) \cdot 0.25 \]
\[ + 0.5 \cdot (-60 - (-90)) \cdot 0.25 = 11.25 \]

\[ v_{\text{nodal}} = 0.5 \cdot (-30 - (-75)) \cdot 0.25 \]
\[ + 0.5 \cdot [60 - 45] \cdot 1 \]
\[ + (-60 - (-75)) \cdot 0.25 = 15 \]
Example 2

Node 2: Nuclear + Hydro

Node 1: Wind (scen. 2) + Thermal – Load

Node 3: Hydro
Wind scenarios

- Wind = 0
  - $p_1 = 0.2$

- Wind = 7000
  - $p_2 = 0.5$

- Wind = 15000
  - $p_3 = 0.3$
## Cost and benefit parameters

<table>
<thead>
<tr>
<th>Entity</th>
<th>Node</th>
<th>Intercept ($a$)</th>
<th>Slope ($b$)</th>
<th>Flexible?</th>
<th>Flex. cost up</th>
<th>Flex. cost down</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>Partly</td>
<td>$a^u = a$</td>
<td>$a^d = a$</td>
</tr>
<tr>
<td>Therm.</td>
<td>1</td>
<td>30</td>
<td>0</td>
<td>Yes</td>
<td>$a^u = a + 6$</td>
<td>$a^d = a$</td>
</tr>
<tr>
<td>Load</td>
<td>1</td>
<td>2000</td>
<td>0</td>
<td>Yes¹</td>
<td>$a^u = a$</td>
<td>$a^d = a$</td>
</tr>
<tr>
<td>Nucl.</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>No</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Hydro</td>
<td>2</td>
<td>0</td>
<td>0.01</td>
<td>Yes</td>
<td>$b^u = 10b$</td>
<td>$b^d = b$</td>
</tr>
<tr>
<td>Hydro</td>
<td>3</td>
<td>0</td>
<td>0.01</td>
<td>Yes</td>
<td>$b^u = 10b$</td>
<td>$b^d = b$</td>
</tr>
</tbody>
</table>

¹Load can be shed at VOLL = $a = 2000$ Euros / MWh.
Example 2 - optimal values, stochastic model

<table>
<thead>
<tr>
<th>Model</th>
<th>$\mathbb{E}[c]$</th>
<th>$\mathbb{E}[\tilde{c}]$</th>
<th>$\mathbb{E}[c + \tilde{c}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>€</td>
<td>Relative</td>
<td>€</td>
</tr>
<tr>
<td>Wait-and-see</td>
<td>66360</td>
<td>100.0 %</td>
<td>0</td>
</tr>
<tr>
<td>Unconstrained</td>
<td>76250</td>
<td>114.9 %</td>
<td>0</td>
</tr>
<tr>
<td>Balanced</td>
<td>76322</td>
<td>115.0 %</td>
<td>1600</td>
</tr>
<tr>
<td>Nodal</td>
<td>82325</td>
<td>124.1 %</td>
<td>2190</td>
</tr>
<tr>
<td>Zonal ($cap_{{1},{2,3}} = 5000$)</td>
<td>116977</td>
<td>176.3 %</td>
<td>117168</td>
</tr>
<tr>
<td>Zonal ($cap_{{1},{2,3}} = 10000$)</td>
<td>79810</td>
<td>120.3 %</td>
<td>2769</td>
</tr>
</tbody>
</table>
### Example 2 - optimal schedules

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind</td>
<td>1</td>
<td>153</td>
<td>-153, 6847, 9849</td>
<td>0</td>
<td>7000, 10000</td>
</tr>
<tr>
<td>Therm.</td>
<td>1</td>
<td>5000</td>
<td>-5000, -5000</td>
<td>5000</td>
<td>-5000, -5000</td>
</tr>
<tr>
<td>Load</td>
<td>1</td>
<td>-15000</td>
<td></td>
<td>-15000</td>
<td></td>
</tr>
<tr>
<td>Nucl.</td>
<td>2</td>
<td>4998</td>
<td></td>
<td>5000</td>
<td></td>
</tr>
<tr>
<td>Hydro</td>
<td>2</td>
<td>155</td>
<td>-153, 245, -155</td>
<td>1500</td>
<td>-1500, -1500</td>
</tr>
<tr>
<td>Hydro</td>
<td>3</td>
<td>4694</td>
<td>306, -2092, -4694</td>
<td>5000</td>
<td>-3500, -5000</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>0</td>
<td>0</td>
<td>1500</td>
<td>-1500, -1500</td>
</tr>
</tbody>
</table>

We see that the unconstrained model overbooks generation.
Myopic model

- Solutions depend on wind power bids in day-ahead market
- Removing constraints in the day-ahead market may lead to infeasibilities, e.g. due to scheduling of non-flexible nuclear in day-ahead market
Myopic model - scheduled flow

**Nodal**

**Balanced**
Myopic model - scheduled generation

- Too much inflexible generation is scheduled with balanced model
Myopic model - expected cost

### Nodal

<table>
<thead>
<tr>
<th>Wind capacity (MWh/h)</th>
<th>Expected cost (1000 Euros)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>740</td>
<td>153</td>
</tr>
<tr>
<td>960</td>
<td>7100</td>
</tr>
<tr>
<td>15000</td>
<td>9600</td>
</tr>
</tbody>
</table>

### Balanced

<table>
<thead>
<tr>
<th>Wind capacity (MWh/h)</th>
<th>Expected cost (1000 Euros)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7100</td>
<td>9600</td>
</tr>
<tr>
<td>15000</td>
<td>15000</td>
</tr>
</tbody>
</table>

### Graphs

- **Nodal**: Graph showing expected cost for total, VOLL, and VOLL+Flex. as a function of wind capacity.
- **Balanced**: Graph showing expected cost for total, VOLL, and VOLL+Flex. as a function of wind capacity.
### Myopic zonal model - expected cost

**cap\{1\},\{2,3\} = 10000**

<table>
<thead>
<tr>
<th>Wind capacity (MWh/h)</th>
<th>Expected cost (1000 Euros)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>224</td>
</tr>
<tr>
<td>224</td>
<td></td>
</tr>
<tr>
<td>320</td>
<td></td>
</tr>
<tr>
<td>572</td>
<td></td>
</tr>
<tr>
<td>738</td>
<td></td>
</tr>
<tr>
<td>1008</td>
<td></td>
</tr>
<tr>
<td>15000</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
</tr>
<tr>
<td><strong>VOLL</strong></td>
<td></td>
</tr>
<tr>
<td><strong>VOLL+Flex.</strong></td>
<td></td>
</tr>
</tbody>
</table>

**cap\{1\},\{2,3\} = 5000**

<table>
<thead>
<tr>
<th>Wind capacity (MWh/h)</th>
<th>Expected cost (1000 Euros)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>211</td>
</tr>
<tr>
<td>211</td>
<td></td>
</tr>
<tr>
<td>313</td>
<td></td>
</tr>
<tr>
<td>572</td>
<td></td>
</tr>
<tr>
<td>738</td>
<td></td>
</tr>
<tr>
<td>1060</td>
<td></td>
</tr>
<tr>
<td>15000</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
</tr>
<tr>
<td><strong>VOLL</strong></td>
<td></td>
</tr>
<tr>
<td><strong>VOLL+Flex.</strong></td>
<td></td>
</tr>
</tbody>
</table>
Pricing (Zakeri et al., 2016)

- The pricing scheme in Pritchard et al. (2010) is revenue adequate in expectation.
- Mathematically this means
  \[ E \left[ \sum_i \left( x_i^* \pi_n(i) + (X_i^* - x_i^*) \lambda_n(i) \right) \right] \leq 0. \]
  (recall payout is positive and collection is negative).
- Intuitively this means that, if this type of market is used repeatedly over many trading periods, the SO will not run a financial deficit over time.
- There may be a deficit in an individual trading period.
Revenue adequacy

- NewSP is revenue adequate in every scenario.
- Mathematically this means for any scenario $\omega$, at optimality
  \[
  \sum_i \left( X_i^\omega \pi^\omega_{n(i)} - d_{n(i)}^\omega \pi^\omega_{n(i)} \right) \leq 0.
  \]
  (recall payout is positive and collection is negative).
- This is natural and the argument is very similar to revenue adequacy of the DCOPF.
Cost recovery in expectation

- NewSP recovers cost for each tranche, including the deviation penalties, in expectation.
- Mathematically this means at optimality

\[
E \left[ \pi_{n(i)}^\omega X_i^\omega - c_iX_i^\omega - rU_iu_i^\omega - rV_iv_i^\omega \right] \geq 0.
\]
- This result follows from SP duality and a technical lemma proving the validity of duals in real time.


