Estimating Parameter Uncertainty in the StoNED Model

Ørjan Mydland[1,2] Jonas Andersson[1] Endre Bjørndal[1]

[1]Norwegian School of Economics, Norway

[2]Høgskolen i Innlandet, Norway

February 13, 2018

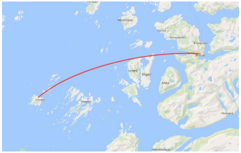
Outline

- Incentive regulation
- The StoNEZD benchmarking model
- Bootstrapping procedure for assessing uncertainty in parameter estimates
- Monte Carlo Study
- Conclusions

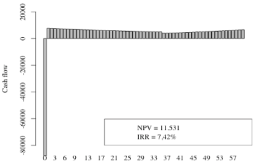
Incentive regulation

- Incentives for efficient organization, operation, and investments
  - Revenue should be independent of the regulated company's own cost
  - Revenue = cost of the "marginal" company, given the company's "output" (volume and quality)
  - Operating income: depends also on the company's efficiency and cost
- Sufficient revenue level to attract both financial and human capital
  - Competitive rate of return on invested capital
  - Accept continual efficiency differences and super-profits
- Yardstick implementation: $R_i = \alpha C_i^* + (1 - \alpha) C_i$
  - C^* typically determined with benchmarking methods: DEA, SFA, StoNED/StoNEZD

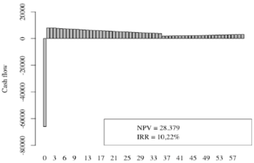
Marginal cost estimates and investment incentives



Alternative 1 (22kV)



Alternative 2 (36kV)



	Project 1	Project 2
Stage 1 (DEA shadow prices)	-12 582	17 409
Stage 2 (env. factors)	-2 317	1 369
Stage 3 (calibration)	26 430	9 601
NPV	11 531	28 379

The StoNEZD model (Johnson and Kuosmanen, 2011)

$$\min_{\gamma,\alpha,\beta,\delta,\epsilon} \sum_i \epsilon_i^2 \tag{1}$$

s.t.

$$\ln x_i = \ln \gamma_i + \sum_k \delta_k z_{ki} + \epsilon_i \qquad \forall i \tag{2}$$

$$\gamma_i = \alpha_i + \sum_r \beta_{ri} y_{ri} \qquad \forall i \tag{3}$$

$$\gamma_i \geq \alpha_j + \sum_r \beta_{rj} y_{ri} \qquad \forall j, i \tag{4}$$

$$\beta_{ri} \geq 0 \qquad \forall r, i \tag{5}$$

Method of moments

$$\hat{\sigma}_u = \sqrt[3]{\frac{\hat{M}_3}{\left(\frac{4}{\pi}-1\right)\sqrt{\frac{2}{\pi}}}}, \tag{6}$$

$$\hat{\sigma}_v = \sqrt{\hat{M}_2 - \left(\frac{\pi-2}{\pi}\right)\hat{\sigma}_u^2}, \tag{7}$$

where $\hat{M}_2 = \sum_i (\hat{\epsilon}_i - \bar{\epsilon})^2/n$ and $\hat{M}_3 = \sum_i (\hat{\epsilon}_i - \bar{\epsilon})^3/n$.

Estimated cost norm

$$\hat{\mu} = \hat{\sigma}_u \sqrt{2/\pi}, \tag{8}$$

$$\hat{C}(\boldsymbol{y}_i, \boldsymbol{z}_i) = \left(\hat{\alpha}_i + \sum_r \hat{\beta}_{ri} y_{ri} \right) e^{-\hat{\mu} + \sum_k \hat{\delta}_k z_{ki}} \tag{9}$$

Parametric bootstrap

1. Use the estimates $\hat{\sigma}_u$ and $\hat{\sigma}_v$ to simulate bootstrap error terms for each of the $i = 1, \ldots, n$ DMUs:

$$\epsilon_i^* = u_i^* + v_i^*$$
$$u_i^* \sim N(0, \hat{\sigma}_u^2)^+$$
$$v_i^* \sim N(0, \hat{\sigma}_v^2)$$

2. Generate new cost numbers based on the simulated residuals and estimated parameter values:

$$x_i^* = \left(\hat{\alpha}_i + \sum_r \hat{\beta}_{ri} y_{ri}\right) e^{\epsilon_i^* + \sum_k \hat{\delta}_k z_{ki}}$$

3. Based on x^*, y, and z, use the StoNEZD procedure to estimate $\alpha^*, \beta^*, \delta^*, \sigma_u^*$ and σ_v^*.
4. Repeat 1-3 for B, say 1000, times. We now have B bootstrap estimates which can be seen as a draw from the sampling distribution of the parameter estimator(s).

Checking the reliability of the bootstrap estimates

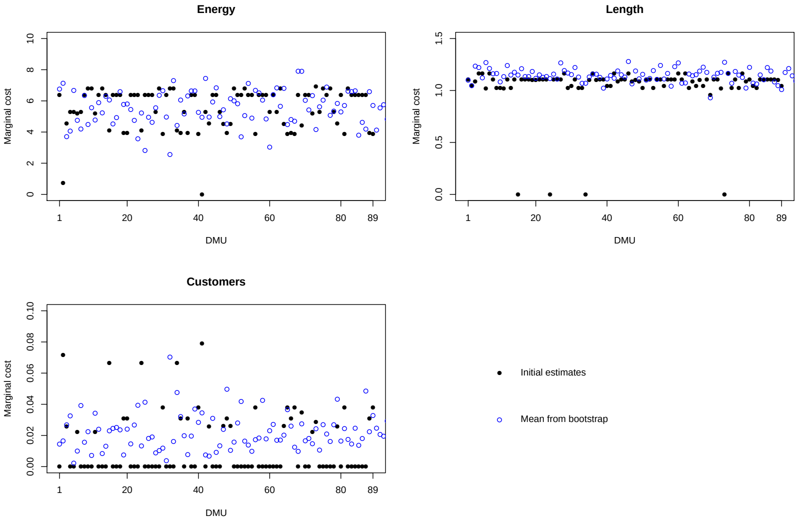
"True" parameter values, α, β, δ, σ_u and σ_v (based on estimates from the Finnish data).

1. Simulate error term for each of the $i = 1, ..., n$ DMUs:
$$\epsilon_i = u_i + v_i$$
$$u_i \sim N(0, \sigma_u^2)^+$$
$$v_i \sim N(0, \sigma_v^2)$$
2. Generate new cost numbers based on the simulated error terms:
$$\tilde{x}_i = \left(\alpha_i + \sum_r \beta_{ri} y_{ri}\right) e^{\epsilon_i + \sum_k \delta_k z_{ki}}$$
3. Based on $\tilde{x}$, y, and z, use the StoNEZD procedure to estimate α, β, δ, σ_u and σ_v.
4. Save the estimates $\hat{\alpha}$, $\hat{\beta}$, $\hat{\delta}$, $\hat{\sigma_u}$ and $\hat{\sigma_v}$.
5. Run the bootstrap procedure to obtain measures of uncertainty for each parameter estimate, e.g. standard deviation. Save them.
6. Repeat 1-5 S, say 1000, times. We now have S estimates for each parameter, each with a corresponding standard deviation.

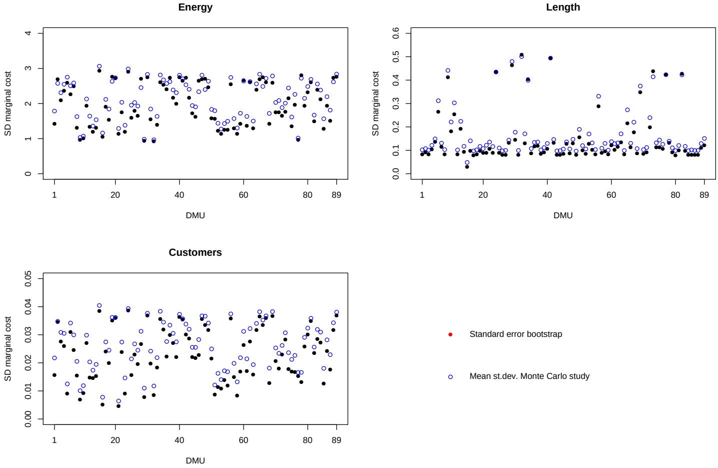
Data for Finnish distribution companies (Kuosmanen, 2012)

Variable	Label	Mean	St. Dev.	Min	Median	Max
Outputs						
Energy transmission (Gwh)	y_1	480	972	15	171	6599
Length of network (km)	y_2	4,135	10,223	51	989	67611
Number of customers	y_2	35,449	71,871	24	11,081	420,473
Inputs						
Total cost (1000 €)	x	8,418	18,048	268	3,102	117,554
Environmental variables						
Proportion of underground cables	z	0.33	0.26	0.01	0.24	1.00

"True" beta values and mean of bootstrap values (B = 1000)



Bootstrap estimates (B = 1000) of st.dev. compared to mean from simulations (S = B = 100)



Conclusions and future research

- We study the performance of the bootstrap for making inference on the estimation of parameters, marginal costs, in the cost function of the StoNEZD model
- Preliminary results are mixed
    - Estimated means versus true means
    - Estimated uncertainty versus simulated uncertainty
- Future work
    - Run the study with larger values for B and S
    - Non-parametric bootstrap