Electricity Auctions in the Presence of Transmission Constraints and Transmission Costs*

Mario Blazquez de Paz†

This version: September 20, 2016

Abstract

Electricity markets are moving through integration around the world. However, our understanding of those markets is still limited. I characterize the equilibrium in an electricity auction when suppliers face transmission constraints and linear tariffs for the injection of electricity into the grid. With *point of connection* tariffs, which are used in the majority of the countries, suppliers pay a tariff for the total electricity injected into the grid. In contrast, with *transmission tariffs*, suppliers only pay a tariff for the electricity sold in the other market. Transmission tariffs outperform point of connection tariffs by maximizing consumer welfare and minimizing transmission losses. The consequences of an increase in transmission capacity differ considerably depending on the tariff. If the transmission tariffs are *zero*, an increase in transmission capacity is pro-competitive. In contrast, if the transmission tariffs are *positive*, an increase in transmission capacity is pro-competitive only when the transmission capacity is low.

KEYWORDS: electricity auctions, wholesale electricity markets, transmission capacity constraints, network tariffs, energy economics.
JEL codes: D43, D44, L13, L94

*I am very grateful to Giacomo Calzolari and Emanuele Tarantino for comments and excellent supervision during my Ph.D. Pär Holmberg and Henrik Horn helped me considerably improve the introduction. Daniel Kovenock made very useful comments on the existence of the equilibrium in the presence of transmission costs. I am also very grateful for fruitful discussions with and comments from Ola Andersson, Claude Crampes, Oscar Erixson, Shon Ferguson, Pär Holmberg, Henrik Horn, Ewa Lazarczyk, Thomas-Olivier Léautier, Chloé Le Coq, Pehr-Johan Norbäck, Andy Philpott, Michele Polo, Mar Reguant, Andrew Rodes, Keith Ruddell, Thomas Tangerås, workshop participants at Toulouse School of Economics and Vaxholm (Stockholm), seminar participants at Bologna University, Research Institute of Industrial Economics (IFN), Complutense University, Salamanca University and conference participants at Mannheim Energy Conference and Industrial Organization: Theory, Empirics and Experiments in Alberobello (Italy). This research was completed within the framework of the IFN research program "The Economics of Electricity Markets". I acknowledge financial support from the Torsten Söderberg Foundation and the Swedish Energy Agency. Fredrik Andersson helped me translate parts of the paper into Swedish. Christina Loennblad helped me proofread the paper.

†Research Institute of Industrial Economics. Mail: mario.blazquezdepaz@ifn.se

1 Introduction

The integration of electricity markets around the world has increased the importance of congestion between countries/states and has initiated a discussion of how to harmonize network tariffs. In general, transmissions from regions with low prices to regions with high prices benefit social welfare. In deregulated electricity markets, more transmission would, in addition, normally improve the market competitiveness. However, it is very costly to expand the transmission capacity. In order to focus the investments to points in the grid where the gains in terms of enhanced market performance will be the largest, one needs a better understanding of how transmission capacity influences the competition between spatially distributed producers. The contribution of this paper is to characterize the outcome of an electricity market auction and how it depends on transmission constraints and the tariffs to access the grid.

The analysis employs a simple duopoly model similar to that in Fabra et al. (2006). In the basic set up, the two suppliers have symmetric production capacities and marginal costs, but are located in two different markets ("North" and "South") that are connected through a transmission line with a limited transmission capacity.[1] Suppliers pay a monetary charge (tariff) to the network owner when using the grid. The charge is linear and it depends on how much power the suppliers inject into the grid (*point of connection tariff*) or transmit through the grid (*transmission tariff*). The majority of European countries (ENTSO-E, 2013) have point of connection tariffs. With the point of connection tariffs scheme, suppliers pay a linear tariff for the electricity injected into the grid, i.e., the electricity sold in their own market and the electricity sold in the other market. In contrast, for transmission tariffs, electricity suppliers would only pay a linear tariff for the electricity sold to the other market. Thus, the total marginal cost of production depends on the fraction exported. When the transmission line is capacity constrained, the equilibrium prices differ across markets. Those differences in prices generate a congestion rent which, as in the majority of countries, I assume to be captured by the transmission system operator.

Each supplier faces a perfectly inelastic demand in each market that is known with certainty when suppliers submit their offer prices. Each supplier submits a single price offer for its entire capacity[2] in a discriminatory price auction such as those used in the UK wholesale electricity market. The assumption of price-inelastic demand can be justified by the fact that the vast majority of consumers purchase electricity under regulated tariffs that are independent of the prices set in the wholesale market, at least in the short run. The assumption that suppliers have perfect information concerning market demand is reasonable when applied to markets where offers are "short lived", such as in Spain, where there are 24 hourly day-ahead markets each day.

In the presence of transmission constraints and *transmission tariffs*, there are two economic forces that determine equilibrium market outcomes. First, the supplier located in the high-demand market faces a high residual demand and thus, it has incentives to

[1] The term "transmission capacity constraint" is used throughout this article in the electrical engineering sense: a transmission line is constrained when the flow of power is equal to the capacity of the line, as determined by engineering standards.

[2] Fabra et al. (2006) show that the equilibrium outcome allocation does not change when firms submit single price offers for their entire capacity and when they submit a set of price-quantity offers.

submit high bids (size effect). Second, the supplier located in the high-demand market faces lower total marginal costs and thus, it has incentives to submit low bids to extract the efficiency rents (cost effect). For the supplier located in the low-demand market, the size and the cost effects work in the opposite direction.

When *transmission tariffs* are *zero*, only the size effect determines the equilibrium. In that case, the supplier located in the high-demand market faces a high residual demand and it submits higher bids than the supplier located in the low-demand market, i.e., its cumulative distribution function stochastically dominates that of the supplier located in the low-demand market. When the transmission tariffs are *positive*, both effects determine the equilibrium and non-stochastic dominance can be established. However, if the transmission tariffs are high enough, the supplier located in the high-demand market submits lower bids than the supplier located in the low-demand market to extract the efficiency rents; given that the majority of consumers are located in that market, consumers' aggregate welfare could increase.[3] In contrast, when suppliers are charged by the power that they inject into the grid (*point of connection* tariffs), given that they pay the same tariff independent of the market in which they are selling electricity, the strategic component of being located in the high-demand market disappears and the equilibrium is only determined by the size effect and the cumulative distribution function of the supplier located in the high-demand market stochastically dominates that of the supplier located in the low-demand market. Moreover, due to demand being inelastic, the tariff is passed through to consumers that are worse off than in the *zero* transmission tariffs scenario and thereby also worse off than in the *positive* transmission tariffs scenario. This is in line with the pass-through literature (Marion and Muehlegger, 2011; Fabra and Reguant, 2014).[4]

An increase in transmission capacity modifies the market size and the total marginal costs and thus also the suppliers' strategies. First, an increase in transmission capacity increases the size of the market for the supplier located in the low-demand market and thus, reduces the residual demand of the supplier located in the high-demand market. Therefore, the competition between suppliers becomes more fierce (size effect). Second, an increase in transmission capacity also increases suppliers' total marginal costs because they can sell more electricity into the other market and given that they pay a tariff for that electricity, there is an increase in their total marginal costs (cost effect). If the transmission tariffs are *zero*, the equilibrium is only determined by the size effect and an increase in transmission capacity reduces the equilibrium prices in both markets, i.e., an increase in transmission is pro-competitive. In contrast, when the transmission tariffs are positive, the equilibrium is determined by both effects. In particular, if the transmission capacity is

[3] If the transmission tariffs are very high, the supplier located in the high-demand market submits lower bids than the supplier located in the low-demand market. However, due to the high tariffs, the suppliers have to submit high bids to cover the total marginal costs, the equilibrium price in both markets is high and consumers' aggregate welfare decreases. Therefore, positive transmission tariffs only increase consumers' aggregate welfare when the tariff is high, but not when it is very high.

[4] The point of connection tariffs consist of two parts, a power charge and an energy charge. The power charge covers for expansion, operation and maintenance of the transmission grid. It is based on annual capacity subscription for injection and outtake of electricity at each connection point. The energy charge is based on the transmission losses in the transmission grid caused by injection and outtake of electricity in different connection points. The power and the energy charges are used to cover the expansion costs of the transmission grid and therefore, they affect the equilibrium outcome allocations and consumer welfare in the long term; however, I do not consider those effects in this paper.

low, an increase in transmission capacity substantially increases the competition between suppliers that move from an isolated market scenario to a connected market scenario; simultaneously, and due to the low capacity of the line, an increase in transmission capacity slightly increases the total marginal costs. Hence, the size effect dominates and an increase in the transmission capacity is pro-competitive. If the transmission capacity is high, an increase in transmission capacity slightly increases the competition between suppliers, but substantially increases their total marginal costs; therefore, the cost effect dominates and an increase in transmission capacity is anti-competitive. Finally, when the point of connection tariffs are implemented, the equilibrium is only determined by the size effect, and an increase in transmission capacity is pro-competitive.

My model contrasts with the previous models of price competition with generation capacity constraints (Kreps and Scheinkman, 1983; Osborne and Pitchik, 1986; Deneckere and Kovenock, 1996; Fabra et al., 2006) where the results are exclusively driven by generation capacity constraints. In the presence of transmission constraints, there are two relevant constraints that explain the results. If the generation capacity is binding, the equilibrium is symmetric even when the realization of demands across markets is asymmetric. If the transmission capacity is binding, the equilibrium is asymmetric even when the suppliers are symmetric in generation capacity and production costs. Therefore, this model provides a complete and novel analysis of the role played by the structural variables of the model (demand realization, generation capacity and transmission capacity) for equilibrium outcome allocations. Moreover, I show that in the mixed strategies equilibrium, the cumulative distribution functions are continuous except in the upper bound of the support where there might exist a mass point, i.e., suppliers adopt smooth cumulative distribution functions which could non-stochastically dominate each other.

This paper also contributes to the literature that analyzes electricity markets. Borenstein et al. (2000) characterize the equilibrium in an electricity network where suppliers compete in quantities as in a Cournot game. Holmberg and Philpott (2012) solve for symmetric supply function equilibria in electricity networks when demand is uncertain ex-ante, but they do not consider any transmission costs. Escobar and Jofré (2010) analyze the effect of transmission losses and transmission costs on equilibrium outcome allocations, but they neglect transmission constraints. Downward et al. (2014) found that the introduction of a tax on suppliers' profit sometimes increases consumer welfare. However, in their analysis, all suppliers produce in the same market and therefore, the results are not driven by any type of size effect similar to the one described in this paper. Hence, this paper is the first to characterize equilibrium outcomes in networks with both transmission constraints and transmission costs. The paper also shows that the interaction between transmission costs and transmission constraints is non-straightforward.

Hogan (1992) introduces the concept of a contract network that maintains short-run efficiency through an optimal spot-price calculation of transmission prices and provides the correct long-term signals to invest in capacity. Chao and Peck (1996) propose a market mechanism for electricity power transmission that consists of tradable transmission capacity rights and a trading rule that also induces short-run efficiency and long-term correct signals. Joskow and Tirole (2000) work out the equilibrium in an electricity market when financial and physical rights are introduced and they find that in a perfect competition scenario, as in Chao and Peck (1996), the financial and physical transmission rights generate the same equilibrium outcome allocations; in contrast, in an imperfect competition scenario, the financial transmission rights outperform the physical transmission rights by generating lower equilibrium prices and increasing efficiency in production. In this paper, I assume that the transmission rights are allocated to the transmission system operator; however, the set up of the model can be modified to analyze the effect of different transmission rights allocations on the equilibrium outcome.

The results of this paper could also be of relevance for the trade literature. For instance, Krugman (1980), Flam and Helpman (1987), Brezis et al. (1993) and Motta et al. (1997) explain differences in prices and profits in international trade models based on product differentiation or product cost advantages. By introducing transport costs and transport constraints, this paper finds related results, even if the product is homogeneous and suppliers have identical production technologies.

The article proceeds as follows. Section 2 describes the model and characterizes the equilibrium in the presence of transmission capacity constraints. Section 3 characterizes the equilibrium in the presence of transmission capacity constraints and zero transmission tariffs. Section 4 characterizes the equilibrium when transmission tariffs are positive. Section 5 compares equilibrium outcomes and consumer welfare when transmission tariffs and point of connection tariffs are implemented. Section 6 concludes the paper. The analysis of point of connection tariffs and all proofs are found in the Appendix.

2 The model

Set up of the model. There exist two electricity markets, market North and market South, that are connected by a transmission line with capacity T. When suppliers transmit electricity through the grid from one market to the other, they face a symmetric[5] and linear[6] transmission tariff t.

There exist two duopolists with capacities k_n and k_s, where subscript n means that the supplier is located in market North and subscript s means that the supplier is located in market South. The suppliers' marginal costs of production are c_n and c_s for production levels less than the capacity, while production above the capacity is impossible (i.e., infinitely costly). Suppliers are symmetric in capacity $k_n = k_s = k > 0$ and symmetric in production costs $c_n = c_s = c = 0$.[7] The level of demand in any period, θ_n in market

[5] In order to reduce the transmission losses, transmission tariffs can include a locational and a seasonal component similar to those added to point of connection tariffs. The locational component of the tariff penalizes the injection of electricity into points of the grid that generate high flows of electricity. The seasonal/period-of-day component of the tariff penalizes the transmission of electricity when the losses are larger. Due to the locational and seasonal elements, suppliers face asymmetric linear tariffs. I characterize the equilibrium for symmetric transmission tariffs; however, the model can easily be modified to introduce this type of asymmetries. For a complete analysis of losses in Europe and a complete description of the algorithm implemented to work out power losses, consult the document "ENTSO-E ITC Transit Losses Data Report 2013". For a comparison of European tariff systems, check out the document "ENTSO-E Overview of transmission tariffs in Europe: Synthesis 2014."

[6] The transmission tariffs are linear in electricity markets. However, the model can be modified to assume convex costs. When the transmission costs are convex, the existence of the equilibrium is guaranteed by Dixon (1984).

[7] In this paper, I analyze the effect of transmission capacity constraints and tariffs to access the grid on the equilibrium. In order to focus on this effect, I assume that suppliers are symmetric in capacity

North and θ_s in market South, is independent across markets[8] and independent of market price, i.e., perfectly inelastic. Moreover, $\theta_i \in [\underline{\theta}_i, \overline{\theta}_i] \subseteq [0, k+T]$, $i = n, s$.

The capacity of the transmission line can be lower than the installed capacity in each market $T \leq k$, i.e., the transmission line could be congested for some realization of demands (θ_s, θ_n).

Timing of the game. Having observed the realization of demands $\theta \equiv (\theta_s, \theta_n)$, each supplier simultaneously and independently submits a bid specifying the minimum price at which it is willing to supply up to its capacity, $b_i \leq P$, $i = n, s$, where P denotes the "market reserve price", possibly determined by regulation.[9] Let $b \equiv (b_s, b_n)$ denote a bid profile. On basis of this profile, the auctioneer calls suppliers into operation. If suppliers submit different bids, the capacity of the lower-bidding supplier is dispatched first. If the capacity of the lower-bidding supplier is not sufficient to satisfy total demand, the higher-bidding supplier's capacity, supplier s, is then dispatched to serve residual demand. If the two suppliers submit equal bids, then supplier i is ranked first with probability ρ_i, where $\rho_n + \rho_s = 1$, $\rho_i = 1$ if $\theta_i > \theta_j$, and $\rho_i = \dfrac{1}{2}$ if $\theta_i = \theta_j$, $i = n, s$, $i \neq j$.[10]

The output allocated to supplier $i, i = n, s$, denoted by $q_i(\theta, b)$, is given by

$$q_i(b; \theta, T) = \begin{cases} \min\{\theta_i + \theta_j, \theta_i + T, k_i\} & \text{if } b_i < b_j \\ \rho_i \min\{\theta_i + \theta_j, \theta_i + T, k_i\} + & \\ \qquad [1 - \rho_i] \max\{0, \theta_i - T, \theta_i + \theta_j - k_j\} & \text{if } b_i = b_j \\ \max\{0, \theta_i - T, \theta_i + \theta_j - k_j\} & \text{if } b_i > b_j \end{cases} \tag{1}$$

The output function plays an important role in determining the equilibrium and thus, it is explained in detail. Below, I describe the construction of supplier n's output function; the output for supplier s is symmetric.

The total demand that can be satisfied by supplier n when it submits the lower bid $(b_n < b_s)$ is defined by $\min\{\theta_n + \theta_s, \theta_n + T, k\}$. The realization of (θ_s, θ_n) determines three different areas (left-hand panel in figure 1).

---

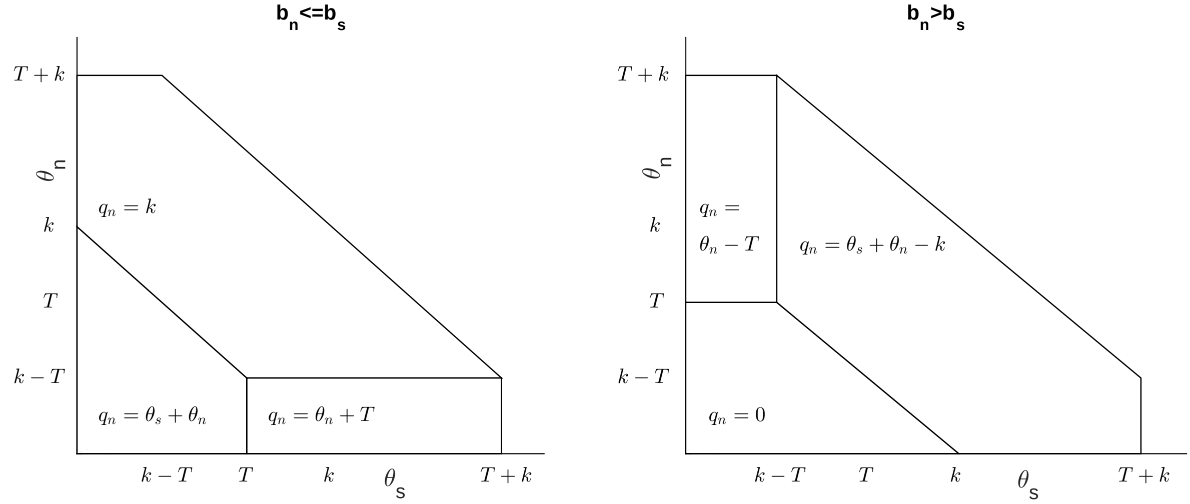
and production costs. The generalization of the equilibrium to introduce asymmetries in capacity and costs complicates the theoretical analysis and the interpretation of the results and is outside the scope of this paper.

[8] In the majority of electricity markets, demand in one market is higher than demand in the other market. Moreover, there exists the possibility of some type of correlation between demands across markets. In this paper, I assume uniform distribution and independence of demand. However, the model can be modified to introduce different distributions of demand and a correlation between demands across markets.

[9] P can be interpreted as the price at which all consumers are indifferent between consuming and not consuming, or a price cap imposed by the regulatory authorities. See von der Fehr and Harbord (1993).

[10] The implemented tie breaking rule is such that if the bids of both suppliers are equal and demand in market i is larger than demand in market j, the auctioneer first dispatches the supplier located in market i. This tie breaking rule minimizes the transmission costs and given that in this model, those costs are the unique ones, it also minimizes the total costs. This tie breaking rule is in line with those used in the literature where the tie breaking rule minimizes the total costs. Moreover, the tie breaking rule ensures the existence of a mixed strategies equilibrium in the Bertrand game with transmission constraints and transmission costs.

Figure 1: Output function for supplier n. ($k_n = k_s = 60, T = 40$)



$$\min\{\theta_n + \theta_s, \theta_n + T, k\} = \begin{cases} \theta_s + \theta_n & \text{if } \theta_n \leq k - \theta_s \text{ and } \theta_s < T \\ \theta_n + T & \text{if } \theta_n < k - T \text{ and } \theta_s > T \\ k & \text{if } \theta_n > k - \theta_s;\ \theta_s \in [0, T] \\ & \text{or if } \theta_n > k - T;\ \theta_s \in [T, k + T] \end{cases}$$

When demand in both markets is low and the transmission line is not congested, supplier n can satisfy total demand $(\theta_s + \theta_n)$. If the demand in market South is larger than the transmission capacity $\theta_s > T$, supplier n cannot satisfy the demand in market South, even when it has enough generation capacity for this; therefore, the total demand that supplier n can satisfy is $(\theta_n + T)$. Finally, if the demand is large enough, the total demand that supplier n can satisfy is its own generation capacity (k).

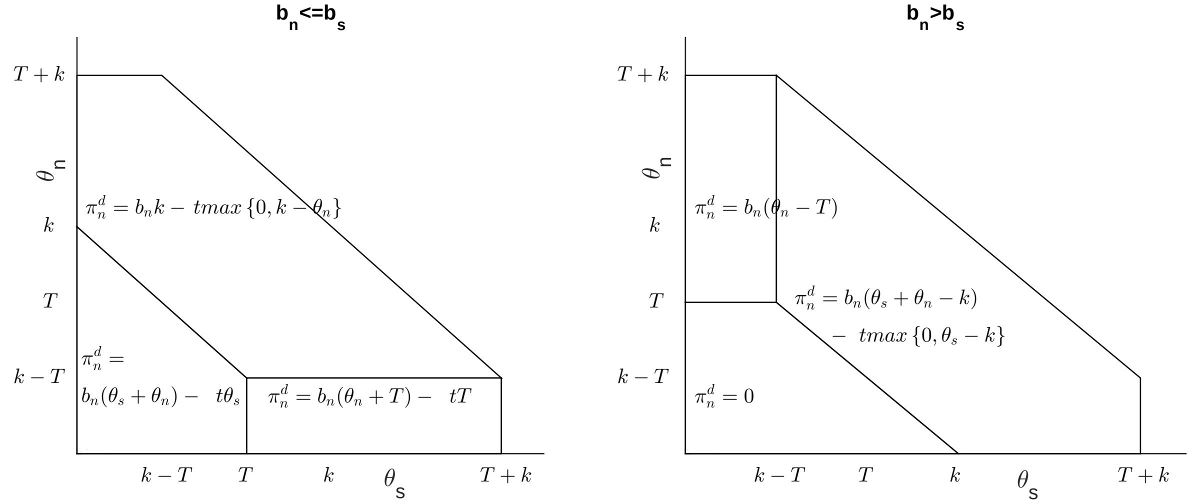
The residual demand that supplier n satisfies when it submits the higher bid $(b_n > b_s)$ is defined by $\max\{0, \theta_n - T, \theta_s + \theta_n - k\}$. The realization of (θ_s, θ_n) determines three different cases (right-hand panel in figure 1).

$$\max\{0, \theta_n - T, \theta_s + \theta_n - k\} = \begin{cases} 0 & \text{if } \theta_n < T;\ \theta_s \in [0, k - T] \\ & \text{or } \theta_n < k - \theta_s;\ \theta_s \in [k - T, k] \\ \theta_n - T & \text{if } \theta_n > T \text{ and } \theta_s \in [0, k - T] \\ \theta_s + \theta_n - k & \text{if } \theta_n > k - \theta_s;\ \theta_s \in [k - T, T + k] \end{cases}$$

When demand in both markets is low and the transmission line is not congested, supplier s satisfies total demand and therefore, the residual demand that remains for supplier n is zero. The total demand that supplier s can satisfy diminishes due to the transmission constraint. As soon as the demand in market North is larger than the transmission capacity $(\theta_n > T)$, it cannot be satisfied by supplier s and thus, some residual demand $(\theta_n - T)$ remains for supplier n. When total demand is large enough, supplier s cannot satisfy total demand and some residual demand $(\theta_s + \theta_n - k)$ remains for supplier n.

Finally, the payments are worked out by the auctioneer. When the auctioneer runs a discriminatory price auction, the price received by a supplier for any positive quantity dispatched by the auctioneer is equal to its own offer price, whenever a bid is wholly or

Figure 2: Profit function for supplier n. ($k_n = k_s = 60, T = 40, t > 0$)



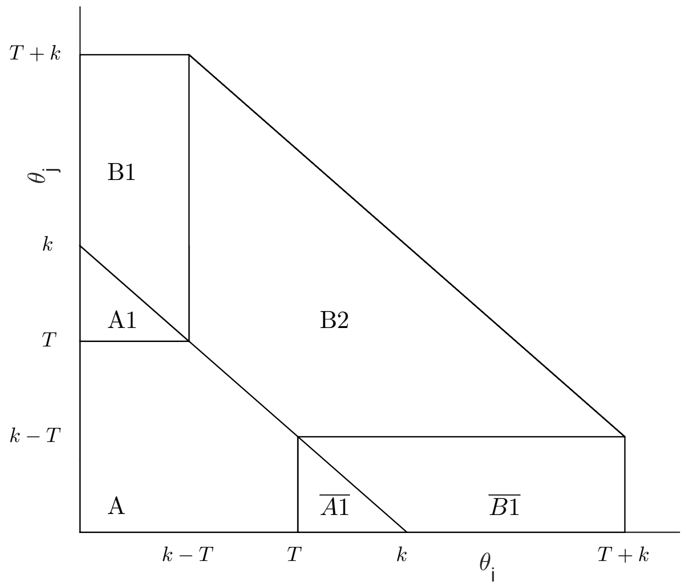
partly accepted. Hence, for a given realization of demands $\theta \equiv (\theta_s, \theta_n)$ and a bid profile $b \equiv (b_s, b_n)$, supplier i's profits, $i = n, s$, can be expressed as

$$\pi_i^d(b;\theta,T,t) = \begin{cases} b_i \min\{\theta_i + \theta_j, \theta_i + T, k\} - \\ \quad t\max\{0, \min\{\theta_j, T, k - \theta_i\}\} & \text{if } b_i \leq b_j \text{ and } \theta_i > \theta_j \\ b_i \max\{0, \theta_i - T, \theta_i + \theta_j - k\} - \\ \quad t\max\{0, \theta_j - k\} & \text{otherwise} \end{cases}$$

Given the relevance of the payoff function determining the equilibrium, I explain it in detail. As for the outcome function, I focus on supplier n's payoff function; the one for supplier s is symmetric. If $b_n \leq b_s$ and $\theta_n > \theta_s$, supplier n is dispatched first and satisfies total demand. Supplier n's payoff function is $\pi_n^d(b;\theta,T) = b_n \min\{\theta_n + \theta_s, \theta_n + T, k\}$. In addition to this expression, due to the transmission tariff, supplier n is charged a transmission tariff t for the power sold in market South. The transmission costs have four different possible values: $t\theta_s$ when the realization of demand in market North is low and the transmission line is not congested; tT when the realization of demand in market North is low and the transmission line is congested; when the realization of demand in market North is high but lower than its generation capacity, the transmission costs are $t(k - \theta_n)$; finally, when demand in market North is larger than the generation capacity k, supplier n cannot sell any electricity in market South and the transmission costs are zero. Hence, after adding the transmission costs, supplier n's payoff is equal to $\pi_n^d(b;\theta,T,t) = b_n \min\{\theta_n + \theta_s, \theta_n + T, k\} - t\ max\{0, min\{\theta_s, T, k - \theta_n\}\}$ (left-hand panel, figure 2).

In the rest of the cases, supplier n is dispatched last and satisfies the residual demand. Supplier n's payoff function is $\pi_n^d(b;\theta,T,t) = b_n \min\{\theta_s + \theta_n, \theta_n + T, k\}$. In addition to this expression, due to the transmission tariff, supplier n is charged a transmission tariff t for the residual demand satisfied in market South. Therefore, after adding the transmission costs, supplier n's payoff is equal to $\pi_n^d(b;\theta,T) = b_n \max\{0, \theta_n - T, \theta_s + \theta_n - k\} - t\max\{0, \theta_s - k\}$ (right-hand panel, figure 2).

Figure 3: *Zero* transmission tariffs. Equilibrium areas ($k_n = k_s = k = 60, T = 40, c = 0$)



3 Effect of transmission capacity constraints

I characterize the equilibrium in the presence of transmission capacity constraints and *zero* transmission tariffs and then I analyze the effect of an increase in transmission capacity.

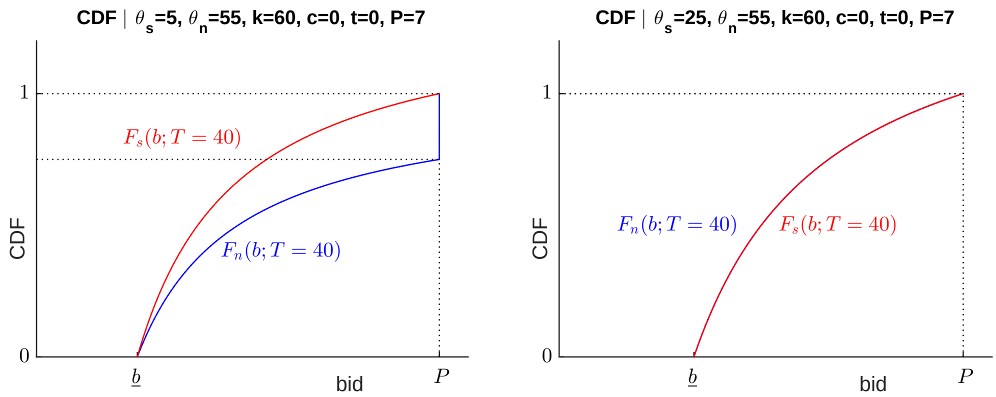
Lemma 1. In the low demand area (area A), the equilibrium is in pure strategies. In the intermediate demand areas (areas $A1, B1$) or in the high demand area (area $B2$), a pure strategies equilibrium does not exist (figure 3).

Proof. In the low demand area (area A), both suppliers have enough capacity to satisfy total demand in both markets and the transmission line is not congested. Therefore, they compete fiercely to be dispatched first in the auction. Hence, the equilibrium is the typical Bertrand equilibrium where both suppliers submit bids equal to their total marginal cost.

In the intermediate demand areas (areas $A1$, $B1$) or in the high demand area (area $B2$), at least one of the suppliers faces a positive residual demand. Therefore, a pure strategies equilibrium does not exist. First, an equilibrium such that $b_i = b_j = c$ does not exist because at least one supplier has the incentive to increase its bid and satisfy the residual demand. Second, an equilibrium such that $b_i = b_j > c$ does not exist because at least one supplier has the incentive to undercut the other to be dispatched first. Finally, an equilibrium such that $b_j > b_i > c$ does not exist because supplier i has the incentive to shade the bid submitted by supplier j. $\square$

A pure strategies equilibrium does not exist in the intermediate or high demand areas. However, the model satisfies the properties established by Dasgupta and Maskin (1986) which guarantee that a mixed strategies equilibrium exists.

Figure 4: *Zero* transmission tariffs. Mixed strategy equilibrium



Lemma 2. In a mixed strategies equilibrium, in the presence of transmission constraints, no supplier submits a bid lower than bid $(\underline{b}_i)$ such that $\underline{b}_i \min\{\theta_i + \theta_j, \theta_i + T, k\} = P \max\{0, \theta_i - T, \theta_i + \theta_j - k\}$. Moreover, the support of the mixed strategies equilibrium for both suppliers is $S = \left[\max\left\{\underline{b}_i, \underline{b}_j\right\}, P\right]$.

Proof. Given that the demand is inelastic, the supplier's profit is maximized when it sets the reservation price. Therefore, the reservation price is the upper-bound of the support.

Each supplier can guarantee for itself the payoff $P \max\{0, \theta_i - T, \theta_i + \theta_j - k\}$, since each supplier can always submit the highest bid and satisfy the residual demand. Therefore, in a mixed strategy equilibrium, no supplier submits a bid that generates a payoff equilibrium lower than $P \max\{0, \theta_i - T, \theta_i + \theta_j - k\}$. Hence, no supplier submits a bid lower than $\underline{b}_i$, where $\underline{b}_i$ solves $\underline{b}_i \min\{\theta_i + \theta_j, \theta_i + T, k\} = P \max\{0, \theta_i - T, \theta_i + \theta_j - k\}$.

No supplier can rationalize submitting a bid lower than $\underline{b}_i, i = n, s$. In the case when $\underline{b}_i = \underline{b}_j$, the support is symmetric. In the case when $\underline{b}_i < \underline{b}_j$, supplier i knows that supplier j never submits a bid lower than $\underline{b}_j$. Therefore, in a mixed strategy equilibrium, supplier i never submits a bid b_i such that $b_i \in \left(\underline{b}_i, \underline{b}_j\right)$, because supplier i can increase its expected payoff choosing a bid b_i such that $b_i \in \left[\underline{b}_j, P\right]$. Hence, the equilibrium strategy support for both suppliers is $S = \left[\max\left\{\underline{b}_i, \underline{b}_j\right\}, P\right]$ □

Using Lemmas one and two, I characterize the equilibrium.

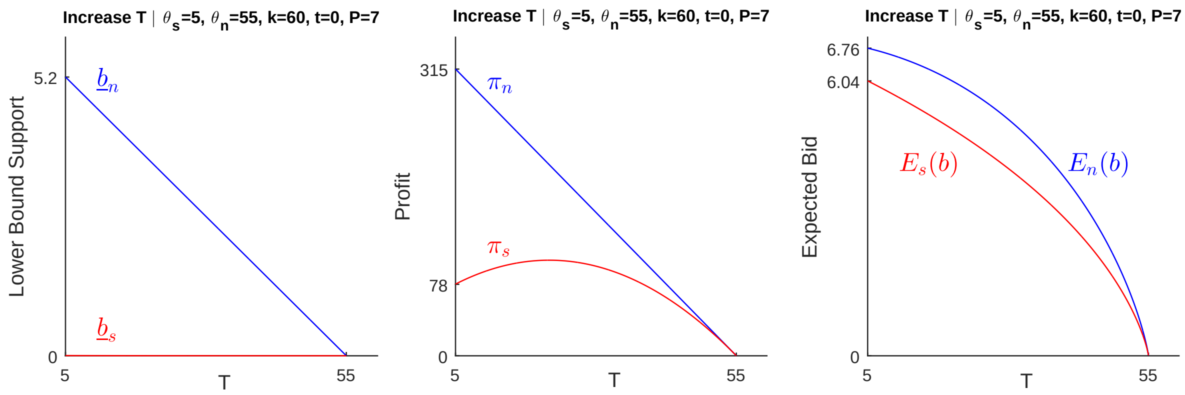
Proposition 1. In the presence of transmission constraints, the characterization of the equilibrium falls into one of the next two categories.

i Low demand (area A). The equilibrium strategies pair is in pure strategies.

ii Intermediate demand (areas $A1$, $B1$) and high demand (area $B2$). The equilibrium strategies pair is in mixed strategies.

In the low demand area, suppliers compete fiercely to be dispatched first in the auction and the equilibrium is the typical Bertrand equilibrium in which both suppliers submit bids equal to their total marginal cost. As soon as the transmission line becomes congested (intermediate demand area), the supplier located in the high-demand market

Figure 5: *Zero* transmission tariffs. Increase in transmission capacity $\triangle T$. Main variables



faces a high residual demand and the supplier located in the low-demand market cannot sell its entire generation capacity. Therefore, the equilibrium is an asymmetric mixed strategies equilibrium where the supplier located in the high-demand market randomizes submitting higher bids with a higher probability, i.e., its cumulative distribution function stochastically dominates the cumulative distribution function of the supplier located in the low-demand market (left-hand panel, figure 4). Finally, in the high demand area, the transmission capacity is not binding, but the generation capacity is. Therefore, both suppliers face the same residual and total demand and the equilibrium is symmetric, i.e., both suppliers randomize using the same cumulative distribution function (right-hand panel, figure 4).

This is in contrast to the models of price competition with generation capacity constraints (Kreps and Scheinkman, 1983; Osborne and Pitchik, 1986; Deneckere and Kovenock, 1996; Fabra et al., 2006) where the results are exclusively driven by generation capacity constraints. In the presence of transmission constraints, there are two relevant constraints that explain the results. If the generation capacity is binding, the equilibrium is symmetric even when the realization of demands is asymmetric. If the transmission capacity is binding, the equilibrium is asymmetric even when the suppliers are symmetric in generation capacity and production costs. Therefore, this model provides a complete and novel analysis of the role played by the structural variables of the model (demand realization, generation capacity and transmission capacity) for equilibrium outcome allocations.

To conclude this section, I analyze the effect of an increase in transmission capacity on the main variables of the model.

Proposition 2. An increase in transmission capacity ($\triangle T$) reduces the lower bound of support $\underline{b}$ and reduces the expected bids for both suppliers (an increase in transmission capacity is pro-competitive). Moreover, an increase in transmission capacity reduces the profit of the supplier located in the high-demand market. However, an increase in transmission capacity modifies the profit of the supplier located in the low-demand market in a non monotonic pattern (table 1 and figure 5).

An increase in transmission tariffs capacity reduces the residual demand and, according to

Table 1: *Zero* transmission tariffs. Increase in transmission capacity $\triangle T$ ($\theta_s = 5, \theta_n = 55, k = 60, c = 0, t = 0, P = 7$). Main variables.

T	$\underline{b}$	π_n	π_s	$E_n(b)$	$E_s(b)$
0	7	385.07	35	7	7
5	5.835	350.1	58.35	6.8963	6.3795
15	4.668	280.08	93.36	6.5587	5.6770
25	3.501	210.06	105.03	5.9261	4.8530
35	2.335	140.1	93.4	4.8981	3.8464
45	1.168	70.08	58.4	3.2589	2.5102
55	0	0	0	0	0

lemma two, the lower bound of the support decreases (left-hand panel, figure 5; column two of table 1). A decrease in the lower bound of the support implies that both suppliers randomize submitting lower bids and therefore, the expected bid decreases for both suppliers (right-hand panel, figure 5; columns five and six of table 1). Finally, an increase in transmission capacity reduces the expected bid and the residual demand of the supplier located in the high-demand market as does its expected profit (central panel, figure 5; column three, table 1.). In contrast, an increase in transmission capacity reduces the expected bid and increases the total demand of the supplier located in the low-demand market. When the transmission capacity is low, the increase in demand dominates the decrease in the expected bid and its expected profit increases. However, when the transmission capacity is large enough, the decrease in bids dominates and its expected profit decreases (central panel, figure 5; column four, table 1).

4 Effect of transmission capacity constraints and transmission tariffs

I characterize the equilibrium in the presence of transmission capacity constraints and *positive* transmission tariffs and then I analyze the effect of an increase in transmission capacity.

Lemma 3. In the low demand area (area A), the equilibrium is in pure strategies. In the intermediate demand area (area $A1$) and when the transmission tariffs are high, the equilibrium is in pure strategies; otherwise, a pure strategies equilibrium does not exist. Moreover, due to the presence of transmission tariffs, the pure strategies equilibrium is asymmetric. In the intermediate demand areas (areas $B1a, B1b$) or in the high-demand areas (area $B2a, B2b$), a pure strategies equilibrium does not exist (figure 6).

Proof. In the low demand area (area A), both suppliers have enough capacity to satisfy total demand and the transmission line is not congested. Therefore, the competition to be dispatched first is fierce. Moreover, the supplier located in the high-demand market (supplier i) faces lower total marginal costs. Hence, the equilibrium is the typical Bertrand equilibrium with asymmetries in costs where the supplier located in the high-demand market extracts the efficiency rents. The *pure strategies equilibrium* is $b_i = b_j = \dfrac{t\theta_i}{\theta_i + \theta_j}$.

The *equilibrium profits* are:

$$\overline{\pi}_j = (\theta_i + \theta_j)\frac{t\theta_i}{\theta_i + \theta_j} - t\theta_i = 0;\ \ \overline{\pi}_i = (\theta_i + \theta_j)\frac{t\theta_i}{\theta_i + \theta_j} - t\theta_j = t(\theta_i - \theta_j) > 0$$

The *equilibrium price* is $\dfrac{t\theta_i}{\theta_i + \theta_j}$

Electricity flows from the high-demand market to the low-demand market, i.e., the electricity losses are minimized.

When the demand belongs to area $A1$, the transmission constraint binds for the supplier located in the low-demand market (supplier j); therefore, only the supplier located in the high-demand market can satisfy total demand. The supplier located in the high-demand market prefers to submit a low bid and extract the efficiency rent instead of submitting a high bid and satisfying the residual demand if $(\theta_i + \theta_j)\dfrac{tT}{\theta_j + T} - t\theta_j \geq P(\theta_i - T)$. In such a case, the pure strategies equilibrium is $b_i = b_j = \dfrac{tT}{\theta_j + T}$, i.e., supplier i prefers to extract the efficiency rents when the transmission tariffs are high enough.

The *equilibrium profits* are:

$$\overline{\pi}_j = (\theta_j + T)\frac{tT}{\theta_j + T} - tT = 0;\ \ \overline{\pi}_i = (\theta_i + \theta_j)\frac{tT}{\theta_j + T} - t\theta_j > 0$$

The *equilibrium price* is $\dfrac{tT}{\theta_i + T}$

The electricity flows from the high-demand market to the low-demand market, i.e., the electricity losses are minimized.

In the rest of the cases, a pure strategies equilibrium does not exist and the proof proceeds as in lemma one. $\square$

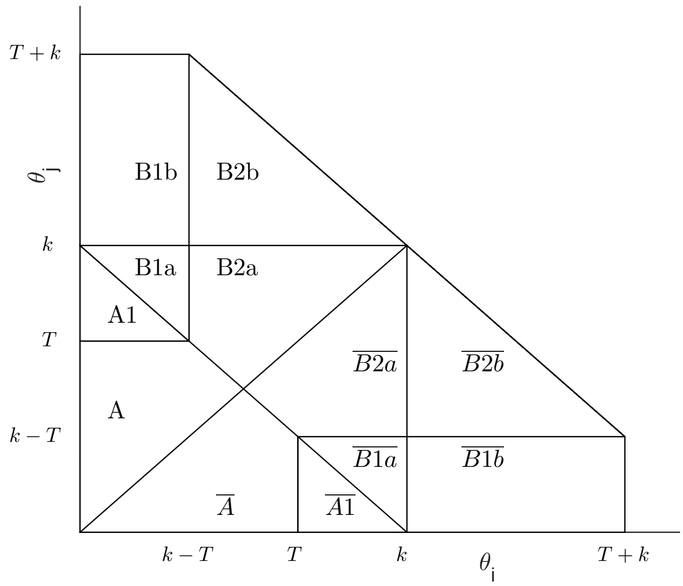
A pure strategy equilibrium does not exist in the intermediate or high-demand areas. However, the implemented tie breaking rule guarantees that the model satisfies the properties established by Dasgupta and Maskin (1986) which ensure that a mixed strategy equilibrium exists.

Lemma 4. In a mixed strategies equilibrium, in the presence of transmission tariffs and *positive* transmission costs, no supplier submits a bid lower than bid ($\underline{b}_i$) such that

$$\begin{aligned}&\underline{b}_i \min\{\theta_i + \theta_j, \theta_i + T, k\} - t\max\{0, min\{\theta_j, T, k - \theta_i\}\} = \\ &P\max\{0, \theta_i - T, \theta_i + \theta_j - k\} - t\max\{0, \theta_j - k\}.\end{aligned}$$

Moreover, the support for the mixed strategies equilibrium for both suppliers is $S = \left[\max\left\{\underline{b}_i, \underline{b}_j\right\}, P\right]$.

Figure 6: *Positive* transmission tariffs. Equilibrium areas ($k_n = k_s = k = 60, T = 40, c = 0, t > 0$)



Proof. The proof proceeds as in lemma two. □

Using lemmas three and four, I characterize the equilibrium.

Proposition 3. In the presence of transmission constraints and transmission tariffs, the characterization of the equilibrium falls into one of the next three categories.

i Low demand (area A). The equilibrium strategies pair is in pure strategies.

ii Intermediate demand (area $A1$). When the transmission tariffs are high, the equilibrium strategies pair is in pure strategies. In contrast, when the transmission tariffs are low, the equilibrium strategies pair is in mixed strategies.

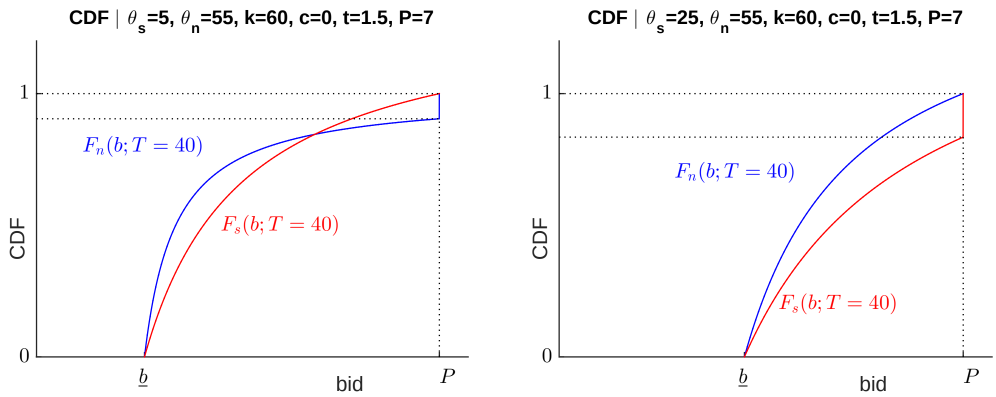
iii Intermediate demand (areas $B1a$, $B1b$) and high demand (areas $B2a$, $B2b$). The equilibrium strategies pair is in mixed strategies.

In the low demand area (area A), suppliers compete fiercely to be dispatched first in the auction and the equilibrium is the typical Bertrand equilibrium with asymmetries in costs where the supplier located in the high-demand market extracts the efficiency rents.

In the intermediate demand area (area $A1$), the transmission capacity binds for the supplier located in the low-demand market; therefore, only the supplier located in the high-demand market can satisfy total demand and the equilibrium crucially depends on the value of the transmission tariffs (low, intermediate or high).[11] If the transmission tariffs are high enough, the supplier located in the high-demand market prefers to submit a low bid to extract the efficiency rents and the equilibrium is in pure strategies. In contrast, when the transmission tariffs are low, the supplier located in the high-demand

[11]Those tariffs are defined in equations 15 and 17 and are summarized in figure 9 in annex two.

Figure 7: *Positive* transmission tariffs. Mixed strategy equilibrium



market prefers to submit a high bid and satisfy the residual demand. In that case, some residual demand remains for the supplier located in the low-demand market, the equilibrium is in mixed strategies and the cumulative distribution function of the supplier located in the high-demand market stochastically dominates the one of the supplier located in the low-demand market, i.e., the equilibrium is similar to that presented in proposition one (left-hand panel, figure 4). Finally, in case of intermediate transmission tariffs, two economic forces drive the result. First, the supplier located in the high-demand market faces a high residual demand and thus, it has incentives to submit high bids (size effect). Second, the supplier located in the high-demand market faces lower total marginal costs and thus, it has incentives to submit low bids to extract the efficiency rents (cost effect). The two effects work in opposite directions and, consequently, one non cumulative distribution function stochastically dominates the other (left-hand panel, figure 7).

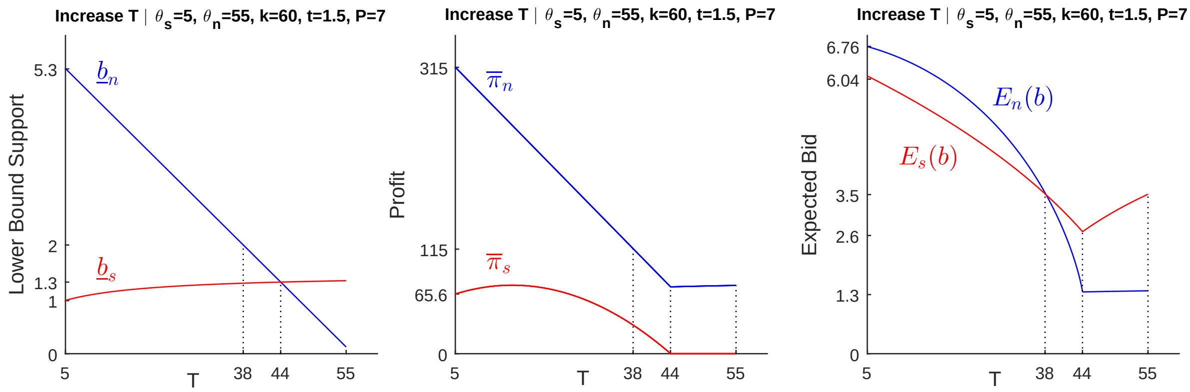
When the demand is intermediate, but larger than in Area $A1$ (areas $B1a$ and $B1b$), the same logic applies and the stochastic or non-stochastic dominance crucially depends on transmission tariffs. Moreover, since both suppliers face a positive residual demand, a pure strategies equilibrium does not exist.

In the high demand areas (areas $B2a$ and $B2b$), the transmission capacity is not binding, but the generation capacity is. Therefore, both suppliers face the same demand. However, due to the transmission tariffs, the supplier located in the high-demand market faces lower total marginal costs and submits lower bids (cost effect). Hence, the cumulative distribution function of the supplier located in the low-demand market stochastically dominates the cumulative distribution function of the supplier located in the high-demand market (right-hand panel, figure 7). This is in contrast to the *zero* transmission tariffs case where both suppliers randomize using the same cumulative distribution function (right-hand panel, figure 4).

Finally, when the demand belongs to the diagonal, both suppliers face the same demand and total marginal costs. If the realization of demands is low, the equilibrium is a symmetric pure strategies equilibrium; otherwise, the equilibrium is a symmetric mixed strategies equilibrium.

To conclude this section, I analyze the effect of an increase in transmission capacity

Figure 8: *Positive* transmission tariffs. Increase in transmission capacity $\triangle T$. Main variables.



on equilibrium outcome allocations.

Proposition 4. An increase in transmission capacity ($\triangle T$) reduces the lower bound of the support of the supplier located in the high-demand market and increases the lower bound of the support of the supplier located in the low-demand market (left-hand panel, figure 8).

i. When the lower bound of the support of the supplier located in the high-demand market is higher than the lower bound of the support of the supplier located in the low-demand market, an increase in transmission capacity reduces the expected bids of both suppliers (an increase in transmission capacity is pro-competitive), reduces the profit of the supplier located in the high-demand market and modifies the profit of the supplier located in the low-demand market in a non-monotonic pattern.

ii. Otherwise, an increase in transmission capacity increases the expected bids of both suppliers (an increase in transmission capacity is anti-competitive), increases the expected profit of the supplier located in the high-demand market and does not modify the expected profit of the supplier located in the low-demand market (table 2; figure 8).

An increase in transmission capacity modifies the market size and the total marginal costs and thus also the suppliers' strategies. First, an increase in transmission capacity increases the size of the market for the supplier located in the low-demand market and thus reduces the residual demand of the supplier located in the high-demand market. Therefore, competition between suppliers becomes more fierce (size effect). Second, an increase in transmission capacity also increases suppliers' total marginal costs because they can sell more electricity into the other market and given that they pay a linear tariff for that electricity, their total marginal costs increase (cost effect). Therefore, an increase in transmission capacity modifies the main variables of the model differently depending on which effect dominates.

When the transmission capacity is sufficiently low ($T \leq 44$ for the numerical examples in table 2 and figure 8), an increase in transmission capacity substantially increases the

Table 2: *Positive* transmission tariffs. Increase in transmission capacity $\triangle T$ ($\theta_s = 5, \theta_n = 55, k = 60, t = 1.5, P = 7$) . Main variables.

T	$\underline{b}$	$\overline{\pi}_n$	$\overline{\pi}_s$	$E_n(b)$	$E_s(b)$
0	7	385.07	35	7	7
5	5.959	350.05	52.09	6.9079	6.4483
15	4.793	280.09	73.36	6.5206	5.7490
25	3.626	210.07	71.28	5.7253	4.9301
35	2.459	140.05	45.86	4.2942	3.9307
45	1.351	73.575	0	1.3569	2.7304
55	1.376	75	0	1.3821	3.5075

competition between suppliers that move from an isolated market scenario to a connected market scenario; simultaneously, and due to the low capacity of the line, an increase in transmission capacity slightly increases the total marginal costs. Hence, the size effect dominates and an increase in transmission capacity induces the same changes in the variables as when the transmission costs are null (proposition two). Therefore, an increase in transmission capacity decreases the lower bound of the support and thus, decreases the expected bid for both suppliers, i.e., an increase in transmission capacity is pro-competitive (right-hand panel, figure 8; columns five and six, table 2); reduces the expected profit of the supplier located in the high-demand market and modifies the profit of the supplier located in the low-demand market in a non-monotonic pattern (central panel, figure 8; columns three and four, table 2).

If the transmission capacity is high enough ($T > 44$), an increase in transmission capacity slightly increases the competition between suppliers, but substantially increases their total marginal costs; therefore, the cost effect dominates. Due to the dominance of the cost effect, an increase in transmission capacity increases the lower bound of the support (left-hand panel, figure 8). An increase in the lower bound of the support entailed that both suppliers randomize submitting higher bids and therefore, the expected bid increases for both suppliers, i.e., an increase in transmission capacity is anti-competitive (right-hand panel, figure 8; columns five and six, table 2). Finally, an increase in transmission capacity increases the expected profit of the supplier located in the high-demand market since it can exploit the efficiency rents more; in contrast, the expected profit of the supplier located in the low-demand market does not change because the increase in profits derived from an increase in the expected bid is compensated by the increase in total marginal costs (central panel, figure 8; columns three and four, table 2).

5 Model comparison and consumer welfare

In this section, I compare equilibrium outcome allocations and their effects on consumer welfare in the presence of transmission constraints when the transmission tariffs are zero and when the suppliers pay a linear transmission tariff. I also compare these results with the equilibrium when suppliers face a point of connection tariff.[12]

[12] In annex four, I have characterized the equilibrium when suppliers face a point of connection tariff.

In the presence of a congested transmission line and zero transmission tariffs (Model I), the supplier located in the high-demand market faces a high residual demand, while the supplier located in the low-demand market cannot sell its entire generation capacity (size effect). Therefore, the supplier located in the high-demand market has incentives to submit higher bids than the supplier located in the low-demand market. Given that the majority of consumers are located in the high-demand market, the aggregate payment that consumers face to acquire electricity is large (column eight, table 3).

When linear transmission tariffs are implemented (Model II), the supplier located in the low-demand market faces high transmission costs and thus, its expected bid is high. In contrast, the supplier located in the high-demand market faces low total marginal costs and for high enough transmission tariffs, it can be more profitable to extract the efficiency rents undercutting the supplier located in the low-demand market (cost effect).[13] These changes in equilibrium prices induce a drastic decrease in the total cost that consumers pay for the purchase of electricity[14] (column eight, table 3). Moreover, the presence of transmission tariffs makes electricity flow from the high to the low-demand market, i.e., the presence of transmission tariffs could reduce the flow of electricity and thus, the transmission losses.

Finally, if suppliers face a point of connection tariff (Model III), they pay the same transmission tariff for the electricity sold in their own market and the electricity sold in the other market. Therefore, the competitive advantage (the cost effect) derived from the location in the high-demand market disappears and equilibrium market outcomes exclusively depend on the size effect. Moreover, given that electricity demand is very inelastic, an increase in suppliers' costs is passed through to consumers that face an increase in equilibrium prices in both markets. Hence, there is a decrease in consumer welfare (column eight, table 3).

The comparison between the three models suggests that the introduction of transmission tariffs could increase aggregate consumer welfare. In contrast, point of connection tariffs always reduce aggregate consumer welfare. However, it is important to emphasize that these results are only valid when the tariffs are symmetric.

The symmetric case is relevant as a benchmark model. In that case, the regulator does not interfere in the market and equilibrium market allocations are only determined by the structural parameters of the model. Moreover, in that scenario, the knowledge of the market and the information required by the regulator to implement those tariffs is minimal. However, as described in the model section, in the majority of countries, point of connection tariffs present some type of asymmetry (seasonal or locational components) to reduce the flow of electricity. Therefore, it is important to characterize the equilibrium when asymmetric transmission and point of connection tariffs are implemented. Moreover, due to the outperformance of symmetric transmission tariffs in terms of consumer welfare and transmission losses, it is a noteworthy economic policy issue to determine whether

[13] As I have previously indicated in the text, this result is only valid when the tariff is high, but not when it is very high.

[14] Downward et al. (2014) found that the introduction of a tax on suppliers' profit induces an increase in consumer welfare. However, in their analysis, the reduction in equilibrium prices is not induced by some type of cost effect similar to that described in this paper, but by a change in firms' strategies to avoid the tax.

Table 3: Effect of the transmission constraint and different tariffs to access the grid on the equilibrium outcome ($\theta_s = 5$, $\theta_n = 55$, $k = 60$, $c = 0$, $P = 7$)

	T	$\underline{b}$	$\overline{\pi}_n$	$\overline{\pi}_s$	$\overline{\pi} = \overline{\pi}_n + \overline{\pi}_s$	$E_n(b)$	$E_s(b)$	$\theta_n E_n(b) + \theta_s E_s(b)$
Model I	40	1.75	105	184	289	4.2	3.2	247
Model II	40	1.87	105	24	129	3.1	3.3	187
Model III	40	2.87	82.5	62	144.5	4.8	4	284

Model I: zero transmission costs. Model II: transmission tariff ($t = 1.5$). Model III: point of connection tariff ($t = 1.5$)

introducing the "correct" asymmetry in point of connection tariffs could lead to the same outcome as when symmetric transmission tariffs are implemented, i.e., asymmetric point of connection tariffs and symmetric transmission tariffs are equivalent.

6 Conclusion

Electricity markets are moving through integration processes around the world. In such a context, there exists an intense debate to analyze the effect of transmission constraints and tariffs to access the grid on suppliers' strategies. The contribution of this paper is to characterize the outcome of an electricity market auction in the presence of transmission constraints and different tariffs to access the grid.

In the presence of transmission constraints and transmission tariffs, there are two economic forces that determine equilibrium market outcomes. First, the supplier located in the high-demand market faces a high residual demand and thus, it has incentives to submit high bids (size effect). Second, the supplier located in the high-demand market faces lower total marginal costs and thus, it has incentives to submit low bids to extract the efficiency rents (cost effect).

When the transmission tariffs are *zero*, only the size effect determines the equilibrium. In that case, the supplier located in the high-demand market faces a high residual demand and it submits higher bids than the supplier located in the low-demand market, i.e., its cumulative distribution function stochastically dominates that of the supplier located in the low-demand market. When transmission tariffs are *positive*, both effects determine the equilibrium and non-stochastic dominance can be established. However, if the transmission tariffs are high enough, the supplier located in the high-demand market submits lower bids than the supplier located in the low-demand market to extract the efficiency rents; given that the majority of consumers are located in that market, consumers' aggregate welfare could increase. In contrast, when suppliers are charged according to the power that they inject into the grid (*point of connection* tariffs), given that they pay the same tariff independent of the market where they are selling electricity, the strategic component of being located in the high-demand market disappears and the equilibrium is only determined by the size effect and the cumulative distribution function of the supplier located in the high-demand market stochastically dominates that of the supplier located in the low-demand market. Moreover, due to the fact that demand is inelastic, the tariff is passed through to consumers that are worse off than in the *zero* transmission tariffs scenario and thereby also worse off than in the *positive* transmission tariffs scenario.

The consequences of an increase in transmission capacity differ considerably due to the transmission tariffs. If the transmission tariffs are *zero*, an increase in transmission capacity is pro-competitive. In contrast, if the transmission tariffs are *positive*, an increase in transmission capacity could be anti-competitive. When *point of connection tariffs* are implemented, an increase in transmission capacity is always pro-competitive.

The models of price competition with capacity constraints are very well-known and provide a complete toolbox to analyze the market performance in many relevant economic situations. The full characterization that I provide in this paper helps us use and adapt price competition models with capacity constraints to analyze relevant economic policy questions on electricity markets affected by transmission constraints, e.g., mergers, investments generation decisions, etcetera.

The size and cost effects described in the paper also appear in models of competition with capacity constraints when the firms face asymmetries in capacity and costs as in the models presented in Kreps and Scheinkman (1983); Osborne and Pitchik (1986); Deneckere and Kovenock (1996) and Fabra et al. (2006). Moreover, due to the size and cost effects, equilibrium firms' cumulative distribution functions do not stochastically dominate each other. This feature of the equilibrium has important implications on prices and consumer welfare. However, our knowledge of these effects is still limited and thus, more study is required to best characterize equilibrium outcome allocations in the presence of size and cost effects.

The symmetric benchmark model presented in the paper is useful to compare equilibrium outcome allocations when different tariffs are implemented. However, this model can easily be modified to analyze models that include some type of seasonal and geographical component in the tariffs. Moreover, due to the outperformance in terms of consumer welfare and transmission losses of symmetric transmission tariffs, it could be relevant to analyze if asymmetric point of connection tariffs could generate the same equilibrium outcomes as symmetric transmission tariffs.

Annex 1. The effect of transmission capacity constraints

Proposition 1. Characterization of the equilibrium in the presence of transmission constraints.

When demand is low (area A, figure 3): $b_n = b_s = c = 0$, the *equilibrium profit* is zero for both firms. No electricity flows through the grid.

When demand is intermediate (areas $A1$ and $B1$, figure 3) or high (area $B2$, figure 3), a pure strategies equilibrium does not exist, as is proved in lemma one; however, the model presented in section two satisfies the properties established by Dasgupta and Maskin (1986) which guarantee that a mixed strategy equilibrium exists. In particular, the discontinuities of $\pi_i, \forall i, j$ are restricted to the strategies such that $b_i = b_j$. Furthermore, it is simple to confirm that by reducing its price from a position where $b_i = b_j$, a firm discontinuously increases its profit. Therefore, $\pi_i(b_i, b_j)$ is everywhere left lower semi-continuous in b_i and hence, weakly lower semi-continuous. Obviously, $\pi_i(b_i, b_j)$ is bounded. Finally, $\pi_i(b_i, b_j) + \pi_j(b_i, b_j)$ is continuous because discontinuous shifts in the clientele from one firm to another only occur where both firms derive the same profit per customer. Therefore, theorem five in Dasgupta and Maskin (1986) applies and hence, a mixed strategy equilibrium exists.

The existence of the equilibrium is guaranteed by Dasgupta and Maskin (1986). However, they did not provide an algorithm to work out the equilibrium. Nevertheless, using the approach proposed by Karlin (1959), Shapley (1957), Shilony (1977), Varian (1980), Kreps and Scheinkman (1984), Osborne and Pitchik (1986), Deneckere and Kovenock (1996) and Fabra et al. (2006), the equilibrium characterization is guaranteed by construction. I use the approach proposed by this branch of the literature to work out the mixed strategy equilibrium. In particular: I first work out the general formulas of the *lower bound of the support*, the *cumulative distribution function*, the *probability distribution function*, the *expected equilibrium price* and the *expected profit*; second, I work out the particular formulas associated with each single area[15] in figure 3.

Lower Bound of the Support. The lower bound of the support is defined according to lemma two.

Cumulative Distribution Function.

In the first step, the payoff function for any firm is:

$$\begin{aligned} \pi_i(b) &= b\left[F_j(b)max\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\} + (1 - F_j(b))min\left\{\theta_i + \theta_j, \theta_i + T, k\right\}\right] = \\ &= -bF_j(b)\left[min\left\{\theta_i + \theta_j, \theta_i + T, k\right\} - max\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\}\right] + \\ &\quad bmin\left\{\theta_i + \theta_j, \theta_i + T, k\right\} \end{aligned} \tag{2}$$

In the second step, $\pi_i(b) = \overline{\pi}_i \forall b \in S_i, i = n, s$, where S_i is the support of the mixed strategies. Then,

[15] The general formulas that I will introduce below fully characterize the equilibrium. However, the equilibrium presents specific characteristics in each single area. In order to fully characterize the equilibrium, I have decided to write down the formulas for each single area.

$$
\begin{aligned}
\overline{\pi}_i &= -bF_j(b)\left[min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right]+ \\
& \quad bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\} \Rightarrow \\
F_j(b) &= \frac{bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-\overline{\pi}_i}{b\left[min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right]} \qquad (3)
\end{aligned}
$$

The third step, at $\underline{b}$, $F_i(\underline{b})=0\forall i=n,s$. Then,

$$
\overline{\pi}_i = \underline{b}min\left\{\theta_i+\theta_j,\theta_i+T,k\right\} \qquad (4)
$$

In the fourth step, plugging 4 into 3, I obtain the mixed strategies for both firms.

$$
\begin{aligned}
F_j(b) &= \frac{bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-\underline{b}min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}}{b\left[min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right]} = \\
&= \frac{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}}{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}}\frac{b-\underline{b}}{b} \quad \forall i=n,s \qquad (5)
\end{aligned}
$$

For further reference:
$L_j(b)=bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\}$,
$H_j(b)=bmax\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}$ and
$C_j=\dfrac{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}}{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}}$.

It is easy to verify that equation $F_j(b)\forall i,j$ is indeed a cumulative distribution function with the following properties. First, in the third step, I have established that $F_j(\underline{b})=0$. Second, $F_j(b)$ is an increasing function in b. At $\underline{b}$, $L_j(\underline{b})=H_j(b)$, for any $b>\underline{b}$, $L_j(\underline{b})<H_j(b)$; moreover, $\dfrac{\partial L_j(b)}{\partial b}>0$, $\dfrac{\partial L_j(\underline{b})}{\partial b}=0$ and $\dfrac{\partial H_j(b)}{\partial b}>0$, therefore, $\dfrac{\partial\left(L_j(b)-L_j(\underline{b})\right)}{\partial b}>\dfrac{\partial\left(L_j(b)-H_j(b)\right)}{\partial b}$; i.e., the numerator increases more than the denominator. It can be show that the cumulative distribution function is increasing using partial derivatives $\dfrac{\partial F_j(b)}{\partial b}=C_j\dfrac{\underline{b}}{b^2}>0$. Third, the cumulative distribution function is concave, $\dfrac{\partial F_j(b)}{\partial b^2}=-C_j\dfrac{\underline{b}}{b^3}<0$. Fourth, $F_j(b)\leq 1\forall b\in S_j$. Fifth, $F_j(b) \lesseqqgtr F_i(b\forall b\in S$; i.e., a stochastic rank can be established between the cumulative distribution functions.[16] Finally, $F_j(b)$ is continuous in the support because $L_j(b)-L_j(\underline{b})$ and $L_j(b)-H_j(b)$ are continuous functions in the support.

Probability Distribution Function.

$$
\begin{aligned}
f_j(b) &= \frac{\partial F_j(b)}{\partial b} \\
&= \frac{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}\underline{b}\left(min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right)}{b^2\left(min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right)^2} \\
&= \frac{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}\underline{b}}{b^2\left(min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right)} \quad \forall i=n,s \qquad (6)
\end{aligned}
$$

[16]Due to the presence of different cases, it is difficult to find a general proof for properties four and five. I will provide proof for both properties for each single realization of the demand below in this proposition.

Expected Equilibrium Bid.

$$\begin{aligned}
E_j(b) &= \int_{\underline{b}}^{P} bf_j(b)\partial b \\
&= \int_{\underline{b}}^{P} \frac{bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\}\underline{b}}{b^2\left(min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right)}\partial b \\
&\quad +P\left(1-F_j(P)\right) \\
&= \frac{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}\underline{b}}{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}}\left[ln(b)\right]_{\underline{b}}^{P} \\
&\quad +P\left(1-F_j(P)\right)\ \ \forall i=n,s
\end{aligned} \tag{7}$$

where $(1-F_j(P))$ in equation 7 is the probability assigned by firm j to the maximum price allowed by the auctioneer.[17]

Expected Profit. The expected profit is defined by equation 4 and is equal to $\overline{\pi}_i = \underline{b}min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}$.

In the rest of the proof, I will work out the *lower bound of the support*, the *cumulative distribution function*, the *probability distribution function*, the *expected equilibrium price* and the *expected profit* for the different possible realizations of demands (θ_s,θ_n).

Area A1.

First, I work out the lower bound of the support on the border between areas $B1$ and $B2$, $\theta_s=k-T$. On the border, $\underline{b}_n$ solves $\underline{b}_n min\left\{\theta_n+\theta_s,\theta_n+T,k\right\}=Pmax\left\{0,\theta_n-T,\theta_s+\theta_n-k\right\}$, therefore $\underline{b}_n=\dfrac{P(\theta_n-T)}{k}$ and $\underline{b}_s$ solves $\underline{b}_s min\left\{\theta_n+\theta_s,\theta_s+T,k\right\}=Pmax\left\{0,\theta_s-T,\theta_s+\theta_n-k\right\}$, therefore $\underline{b}_s=\dfrac{P(\theta_n+\theta_s-k)}{\theta_s+T}$. Plugging the value of θ_s on the border between these areas into $\underline{b}_s$ formula, I obtain $\underline{b}_s=\dfrac{P(\theta_n+k-T-k)}{k-T+T}=\dfrac{P(\theta_n-T)}{k}=\underline{b}_n$. Therefore, on the border between these areas, $\underline{b}_s=\underline{b}_n=\dfrac{P(\theta_n-T)}{k}$.

In areas $A1$ and $B1$, $\underline{b}_n>\underline{b}_s$. In area $A1$, taking partial derivatives $\dfrac{\partial\underline{b}_n}{\partial\theta_s}=\dfrac{-P(\theta_n-T)}{(\theta_n+\theta_s)^2}<0$ and $\dfrac{\partial\underline{b}_s}{\partial\theta_s}=\dfrac{P(k+T-\theta_n)}{(\theta_s+T)^2}>0$. In area $B1$, taking partial derivatives $\dfrac{\partial\underline{b}_n}{\partial\theta_s}=0$ and $\dfrac{\partial\underline{b}_s}{\partial\theta_s}=\dfrac{P(k+T-\theta_n)}{(\theta_s+T)^2}>0$. Therefore, in areas $A1$ and $B1$, $\underline{b}_n>\underline{b}_s$. Hence, $S=\left[max\left\{\underline{b}_n,\underline{b}_s\right\},P\right]=\left[\underline{b}_n,P\right]$. In particular, in area $A1$, $S=\left[\dfrac{P(\theta_n-T)}{(\theta_n+\theta_s)},P\right]$ and

[17] When the transmission line is congested, the mixed strategy equilibrium is asymmetric. In such an equilibrium, the cumulative distribution function for the firm located in the low-demand market is continuous in the upper bound of the support. In contrast, the cumulative distribution function of the firm located in the high-demand region has a mass point in the upper-bound of the support, which means that the firm located in the high-demand market submits the maximum bid allowed by the auctioneer with a positive probability $(1-F_j(P))$. Hence, in order to work out the expected value, in addition to the integral, it is necessary to add the term $P\left(1-F_j(P)\right)$. Figure 4 illustrates these characteristics.

in area $B1$, $S = \left[\frac{P(\theta_n - T)}{k}, P\right]$.

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \frac{\theta_n + \theta_s}{\theta_s + T}\frac{b - \underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \frac{\theta_s + T}{\theta_s + T}\frac{b - \underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Given than $\theta_n + \theta_s \geq T + \theta_s$, it is straightforward to check that $\frac{\theta_s + T}{\theta_s + T}\frac{b - \underline{b}}{b} \leq \frac{\theta_n + \theta_s}{\theta_s + T}\frac{b - \underline{b}}{b} \forall b \in (\underline{b}, P)$. Therefore, $F_n(b) \leq F_s(b) \forall b \in [\underline{b}, P]$, i.e., $F_n(b)$ stochastically dominates $F_s(b)$.

Moreover,

$$\begin{aligned} F_s(P) &= \frac{\theta_n + \theta_s}{\theta_s + T}\frac{P - \frac{P(\theta_n - T)}{\theta_n + \theta_s}}{P} = 1 \\ F_n(P) &= \frac{\theta_s + T}{\theta_s + T}\frac{P - \frac{P(\theta_n - T)}{\theta_n + \theta_s}}{P} = \frac{(\theta_s + T)}{(\theta_n + \theta_s)} < 1 \end{aligned}$$

Third, the probability distribution function is equal to:

$$\begin{aligned} f_s(b) &= \frac{\partial F_s(b)}{\partial b} = \frac{\theta_n + \theta_s}{\theta_s + T}\frac{\underline{b}}{b^2} \\ f_n(b) &= \frac{\partial F_n(b)}{\partial b} = \frac{\theta_s + T}{\theta_s + T}\frac{\underline{b}}{b^2} \end{aligned}$$

Fourth, the expected bid is determined by:

$$\begin{aligned} E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} \frac{\theta_n + \theta_s}{\theta_s + T}\frac{\underline{b}}{b}\partial b = \frac{\theta_n + \theta_s}{\theta_s + T}\underline{b}\left[ln(b)\right]_{\underline{b}}^{P} \\ E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_n)\partial b = \int_{\underline{b}}^{P} \frac{\underline{b}}{b^2}\partial b = \frac{\theta_s + T}{\theta_s + T}\underline{b}\left[ln(b)\right]_{\underline{b}}^{P} + (1 - F_n(P))\,P \end{aligned}$$

Fifth, the expected profit is defined by equation 4 and is equal to $\overline{\pi}_n = \underline{b}(\theta_s + \theta_n)$ and $\overline{\pi}_s = \underline{b}(\theta_s + T)$.

Area B1.

First, the lower bound of the support is $S = \left[\frac{P(\theta_n - T)}{k}, P\right]$.

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \frac{k}{T+k-\theta_n}\frac{b-\underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \frac{\theta_s + T}{T+k-\theta_n}\frac{b-\underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Given than $k \geq T + \theta_s$, it is straightforward to check that $\frac{k}{T+k-\theta_n}\frac{b-\underline{b}}{b} \leq \frac{\theta_s+T}{T+k-\theta_n}\frac{b-\underline{b}}{b} \forall b \in (\underline{b}, P)$. Therefore, $F_n(b) \leq F_s(b) \forall b \in [\underline{b}, P]$, i.e., $F_n(b)$ stochastically dominates $F_s(b)$.

Moreover,

$$\begin{aligned} F_s(P) &= \frac{k}{T+k-\theta_n}\frac{P - \frac{P(\theta_n - T)}{k}}{P} = 1 \\ F_n(P) &= \frac{\theta_s+T}{T+k-\theta_n}\frac{P - \frac{P(\theta_n - T)}{k}}{P} = \frac{\theta_s+T}{k} < 1 \end{aligned}$$

Third, the probability distribution function is equal to:

$$\begin{aligned} f_s(b) &= \frac{\partial F_s(b)}{\partial b} = \frac{k}{T+k-\theta_n}\frac{\underline{b}}{b^2} \\ f_n(b) &= \frac{\partial F_n(b)}{\partial b} = \frac{\theta_s+T}{T+k-\theta_n}\frac{\underline{b}}{b^2} \end{aligned}$$

Fourth, the expected bid is determined by:

$$\begin{aligned} E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} \frac{k}{T+k-\theta_n}\frac{\underline{b}}{b}\partial b = \frac{k}{T+k-\theta_n}\underline{b}\left[ln(b)\right]_{\underline{b}}^{P} \\ E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_n)\partial b = \int_{\underline{b}}^{P} \frac{\theta_s+T}{T+k-\theta_n}\frac{\underline{b}}{b}\partial b + (1 - F_n(P))\, P \\ &= \frac{\theta_s+T}{T+k-\theta_n}\underline{b}\left[ln(b)\right]_{\underline{b}}^{P} + (1 - F_n(P))\, P \end{aligned}$$

Fifth, the expected profit is defined by equation 4 and is equal to $\overline{\pi}_n = \underline{b}k$ and $\overline{\pi}_s = \underline{b}(\theta_s + T)$.

Area B2.

First, the lower bound of the support is $S = [max\{\underline{b}_n, \underline{b}_s\}, P] = \left[\frac{P(\theta_s + \theta_n - k)}{k}, P\right]$.

Second, I work out the cumulative distribution function.

$$F_i(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{k}{2k - \theta_i - \theta_j} \dfrac{b - \underline{b}}{b} & \text{if } b \in (\underline{b}, P) \quad \forall i = s, n \\ 1 & \text{if } b = P \end{cases}$$

Third, the probability distribution function is equal to:

$$f_i(b) = \frac{\partial F_i(b)}{\partial b} = \frac{k}{2k - \theta_i - \theta_j} \frac{\underline{b}}{b^2} \quad \forall i = s, n$$

Fourth, the expected bid is determined by:

$$E_i(b) = \int_{\underline{b}}^{P} b f_i(b_i) \partial b = \int_{\underline{b}}^{P} \frac{k}{2k - \theta_n - \theta_s} \frac{\underline{b}}{b} \partial b = \frac{k}{2k - \theta_n - \theta_s} \underline{b} \left[ln(b)\right]_{\underline{b}}^{P} \quad \forall i = s, n$$

Fifth, the expected profit is defined by equation 4 and is equal to $\overline{\pi}_n = \overline{\pi}_s = \underline{b}k$.

Proposition 2. The effect of an increase in transmission capacity.

Area A1.

$$\frac{\partial \underline{b}}{\partial T} = \frac{-P}{(\theta_s + \theta_n)} < 0$$

$$\frac{\partial F_n(P)}{\partial T} = \frac{1}{(\theta_s + \theta_n)} > 0$$

$$\begin{aligned} \frac{\partial E_n(b)}{\partial T} &= \frac{\partial \underline{b}}{\partial T} \left[ln\left(\frac{P}{\underline{b}}\right)\right] + \underline{b}\left[\frac{\underline{b}}{P} \frac{-\frac{\partial \underline{b}}{\partial T} P}{\underline{b}^2}\right] - \frac{\partial F_n(P)}{\partial T} \\ &= \frac{\partial \underline{b}}{\partial T} \left[ln\left(\frac{P}{\underline{b}}\right) - 1\right] - \frac{\partial F_n(P)}{\partial T} < 0 \Leftrightarrow ln\left(\frac{P}{\underline{b}}\right) > 1 \end{aligned}$$

$$\begin{aligned} \frac{\partial E_s(b)}{\partial T} &= \frac{\partial \underline{b}}{\partial T} \frac{\theta_s + \theta_n}{\theta_s + T} \left[ln\left(\frac{P}{\underline{b}}\right)\right] - \underline{b}\frac{\theta_s + \theta_n}{(\theta_s + T)^2} \left[ln\left(\frac{P}{\underline{b}}\right)\right] + \underline{b}\frac{\theta_s + \theta_n}{\theta_s + T}\left[\frac{\underline{b}}{P} \frac{-\frac{\partial \underline{b}}{\partial T} P}{\underline{b}^2}\right] \\ &= \frac{\partial \underline{b}}{\partial T} \frac{\theta_s + \theta_n}{\theta_s + T} \left[ln\left(\frac{P}{\underline{b}}\right) - 1\right] - \underline{b}\frac{\theta_s + \theta_n}{(\theta_s + T)^2} \left[ln\left(\frac{P}{\underline{b}}\right)\right] < 0 \Leftrightarrow ln\left(\frac{P}{\underline{b}}\right) > 1 \end{aligned}$$

$$\frac{\partial \overline{\pi}_n}{\partial T} = -P < 0$$

$$\frac{\partial \overline{\pi}_s}{\partial T} = \frac{-P}{(\theta_s+\theta_n)}(\theta_s+T) + \frac{P(\theta_n - T)}{(\theta_s+\theta_n)} = \frac{P(\theta_n - 2T - \theta_s)}{(\theta_s+\theta_n)} > 0 \Leftrightarrow \theta_n > 2T + \theta_s$$

Area B1.

$$\frac{\partial \underline{b}}{\partial T} = \frac{-P}{k} < 0$$

$$\frac{\partial F_n(P)}{\partial T} = \frac{1}{k} > 0$$

$$\begin{aligned}\frac{\partial E_n(b)}{\partial T} &= \frac{\partial \underline{b}}{\partial T}\frac{\theta_s+T}{k+T-\theta_n}\left[ln\left(\frac{P}{\underline{b}}\right)\right] + \underline{b}\frac{k+T-\theta_n-\theta_s-T}{(k+T-\theta_n)^2}\left[ln\left(\frac{P}{\underline{b}}\right)\right] \\ &+\underline{b}\frac{\theta_s+T}{k+T-\theta_n}\left[\frac{\underline{b}}{P}\frac{-\frac{\partial \underline{b}}{\partial T}P}{\underline{b}^2}\right] - \frac{\partial F_n(P)}{\partial T} \\ &= \frac{\partial \underline{b}}{\partial T}\frac{\theta_s+T}{k+T-\theta_n}\left[ln\left(\frac{P}{\underline{b}}\right) - 1\right] + \underline{b}\frac{k-\theta_s-\theta_n}{(k+T-\theta_n)^2}\left[ln\left(\frac{P}{\underline{b}}\right)\right] \\ &-\frac{\partial F_n(P)}{\partial T} < 0 \Leftrightarrow ln\left(\frac{P}{\underline{b}}\right) > 1\end{aligned}$$

$$\begin{aligned}\frac{\partial E_s(b)}{\partial T} &= \frac{\partial \underline{b}}{\partial T}\frac{k}{k+T-\theta_n}\left[ln\left(\frac{P}{\underline{b}}\right)\right] - \underline{b}\frac{k}{(k+T-\theta_n)^2}\left[ln\left(\frac{P}{\underline{b}}\right)\right] \\ &+\underline{b}\frac{k}{k+T-\theta_n}\left[\frac{\underline{b}}{P}\frac{-\frac{\partial \underline{b}}{\partial T}P}{\underline{b}^2}\right] \\ &= \frac{\partial \underline{b}}{\partial T}\frac{k}{k+T-\theta_n}\left[ln\left(\frac{P}{\underline{b}}\right) - 1\right] \\ &-\underline{b}\frac{k}{(k+T-\theta_n)^2}\left[ln\left(\frac{P}{\underline{b}}\right)\right] < 0 \Leftrightarrow ln\left(\frac{P}{\underline{b}}\right) > 1\end{aligned}$$

$$\frac{\partial \overline{\pi}_n}{\partial T} = -P < 0$$

$$\frac{\partial \overline{\pi}_s}{\partial T} = \frac{-P}{k}(\theta_s+T) + \frac{P(\theta_n - T)}{k} = \frac{P(\theta_n - 2T - \theta_s)}{k} > 0 \Leftrightarrow \theta_n > 2T + \theta_s$$

Annex 2. The effect of transmission capacity constraints and transmission losses

Proposition 3. Characterization of the equilibrium in the presence of transmission constraints and transmission costs.

The tie breaking rule implemented in the model determines that in case of a tie, the supplier located in the high demand market is dispatched first, i.e., the transmission costs are minimized. Therefore, the model satisfies the properties established by Dasgupta and Maskin (1986) which guaranteed that a mixed strategy equilibrium exists.

I proceed as in proposition one: first, I work out the general formulas of the *lower bound of the support*, the *cumulative distribution function*, the *probability distribution function*, the *expected equilibrium price* and the *expected profit*; second, I work out the particular formulas associated with each single area in figure 6.

Lower Bound of the Support. The lower bound of the support is defined according to lemma four.

Cumulative Distribution Function.

For further reference:

$$\begin{aligned} H_i(\theta,P,T,t) &= max\{0,\theta_i - T, \theta_j + \theta_i - k\} \\ Ht_i(\theta,P,T,t) &= max\{0,\theta_j - k\} \\ L_i(\theta,P,T,t) &= min\{\theta_i + \theta_j, \theta_i + T, k\} \\ Lt_i(\theta,P,T,t) &= max\{0, min\{\theta_i, T, k-\theta_i\}\} \end{aligned}$$

In the first step, the payoff function for any firm is:

$$\begin{aligned} \pi_i(b) &= F_j(b)\left[b\left(H_i(\theta,P,T,t)\right) - t\left(Ht_i(\theta,P,T,t)\right)\right] + \\ &\quad (1-F_j(b))\left[b\left(L_i(\theta,P,T,t)\right) - t\left(Lt_i(\theta,P,T,t)\right)\right] = \\ &= -F_j(b)\left[b\left(L_i(\theta,P,T,t)\right) - t\left(Lt_i(\theta,P,T,t)\right) - b\left(H_i(\theta,P,T,t)\right) + t\left(Ht_i(\theta,P,T,t)\right)\right] \\ &\quad b\left(L_i(\theta,P,T,t)\right) - t\left(Lt_i(\theta,P,T,t)\right) \end{aligned} \tag{8}$$

In the second step, $\pi_i(b) = \overline{\pi}_i \forall b \in S_i, i = n, s$, where S_i is the support of the mixed strategy. Then,

$$\begin{aligned} &= -F_j(b)\left[b\left(L_i(\theta,P,T,t)\right) - t\left(Lt_i(\theta,P,T,t)\right) - b\left(H_i(\theta,P,T,t)\right) + t\left(Ht_i(\theta,P,T,t)\right)\right] \\ &\quad b\left(L_i(\theta,P,T,t)\right) - t\left(Lt_i(\theta,P,T,t)\right) \Rightarrow \\ F_j(b) &= \frac{b\left(L_i(\theta,P,T,t)\right) - t\left(Lt_i(\theta,P,T,t)\right) - \overline{\pi}_i}{b\left[L_i(\theta,P,T,t) - H_i(\theta,P,T,t)\right] - t\left[Lt_i(\theta,P,T,t) - Ht_i(\theta,P,T,t)\right]} \end{aligned} \tag{9}$$

In the third step, at $\underline{b}$, $F_i(\underline{b}) = 0 \forall i = n, s$. Then,

$$\overline{\pi}_i = b\left(L_i(\theta,P,T,t)\right) - t\left(Lt_i(\theta,P,T,t)\right) \tag{10}$$

Fourth step, plugging 10 into 9, I obtain the mixed strategies for both firms.

$$\begin{aligned} F_j(b) &= \frac{(b-\underline{b})L_i(\theta,P,T,t)}{b\,[L_i(\theta,P,T,t)-H_i(\theta,P,T,t)]-t\,[Lt_i(\theta,P,T,t)-Ht_i(\theta,P,T,t)]} = \\ & \forall i = n, s \end{aligned} \tag{11}$$

Probability Distribution Function.

$$\begin{aligned} f_j(b) &= \frac{\partial F_j(b)}{\partial b} \\ &= \frac{L_i(\cdot)\,[\underline{b}\,[L_i(\theta,P,T,t)-H_i(\theta,P,T,t)]-t\,[Lt_i(\theta,P,T,t)-Ht_i(\theta,P,T,t)]]}{[b\,[L_i(\theta,P,T,t)-H_i(\theta,P,T,t)]-t\,[Lt_i(\theta,P,T,t)-Ht_i(\theta,P,T,t)]]^2} \\ & \forall i = n, s \end{aligned} \tag{12}$$

For further reference:

$$\begin{aligned} n(\cdot) &= L_i(\cdot)\,[\underline{b}\,[L_i(\theta,P,T,t)-H_i(\theta,P,T,t)]-t\,[Lt_i(\theta,P,T,t)-Ht_i(\theta,P,T,t)]] \\ d_1(\cdot) &= [L_i(\theta,P,T,t)-H_i(\theta,P,T,t)] \\ d_2(\cdot) &= [Lt_i(\theta,P,T,t)-Ht_i(\theta,P,T,t)] \end{aligned}$$

Expected Equilibrium Bid.

$$\begin{aligned} E_j(b) &= \int_{\underline{b}}^{P} b f_j(b)\partial b \\ &= \int_{\underline{b}}^{P} \frac{b\,(n(\cdot))}{[b\,(d_1(\cdot))-t\,(d_2(\cdot))]^2}\partial b + P(1-F_j(P)) \;\; \forall i = n, s \end{aligned}$$

I solve this equation by substitution of variables. In particular:

$$\begin{aligned} U &= [b\,(d_1(\cdot))-t\,(d_2(\cdot))] \Rightarrow b = \frac{U+t\,(d_2(\cdot))}{d_1(\cdot)} \\ \frac{\partial U}{\partial b} &= d_1 \Rightarrow \partial b = \frac{\partial U}{\partial d_1} \end{aligned}$$

Therefore:

$$\begin{aligned} E_j(b) &= \int_{\underline{b}}^{P} \frac{\left(\frac{U+t\,(d_2(\cdot))}{d_1(\cdot)}\right) n(\cdot)}{U^2} \frac{\partial U}{d_1(\cdot)} + P(1-F_j(P)) \\ &= \frac{n(\cdot)}{d_1(\cdot)}\left[\int_{\underline{b}}^{P} \frac{U\partial U}{U^2} + \int_{\underline{b}}^{P} \frac{t\,(d_2(\cdot))\,\partial U}{U^2}\right] + P(1-F_j(P)) \\ &= \frac{n(\cdot)}{d_1(\cdot)^2}\left[ln(U) - \frac{t\,(d_2(\cdot))}{U}\right]_{\underline{b}}^{P} + P(1-F_j(P)) \end{aligned}$$

Substituting again:

$$\begin{aligned} E_j(b) &= \frac{n(\cdot)}{d_1(\cdot)^2} \\ &\left[ln\left(\frac{P\left(d_1(\cdot)\right)-t\left(d_2(\cdot)\right)}{\underline{b}\left(d_1(\cdot)\right)-t\left(d_2(\cdot)\right)}\right)-\frac{t\left(d_2(\cdot)\right)}{P\left(d_1(\cdot)\right)-t\left(d_2(\cdot)\right)}+\frac{t\left(d_2(\cdot)\right)}{\underline{b}\left(d_1(\cdot)\right)-t\left(d_2(\cdot)\right)}\right] \\ &+P(1-F_j(P)) \end{aligned} \tag{13}$$

In the rest of the proof, I will work out the *lower bound of the support*, the *cumulative distribution function*, the *probability distribution function*, the *expected equilibrium price* and the *expected profit* for the different possible realizations of demands (θ_s, θ_n) (figure 6).

Area A1.

First, the lower bound of the support is:

$$\begin{aligned} \underline{b}_n\theta_n+\underline{b}_n\theta_s-t\theta_s=P(\theta_n-T)\Rightarrow\underline{b}_n &= \frac{P(\theta_n-T)+t\theta_s}{\theta_n+\theta_s} \\ \underline{b}_s\theta_s+\underline{b}_sT-tT=0\Rightarrow\underline{b}_s &= \frac{tT}{\theta_s+T} \end{aligned} \tag{14}$$

For further reference, it is important to work out the transmission cost that equalizes $\underline{b}_s$ and $\underline{b}_n$.

$$\begin{aligned} &\hat{t}\mid\underline{b}_n=\frac{P(\theta_n-T)+\hat{t}\theta_s}{\theta_n+\theta_s}=\frac{\hat{t}T}{\theta_s+T}=\underline{b}_s\Leftrightarrow \\ &\hat{t}=\frac{P(\theta_n-T)(\theta_s+T)}{T(\theta_n+\theta_s)-\theta_s(\theta_s+T)} \end{aligned} \tag{15}$$

Second, I work out the cumulative distribution function.

$$F_s(b)=\begin{cases}0 & \text{if } b<\underline{b}\\ \dfrac{(b-\underline{b})(\theta_n+\theta_s)}{b\left[(\theta_s+\theta_n)-(\theta_n-T)\right]-tmin\left\{\theta_s,k-\theta_n\right\}} & \text{if } b\in(\underline{b},P)\\ 1 & \text{if } b=P\end{cases}$$

$$F_n(b)=\begin{cases}0 & \text{if } b<\underline{b}\\ \dfrac{(b-\underline{b})(\theta_s+T)}{b(\theta_s+T)-tT} & \text{if } b\in(\underline{b},P)\\ 1 & \text{if } b=P\end{cases}$$

Moreover,

$$\begin{aligned} \text{If } \underline{b}_n\geq\underline{b}_s\Rightarrow F_s(P) &= 1 \\ F_n(P) &= \frac{(P(\theta_s+T)-t\theta_s)(\theta_s+T)}{(P(\theta_s+T)-tT)(\theta_s+\theta_n)} \\ \text{If } \underline{b}_n<\underline{b}_s\Rightarrow F_s(P) &= \frac{(P(\theta_s+T)-tT)(\theta_s+\theta_n)}{(P(\theta_s+T)-t\theta_s)(\theta_s+T)} \\ F_n(P) &= 1 \end{aligned} \tag{16}$$

It is easy to check that $F_i(b) \forall i = n, s$ is increasing and concave in the domain of the function.

$$\begin{aligned}
\frac{\partial F_s(b)}{\partial b} &= \frac{(\theta_s+\theta_n)(\underline{b}(\theta_s+T)-t\theta_s)}{(b(\theta_s+\theta_n)-t\theta_s-b(\theta_n-T))^2} > 0 \\
\frac{\partial F_s(b)}{\partial b^2} &= \frac{-2(\theta_s+T)(b(\theta_s+\theta_n)-t\theta_s-b(\theta_n-T)(\theta_s+\theta_n)(\underline{b}(\theta_s+T)-t\theta_s)}{(b(\theta_s+\theta_n)-t\theta_s-b(\theta_n-T))^4} < 0 \\
\frac{\partial F_n(b)}{\partial b} &= \frac{(\theta_s+T)(\underline{b}(\theta_s+T)-tT)}{(b(\theta_s+T)-tT)^2} > 0 \\
\frac{\partial F_n(b)}{\partial b^2} &= \frac{-2(\theta_s+T)(b(\theta_s+T)-tT)(\theta_s+T)(\underline{b}(\theta_s+T)-tT)}{(b(\theta_s+T)-tT)^4} < 0
\end{aligned}$$

To analyze the stochastic dominance relation between $F_s(b)$ and $F_n(b)$, I first need to work out the slope of the cumulative distribution function in the lower bound of the support.

$$\begin{aligned}
\left.\frac{\partial F_s(b)}{\partial b}\right|_{b=\underline{b}} &= \frac{(\theta_s+\theta_n)}{(\underline{b}(\theta_s+\theta_n)-t\theta_s-\underline{b}(\theta_n-T))} \\
\left.\frac{\partial F_n(b)}{\partial b}\right|_{b=\underline{b}} &= \frac{(\theta_s+T)}{(\underline{b}(\theta_s+T)-tT)}
\end{aligned}$$

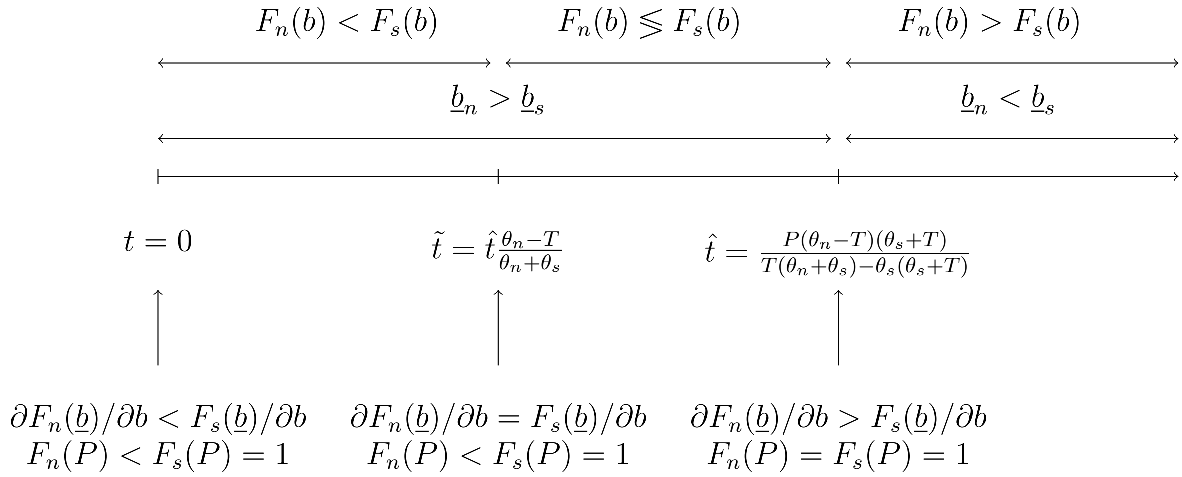
For further reference, I work out the transmission cost that equalizes $\left.\frac{\partial F_s(b)}{\partial b}\right|_{b=\underline{b}}$ and $\left.\frac{\partial F_n(b)}{\partial b}\right|_{b=\underline{b}}$.

$$\begin{aligned}
&\tilde{t} \mid \left.\frac{\partial F_n(b)}{\partial b}\right|_{b=\underline{b}} = \frac{(\theta_s+T)}{\underline{b}(\theta_s+T-tT)} = \frac{(\theta_s+\theta_n)}{(\underline{b}(\theta_s+\theta_n)-t\theta_s-\underline{b}(\theta_n-T))} = \left.\frac{\partial F_s(b)}{\partial b}\right|_{b=\underline{b}} \Leftrightarrow \\
&\tilde{t} = \frac{\underline{b}(\theta_n-T)(\theta_s+T)}{T(\theta_n+\theta_s)-\theta_s(\theta_s+T)} \qquad (17)
\end{aligned}$$

Given that $\underline{b} < P$, it is straightforward to check that $\tilde{t} < \hat{t}$. To work out a close form solution of $\tilde{t}$, it is enough to plug the value of $\underline{b}$ computed in 14 into 17. For the particular values of demand (θ_s, θ_n) that belong to area $A1$, the formula is defined by $\tilde{t} = \hat{t}\frac{(\theta_n - T)}{(\theta_s + \theta_n)}$.

Using the properties of the cumulative distribution function and the value of the slope of the cumulative distribution functions in the lower-bound of the support, the stochastic dominance relation between suppliers' cumulative distribution functions can be established. Using equations 16 and 17 illustrated in figure 9, it is easy to check that when $t \in [0, \tilde{t}]$, the slope of the cumulative distribution function in the lower-bound of the support of the supplier located in the high-demand market is lower than that of the supplier located in the low-demand market. Moreover, the value of the cumulative distribution function in the upper-bound of the support of the supplier located in the high-demand market is lower than that of the supplier located in the low-demand market. Given that the cumulative distribution function is continuous, increasing and concave, it can be inferred that $F_n(b) < F_s(b) \forall b \in [\underline{b}, P]$, i.e., $F_n(b)$ stochastically dominates $F_s(b)$. When

Figure 9: Relation between transmission cost and stochastic dominance



$t \in (\tilde{t}, \hat{t}]$, the slope of the cumulative distribution function in the lower-bound of the support of the supplier located in the high-demand market is higher than that of the supplier located in the low-demand market. Moreover, the value of the cumulative distribution function in the upper-bound of the support is lower for the supplier located in the high-demand market when $t \in (\tilde{t}, \hat{t})$ and it is the same for both suppliers when $t = \hat{t}$. Given that the cumulative distribution function is continuous, increasing and concave, it can neither be concluded that $F_n(b) < F_s(b)$ nor that$F_n(b) > F_s(b) \forall b \in [\underline{b}, P]$, i.e., none of the cumulative distribution functions stochastically dominates the other. Finally, when $t \geq \hat{t}$, using the same procedure as above, it can be established that $F_n(b) > F_s(b) \forall b \in [\underline{b}, P]$, i.e., $F_s(b)$ stochastically dominates $F_n(b)$.

It is important to emphasize that in this model, the change in suppliers' strategies is captured by a smooth change in suppliers' cumulative distribution functions that are continuous in the support. This is in contrast to previous models of price competition with capacity constraints (Deneckere and Kovenock, 1996) where the cumulative distribution function in the support could be discontinuous.

Third, the probability distribution function is equal to:

$$f_s(b) = \frac{\partial F_s(b)}{\partial b} = \frac{(\theta_n + \theta_s)(\underline{b}(\theta_s + T) - t\theta_s)}{(b(\theta_s + T) - t\theta_s)^2}$$
$$f_n(b) = \frac{\partial F_n(b)}{\partial b} = \frac{(\theta_s + T)(\underline{b}(\theta_s + T) - tT)}{(b(\theta_s + T) - tT)^2}$$

Fourth, the expected bid is determined by:

$$
\begin{aligned}
E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{(\theta_n+\theta_s)(\underline{b}(\theta_s+T)-t\theta_s)}{(b(\theta_s+T)-t\theta_s)^2} + (1-F_s(P))P \\
&= \frac{(\theta_n+\theta_s)(\underline{b}(\theta_s+T)-t\theta_s)}{(\theta_s+T)^2} \\
&\quad \left[ln\left(\frac{P(\theta_s+T)-t\theta_s}{\underline{b}(\theta_s+T)-t\theta_s}\right) - \frac{t\theta_s}{P(\theta_s+T)-t\theta_s} + \frac{t\theta_s}{\underline{b}(\theta_s+T)-t\theta_s}\right] \\
&\quad +(1-F_s(P))P \\
E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{(\theta_s+T)(\underline{b}(\theta_s+T)-tT)}{(b(\theta_s+T)-tT)^2} + (1-F_n(P))P \\
&= \frac{(\underline{b}(\theta_s+T)-tT)}{(\theta_s+T)} \\
&\quad \left[ln\left(\frac{P(\theta_s+T)-tT}{\underline{b}(\theta_s+T)-tT}\right) - \frac{tT}{P(\theta_s+T)-tT} + \frac{tT}{\underline{b}(\theta_s+T)-tT}\right] \\
&\quad +(1-F_n(P))P \qquad (18)
\end{aligned}
$$

In equation 18, I have solved this by substituting the variables:

$$
\begin{aligned}
U &= b(\theta_s+T)-t\theta_s \Rightarrow b = \frac{U+t\theta_s}{\theta_s+T} \\
\frac{\partial U}{\partial b} &= \theta_s+T \Rightarrow \partial b = \frac{\partial U}{\theta_s+T} \\
&\text{and} \\
U &= b(\theta_s+T)-tT \Rightarrow b = \frac{U+tT}{\theta_s+T} \\
\frac{\partial U}{\partial b} &= \theta_s+T \Rightarrow \partial b = \frac{\partial U}{\theta_s+T}
\end{aligned}
$$

Fifth, the expected profit is defined by equation 10 and is equal to $\overline{\pi}_n = \underline{b}(\theta_s+\theta_n)-t\theta_s$ and $\overline{\pi}_s = \underline{b}(\theta_s+T)-tT$.

Area B1a.

First, the lower bound of the support is:

$$
\begin{aligned}
\underline{b}_n\theta_n + \underline{b}_n(k-\theta_n) - t(k-\theta_n) = P(\theta_n-T) \Rightarrow \underline{b}_n &= \frac{P(\theta_n-T)+t(k-\theta_n)}{k} \\
\underline{b}_s\theta_s + \underline{b}_sT - tT = P(\theta_s+\theta_n-k) \Rightarrow \underline{b}_s &= \frac{P(\theta_s+\theta_n-k)+tT}{\theta_s+T} \qquad (19)
\end{aligned}
$$

For further reference, it is important to work out the transmission cost that equalizes $\underline{b}_s$ and $\underline{b}_n$.

$$
\begin{aligned}
&\hat{t} \mid \underline{b}_n = \frac{P(\theta_n-T)+\hat{t}(k-\theta_n)}{k} = \frac{P(\theta_s+\theta_n-k)+\hat{t}T}{\theta_s+T} = \underline{b}_s \Leftrightarrow \\
&\hat{t} = \frac{P[(\theta_n-T)(\theta_s+T)-k(\theta_s+\theta_n-k)]}{kT-(k-\theta_n)(\theta_s+T)} \qquad (20)
\end{aligned}
$$

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})k}{b(k+T-\theta_n)-t(k-\theta_n)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})(\theta_s+T)}{b(k+T-\theta_n)-tT} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned} \text{If } \underline{b}_n \geq \underline{b}_s \Rightarrow F_s(P) &= 1 \\ F_n(P) &= \frac{(P(k+T-\theta_n)-t(k-\theta_n))(\theta_s+T)}{(P(k+T-\theta_n)-tT)k} \\ \text{If } \underline{b}_n < \underline{b}_s \Rightarrow F_s(P) &= \frac{(P(k+T-\theta_n)-tT)k}{(P(k+T-\theta_n)-t(k-\theta_n))(\theta_s+T)} \\ F_n(P) &= 1 \end{aligned}$$

It is easy to check that $F_i(b) \forall i = n, s$ is increasing and concave in the domain of the cumulative distribution function.

$$\begin{aligned} \frac{\partial F_s(b)}{\partial b} &= \frac{(\theta_s+T)(\underline{b}(\theta_s+T)-tT)}{(b(\theta_s+T)-tT)^2} > 0 \\ \frac{\partial F_s(b)}{\partial b^2} &= \frac{-2(b(\theta_s+T)-tT)(\theta_s+T)(\theta_s+T)(\underline{b}(\theta_s+T)-tT)}{(b(\theta_s+T)-tT)^4} < 0 \\ \frac{\partial F_n(b)}{\partial b} &= \frac{k(\underline{b}(k+T-\theta_n)-t(k-\theta_n)}{(b(k+T-\theta_n)-t(k-\theta_n))^2} > 0 \\ \frac{\partial F_n(b)}{\partial b^2} &= \frac{-2((b(k+T-\theta_n)-t(k-\theta_n))(k+T-\theta_n)(k(\underline{b}(k+T-\theta_n)-t(k-\theta_n)))}{(b(k+T-\theta_n)-t(k-\theta_n))^4} < 0 \end{aligned}$$

To analyze the stochastic dominance relation between $F_s(b)$ and $F_n(b)$, I first need to work out the slope of the cumulative distribution function in the lower bound of the support.

$$\begin{aligned} \left.\frac{\partial F_s(b)}{\partial b}\right|_{b=\underline{b}} &= \frac{k}{(\underline{b}k-t(k-\theta_n)-\underline{b}(\theta_n-T))} \\ \left.\frac{\partial F_n(b)}{\partial b}\right|_{b=\underline{b}} &= \frac{(\theta_s+T)}{(\underline{b}(\theta_s+T)-tT))-\underline{b}(\theta_s+\theta_n-k)} \end{aligned}$$

For further reference, I work out the transmission cost that equalizes $\left.\dfrac{\partial F_s(b)}{\partial b}\right|_{b=\underline{b}}$ and $\left.\dfrac{\partial F_n(b)}{\partial b}\right|_{b=\underline{b}}$.

$$\begin{aligned}
\tilde{t} \mid \left.\frac{\partial F_n(b)}{\partial b}\right|_{b=\underline{b}} &= \frac{(\theta_s+T)}{(\underline{b}(\theta_s+T)-tT))-\underline{b}(\theta_s+\theta_n-k)} = \\
& \frac{k}{(\underline{b}k-t(k-\theta_n)-\underline{b}(\theta_n-T))} = \left.\frac{\partial F_s(b)}{\partial b}\right|_{b=\underline{b}} \Leftrightarrow \\
\tilde{t} &= \frac{\underline{b}[(\theta_n-T)(\theta_s+T)-k(\theta_s+\theta_n-k)]}{kT-(k-\theta_n)(\theta_s+T)} \qquad (21)
\end{aligned}$$

Given that $\underline{b} < P$, it is straightforward to check that $\tilde{t} < \hat{t}$. To work out a close form solution of $\tilde{t}$, it is enough to plug in the value of $\underline{b}$ computed in 19 into 21. For the particular values of demand (θ_s, θ_n) that belong to area $B1a$, the formula is defined by $\tilde{t} = \dfrac{P(\theta_n-T)((\theta_n-T)(\theta_s+T)-k(\theta_s+\theta_n-k))}{k(kT-(k-\theta_n)(\theta_s+T))-(k-\theta_n)((\theta_n-T)(\theta_s+T)-k(\theta_n+\theta_s-k)}$.

Using the same arguments as in area $A1$, it is easy to establish the stochastic dominance rank between the cumulative distribution functions. When $t \in [0, \tilde{t}]$, $F_n(b)$ stochastically dominates $F_s(b)$; when $t \in (\tilde{t}, \hat{t})$, a non-stochastic dominance rank can be established; finally, when $t \leq \hat{t}$, $F_s(b)$ stochastically dominates $F_n(b)$.

Third, the probability distribution function is equal to:

$$f_s(b) = \frac{\partial F_s(b)}{\partial b} = \frac{k(\underline{b}(k+T-\theta_n)-t(k-\theta_n))}{(b(k+T-\theta_n)-t(k-\theta_n))^2}$$
$$f_n(b) = \frac{\partial F_n(b)}{\partial b} = \frac{(\theta_s+T)(\underline{b}(\theta_s+T)-tT)}{(b(\theta_s+T)-tT)^2}$$

Fourth, the expected bid is determined by:

$$\begin{aligned}
E_s(b) &= \int_{\underline{b}}^{P} bf_s(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{k(\underline{b}(k+T-\theta_n)-t(k-\theta_n))}{(b(k+T-\theta_n)-t(k-\theta_n))^2} + (1-F_s(P))P \\
&= \frac{k(\underline{b}(k+T-\theta_n)-t(k-\theta_n))}{(k+T-\theta_n)^2} \\
& \left[ln\left(\frac{P(k+T-\theta_n)-t(k-\theta_n)}{\underline{b}(k+T-\theta_n)-t(k-\theta_n)}\right)\right] \\
& \left[-\frac{t(k-\theta_n)}{P(k+T-\theta_n)-t(k-\theta_n)} + \frac{t(k-\theta_n)}{\underline{b}(k+T-\theta_n)-t(k-\theta_n)}\right] \\
& +(1-F_s(P))P \\
E_n(b) &= \int_{\underline{b}}^{P} bf_n(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{(\theta_s+T)(\underline{b}(k+T-\theta_n)-tT)}{(b(k+T-\theta_n)-tT)^2} + (1-F_n(P))P \\
&= \frac{(\theta_s+T)(\underline{b}(k+T-\theta_n)-tT)}{(k+T-\theta_n)^2} \\
& \left[ln\left(\frac{P(k+T-\theta_n)-tT}{\underline{b}(k+T-\theta_n)-tT}\right) - \frac{tT}{P(k+T-\theta_n)-tT} + \frac{tT}{\underline{b}(k+T-\theta_n)-tT}\right] \\
& +(1-F_n(P))P \qquad (22)
\end{aligned}$$

In equations 22, I have solved by substituting the variables:

$$\begin{aligned}
U &= b(k+T-\theta_n) - t(k-\theta_n) \Rightarrow b = \frac{U + t(k-\theta_n)}{k+T-\theta_n} \\
\frac{\partial U}{\partial b} &= k+T-\theta_n \Rightarrow \partial b = \frac{\partial U}{k+T-\theta_n} \\
\text{and} & \\
U &= b(k+T-\theta_n) - tT \Rightarrow b = \frac{U+tT}{k+T-\theta_n} \\
\frac{\partial U}{\partial b} &= k+T-\theta_n \Rightarrow \partial b = \frac{\partial U}{k+T-\theta_n}
\end{aligned}$$

Fifth, the expected profit is defined by equation 10 and is equal to $\overline{\pi}_n = \underline{b}k - t(k-\theta_n)$ and $\overline{\pi}_s = \underline{b}(\theta_s + T) - tT$.

Area B1b.

First, the lower bound of the support is:

$$\begin{aligned}
\underline{b}_n k = P(\theta_n - T) \Rightarrow \underline{b}_n &= \frac{P(\theta_n - T)}{k} \\
\underline{b}_s\theta_s + \underline{b}_s T - tT = P(\theta_s + \theta_n - k) - t(\theta_n - k) \Rightarrow \underline{b}_s &= \frac{P(\theta_s+\theta_n-k) + t(k+T-\theta_n)}{\theta_s + T}
\end{aligned} \tag{23}$$

For further reference, it is important to work out the transmission cost that equalizes $\underline{b}_s$ and $\underline{b}_n$.

$$\begin{aligned}
&\hat{t} \mid \underline{b}_n = \frac{P(\theta_n - T)}{k} = \frac{P(\theta_s+\theta_n-k) + \hat{t}T - \hat{t}(\theta_n - k)}{\theta_s+T} = \underline{b}_s \Leftrightarrow \\
&\hat{t} = \frac{P[(\theta_n - T)(\theta_s+T) - k(\theta_s+\theta_n-k)]}{kT - k(\theta_n - k)}
\end{aligned} \tag{24}$$

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})k}{b(k+T-\theta_n)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})(\theta_s+T)}{b(k+T-\theta_n) - t(T+k-\theta_n)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned}
\text{If } \underline{b}_n \geq \underline{b}_s \Rightarrow F_s(P) &= 1 \\
F_n(P) &= \frac{P(k+T-\theta_n)(\theta_s+T)}{(P-t)(k+T-\theta_n)k} \\
\text{If } \underline{b}_n < \underline{b}_s \Rightarrow F_s(P) &= \frac{(P-t)(k+T-\theta_n)k}{P(k+T-\theta_n)(\theta_s+T)} \\
F_n(P) &= 1
\end{aligned}$$

It is easy to check that $F_i(b) \forall i = n, s$ is increasing and concave in the domain of the cumulative distribution function.

$$\begin{aligned}
\frac{\partial F_s(b)}{\partial b} &= \frac{\underline{b}k(k+T-\theta_n)}{(b(k+T-\theta_n))^2} > 0 \\
\frac{\partial F_s(b)}{\partial b^2} &= \frac{-2b(k+T-\theta_n)^2 \underline{b}k(k+T-\theta_n)}{(b(k+T-\theta_n))^4} < 0 \\
\frac{\partial F_n(b)}{\partial b} &= \frac{(k+T-\theta_n)(\theta_s+T)(\underline{b}-t)}{((k+T-\theta_n)(b-t))^2} > 0 \\
\frac{\partial F_n(b)}{\partial b^2} &= \frac{-2((k+T-\theta_n)^2(b-t))(k+T-\theta_n)(\theta_s+T)(\underline{b}-t)}{((k+T-\theta_n)(b-t))^4} < 0
\end{aligned}$$

To analyze the stochastic dominance relation between $F_s(b)$ and $F_n(b)$, I need to work out first the slope of the cumulative distribution function in the lower bound of the support.

$$\begin{aligned}
\left.\frac{\partial F_s(b)}{\partial b}\right|_{b=\underline{b}} &= \frac{k}{(\underline{b}k - \underline{b}(\theta_n - T))} \\
\left.\frac{\partial F_n(b)}{\partial b}\right|_{b=\underline{b}} &= \frac{(\theta_s+T)}{(\underline{b}(\theta_s+T)-tT)) - (\underline{b}(\theta_s+\theta_n-k)-t(\theta_n-k))}
\end{aligned}$$

For further reference, I work out the transmission cost that equalizes $\left.\frac{\partial F_s(b)}{\partial b}\right|_{b=\underline{b}}$ and $\left.\frac{\partial F_n(b)}{\partial b}\right|_{b=\underline{b}}$.

$$\begin{aligned}
\tilde{t} \mid \left.\frac{\partial F_n(b)}{\partial b}\right|_{b=\underline{b}} &= \frac{(\theta_s+T)}{(\underline{b}(\theta_s+T)-tT)) - (\underline{b}(\theta_s+\theta_n-k)-t(\theta_n-k))} = \\
& \frac{k}{(\underline{b}k - \underline{b}(\theta_n - T))} = \left.\frac{\partial F_s(b)}{\partial b}\right|_{b=\underline{b}} \Leftrightarrow \\
\tilde{t} &= \frac{\underline{b}[(\theta_n-T)(\theta_s+T)-k(\theta_s+\theta_n-k)]}{kT-k(\theta_n-k)} \qquad (25)
\end{aligned}$$

Given that $\underline{b} < P$, it is straightforward to check that $\tilde{t} < \hat{t}$. To work out a close form solution of $\tilde{t}$, it is enough to plug the value of $\underline{b}$ computed in 23 into 25. For the particular values of demand (θ_s, θ_n) that belong to area $B1b$, the formula is defined by $\tilde{t} = \hat{t}\frac{(\theta_n - T)}{k}$.

Using the same arguments as in areas $A1$ and $B1a$, it is easy to establish the stochastic dominance rank between the cumulative distribution functions. When $t \in [0, \tilde{t}]$, $F_n(b)$ stochastically dominates $F_s(b)$; when $t \in (\tilde{t}, \hat{t})$, a non-stochastic dominance rank can be established; finally, when $t \leq \hat{t}$, $F_s(b)$ stochastically dominates $F_n(b)$.

Third, the probability distribution function is equal to:

$$
\begin{aligned}
f_s(b) = \frac{\partial F_s(b)}{\partial b} &= \frac{\underline{b}k}{b^2(k + T - \theta_n)} \\
f_n(b) = \frac{\partial F_n(b)}{\partial b} &= \frac{(\underline{b} - t)(\theta_s + T)}{(b - t)^2(k + T - \theta_n)}
\end{aligned}
$$

Fourth, the expected bid is determined by:

$$
\begin{aligned}
E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{\underline{b}k}{b^2(k + T - \theta_n)} + (1 - F_s(P))P \\
&= \frac{\underline{b}k}{(k + T - \theta_n)}\left[ln\left(\frac{P}{\underline{b}}\right)\right] + (1 - F_s(P))P \\
E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{(\underline{b} - t)(\theta_s + T)}{(b - t)^2(k + T - \theta_n)} + (1 - F_n(P))P \\
&= \frac{(\underline{b} - t)(\theta_s + T)}{(k + T - \theta_n)}\left[ln\left(\frac{P - t}{\underline{b} - t}\right) - \frac{t}{P - t} + \frac{t}{\underline{b} - t}\right] + (1 - F_n(P))P \qquad (26)
\end{aligned}
$$

In equations 26, I have solved by substituting the variables:

$$
\begin{aligned}
U &= b - t \Rightarrow b = U + t \\
\frac{\partial U}{\partial b} &= 1 \Rightarrow \partial b = \partial U
\end{aligned}
$$

Fifth, the expected profit is defined by equation 10 and is equal to $\overline{\pi}_n = \underline{b}k$ and $\overline{\pi}_s = \underline{b}(\theta_s + T) - tT$.

Area $B2a$.

First, the lower bound of the support is:

$$
\begin{aligned}
\underline{b}_n\theta_n + \underline{b}_n(k - \theta_n) - t(k - \theta_n) = P(\theta_s + \theta_n - k) \Rightarrow \underline{b}_n &= \frac{P(\theta_s + \theta_n - k) + t(k - \theta_n)}{k} \\
\underline{b}_s\theta_s + \underline{b}_s(k - \theta_s) - t(k - \theta_s) = P(\theta_s + \theta_n - k) \Rightarrow \underline{b}_s &= \frac{P(\theta_s + \theta_n - k) + t(k - \theta_s)}{k}
\end{aligned}
$$

In area $B2a$, $\theta_n > \theta_s$. Hence, it is straightforward to show that

$$
\underline{b}_n = \frac{P(\theta_s + \theta_n - k) + t(k - \theta_n)}{k} \leq \frac{P(\theta_s + \theta_n - k) + t(k - \theta_s)}{k} = \underline{b}_s \forall t \geq 0.
$$

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})k}{b(2k-\theta_n-\theta_s)-t(k-\theta_n)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$
$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})k}{b(2k-\theta_n-\theta_s)-t(k-\theta_s)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned} F_s(P) &= \frac{P(2k-\theta_n-\theta_s)-t(k-\theta_s)}{P(2k-\theta_n-\theta_s)-t(k-\theta_n)} \\ F_n(P) &= 1 \end{aligned}$$

It is easy to check that $F_i(b) \forall i = n, s$ is increasing and concave in the domain of the cumulative distribution function.

$$\begin{aligned} \frac{\partial F_s(b)}{\partial b} &= \frac{\underline{b}k(k-\theta_n) - tk(k-\theta_n) + \underline{b}k(k-\theta_s)}{(b(2k-\theta_n-\theta_s)-t(k-\theta_n))^2} > 0 \\ \frac{\partial F_s(b)}{\partial b^2} &= \frac{-2(2k-\theta_n-\theta_s)(b(2k-\theta_n-\theta_s)-t(k-\theta_n))[\underline{b}k(k-\theta_n) - tk(k-\theta_n) + \underline{b}k(k-\theta_s)]}{(b(2k-\theta_n-\theta_s)-t(k-\theta_n))^4} < 0 \\ \frac{\partial F_n(b)}{\partial b} &= \frac{\underline{b}k(k-\theta_s) - tk(k-\theta_s) + \underline{b}k(k-\theta_n)}{(b(2k-\theta_n-\theta_s)-t(k-\theta_s))^2} > 0 \\ \frac{\partial F_n(b)}{\partial b^2} &= \frac{-2(2k-\theta_n-\theta_s)(b(2k-\theta_n-\theta_s)-t(k-\theta_s))[\underline{b}k(k-\theta_s) - tk(k-\theta_s) + \underline{b}k(k-\theta_n)]}{(b(2k-\theta_n-\theta_s)-t(k-\theta_s))^4} < 0 \end{aligned}$$

Given that $\theta_n > \theta_s$, it is easy to check that

$$F_n(b) = \frac{(b-\underline{b})k}{b(2k-\theta_n-\theta_s)-t(k-\theta_s)} \geq \frac{(b-\underline{b})k}{b(2k-\theta_n-\theta_s)-t(k-\theta_n)} = F_s(b) \forall b \in [0, P],$$

when $t \geq 0$,

i.e., $F_s(b)$ stochastically dominates $F_n(b)$.

Third, the probability distribution is equal to:

$$\begin{aligned} f_s(b) = \frac{\partial F_s(b)}{\partial b} &= \frac{k(\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_n))}{(b(2k-\theta_n-\theta_s)-t(k-\theta_n))^2} \\ f_n(b) = \frac{\partial F_n(b)}{\partial b} &= \frac{k(\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_s))}{(b(2k-\theta_n-\theta_s)-t(k-\theta_s))^2} \end{aligned}$$

Fourth, the expected bid is determined by:

$$
\begin{aligned}
E_s(b) &= \int_{\underline{b}}^{P} bf_s(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{k(\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_n))}{(b(2k-\theta_n-\theta_s)-t(k-\theta_n))^2} + (1-F_s(P))P \\
&= \frac{k(\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_n))}{(b(2k-\theta_n-\theta_s)-t(k-\theta_n))^2} \\
&\quad \left[ln\left(\frac{P(2k-\theta_n-\theta_s)-t(k-\theta_n)}{\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_n)}\right)\right] \\
&\quad \left[-\frac{t(k-\theta_n)}{P(k+T-\theta_n)-t(k-\theta_n)} + \frac{t(k-\theta_n)}{\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_n)}\right] \\
&\quad +(1-F_s(P))P \\
E_n(b) &= \int_{\underline{b}}^{P} bf_n(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{k(\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_s))}{(b(2k-\theta_n-\theta_s)-t(k-\theta_s))^2} + (1-F_n(P))P \\
&= \frac{k(\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_s))}{(b(2k-\theta_n-\theta_s)-t(k-\theta_s))^2} \\
&\quad \left[ln\left(\frac{P(2k-\theta_n-\theta_s)-t(k-\theta_s)}{\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_s)}\right)\right] \\
&\quad \left[-\frac{t(k-\theta_s)}{P(k+T-\theta_n)-t(k-\theta_s)} + \frac{t(k-\theta_s)}{\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_s)}\right] \\
&\quad +(1-F_n(P))P
\end{aligned}
\tag{27}
$$

where in equation 27, I have solved by substituting variables:

$$
\begin{aligned}
U &= b(2k-\theta_n-\theta_s)-t(k-\theta_n) \Rightarrow b = \frac{U+t(k-\theta_n)}{2k-\theta_n-\theta_s} \\
\frac{\partial U}{\partial b} &= 2k-\theta_n-\theta_s \Rightarrow \partial b = \frac{\partial U}{2k-\theta_n-\theta_s} \\
&\text{and} \\
U &= b(2k-\theta_n-\theta_s)-t(k-\theta_s) \Rightarrow b = \frac{U+t(k-\theta_s)}{2k-\theta_n-\theta_s} \\
\frac{\partial U}{\partial b} &= 2k-\theta_n-\theta_s \Rightarrow \partial b = \frac{\partial U}{2k-\theta_n-\theta_s}
\end{aligned}
$$

Fifth, the expected profit is defined by equation 10 and is equal to $\overline{\pi}_n = \underline{b}k - t(k-\theta_n)$ and $\overline{\pi}_s = \underline{b}k - t(k-\theta_s)$.

Area B2b.

First, the lower bound of the support is:

$$
\begin{aligned}
\underline{b}_n k = P(\theta_s+\theta_n-k) \Rightarrow \underline{b}_n &= \frac{P(\theta_s+\theta_n-k)}{k} \\
\underline{b}_s\theta_s + \underline{b}_s(k-\theta_s) - t(k-\theta_s) &= \\
P(\theta_s+\theta_n-k) - t(\theta_n-k) \Rightarrow \underline{b}_s &= \frac{P(\theta_s+\theta_n-k)+t(2k-\theta_n-\theta_s)}{k}
\end{aligned}
$$

It is straightforward to show that

$$\underline{b}_n = \frac{P(\theta_s + \theta_n - k)}{k} \leq \frac{P(\theta_s + \theta_n - k) + t(2k - \theta_n - \theta_s)}{k} = \underline{b}_s \forall t \geq 0.$$

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b - \underline{b})k}{b(2k - \theta_n - \theta_s)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b - \underline{b})k}{(b - t)(2k - \theta_n - \theta_s)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned} F_s(P) &= \frac{P(2k - \theta_n - \theta_s) - t(2k - \theta_n - \theta_s)}{P(2k - \theta_n - \theta_s)} \\ F_n(P) &= 1 \end{aligned}$$

It is easy to check that $F_i(b) \forall i = n, s$ is increasing and concave in the domain of the cumulative distribution function.

$$\begin{aligned} \frac{\partial F_s(b)}{\partial b} &= \frac{\underline{b}k(2k - \theta_n - \theta_s)}{(b(2k - \theta_n - \theta_s))^2} > 0 \\ \frac{\partial F_s(b)}{\partial b^2} &= \frac{-2(b(2k - \theta_n - \theta_s))(2k - \theta_n - \theta_s)[\underline{b}k(2k - \theta_n - \theta_s)]}{(b(2k - \theta_n - \theta_s))^4} < 0 \\ \frac{\partial F_n(b)}{\partial b} &= \frac{(\underline{b} - t)k(2k - \theta_n - \theta_s)}{((b - t)(2k - \theta_n - \theta_s))^2} > 0 \\ \frac{\partial F_n(b)}{\partial b^2} &= \frac{-2((b - t)(2k - \theta_n - \theta_s))(2k - \theta_n - \theta_s)[(\underline{b} - t)k(2k - \theta_n - \theta_s)]}{((b - t)(2k - \theta_n - \theta_s))^4} < 0 \end{aligned}$$

It is easy to check that

$$F_n(b) = \frac{(b - \underline{b})k}{(b - t)(2k - \theta_n - \theta_s)} \geq \frac{(b - \underline{b})k}{b(2k - \theta_n - \theta_s)} = F_s(b) \forall b \in [0, P], \text{ when } t \geq 0,$$

i.e., $F_s(b)$ stochastically dominates $F_n(b)$.

Third, the probability distribution function is equal to:

$$f_s(b) = \frac{\partial F_s(b)}{\partial b} = \frac{\underline{b}k}{b^2(2k - \theta_n - \theta_s)}$$

$$f_n(b) = \frac{\partial F_n(b)}{\partial b} = \frac{(\underline{b} - t)k}{(b - t)^2(2k - \theta_n - \theta_s)}$$

Fourth, the expected bid is determined by:

$$\begin{aligned}
E_s(b) &= \int_{\underline{b}}^{P} bf_s(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{\underline{b}k}{b^2(2k-\theta_n-\theta_s)} + (1-F_s(P))P \\
&= \frac{\underline{b}k}{(2k-\theta_n-\theta_s)}\left[ln\left(\frac{P}{\underline{b}}\right)\right] + (1-F_s(P))P \\
E_n(b) &= \int_{\underline{b}}^{P} bf_n(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{(\underline{b}-t)k}{(b-t)^2(2k-\theta_n-\theta_s)} + (1-F_n(P))P \\
&= \frac{(\underline{b}-t)k}{(2k-\theta_n-\theta_s)}\left[ln\left(\frac{P-t}{\underline{b}-t}\right) - \frac{t}{P-t} + \frac{t}{\underline{b}-t}\right] + (1-F_n(P))P \qquad (28)
\end{aligned}$$

where in equations 28, I have solved by substituting the variables:

$$\begin{aligned}
U &= b-t \Rightarrow b = U+t \\
\frac{\partial U}{\partial b} &= 1 \Rightarrow \partial b = \partial U
\end{aligned}$$

Fifth, the expected profit is defined by equation 10 and is equal to $\overline{\pi}_n = \underline{b}k$ and $\overline{\pi}_s = \underline{b}k - t(k-\theta_s)$.

Proposition 4. The effect of an increase in transmission capacity.

In the presence of transmission capacity constraints and transmission costs, the "size" and "cost" mechanisms determine the equilibrium. These two mechanisms work in opposite directions which has important implications for equilibrium outcome allocations. Hence, an increase in transmission capacity modifies the relevant model variables (lower bound of the support, expected bids and expected profits) in a non-monotonic pattern. Therefore, no clear conclusions can be obtained through the analysis of the partial derivatives.

In this section, I present the static comparative in order to illustrate the difficulties to obtain a formal analysis from the analytical solutions. I present the results for area $A1$, the analysis is the same for the rest of the areas.

Area A1.

$$\begin{aligned}
\frac{\partial \underline{b}_n}{\partial T} &= \frac{-P}{(\theta_s+\theta_n)} < 0 \\
\frac{\partial \underline{b}_s}{\partial T} &= \frac{t(\theta_s+T)-tT}{(\theta_s+T)^2} = \frac{t\theta_s}{(\theta_s+T)^2} > 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial F_n(P)}{\partial T} &= \frac{(2P(\theta_s+T)-t\theta_s)\left((P(\theta_s+T)-tT)(\theta_n+\theta_s)\right)}{\left((P(\theta_s+T)-tT)(\theta_n+\theta_s)\right)^2} + \\
&\quad \frac{t(\theta_n+\theta_s)(P(\theta_s+T)-t\theta_s)(\theta_s+T)}{\left((P(\theta_s+T)-tT)(\theta_n+\theta_s)\right)^2} > 0
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial E_n(b)}{\partial T} = \; & \frac{\frac{\partial \underline{b}}{\partial T}(\theta_s + T) + (\underline{b} - t)(\theta_s + T) - \underline{b}(\theta_s + T) + tT}{(\theta_s + T)^2} \\
& \left[ln\left(\frac{P(\theta_s + T) - tT}{\underline{b}(\theta_s + T) - tT}\right) - \frac{tT}{P(\theta_s + T) - tT} + \frac{tT}{\underline{b}(\theta_s + T) - tT}\right] + \\
& \frac{\underline{b}(\theta_s + T) - tT}{\theta_s + T} \\
& \left[\frac{\underline{b}(\theta_s + T) - tT}{P(\theta_s + T) - tT}\right] \\
& \left[\frac{(P - t)(\underline{b}(\theta_s + T) - tT) - \left(\frac{\partial \underline{b}}{\partial T}(\theta_s + T) + \underline{b} - t\right)(P(\theta_s + T) - tT)}{(\underline{b}(\theta_s + T) - tT)^2}\right] + \\
& \frac{\underline{b}(\theta_s + T) - tT}{\theta_s + T}\left[-\frac{t(P(\theta_s + T) - tT) - (P - t)tT}{(P(\theta_s + T) - tT)^2}\right] + \\
& \frac{\underline{b}(\theta_s + T) - tT}{\theta_s + T}\left[\frac{t(\underline{b}(\theta_s + T) - tT) - \left(\frac{\partial \underline{b}}{\partial T}(\theta_s + T) + \underline{b} - t\right)tT}{(\underline{b}(\theta_s + T) - tT)^2}\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E_s(b)}{\partial T} = \; & \frac{\frac{\partial \underline{b}}{\partial T}(\theta_s + T)^3(\theta_s + \theta_n) + \underline{b}(\theta_n + \theta_s)(\theta_s + T)^2 - 2(\theta_s + T)\left[(\theta_s + \theta_n)(\underline{b}(\theta_s + T) - t\theta_s)\right]}{(\theta_s + T)^4} \\
& \left[ln\left(\frac{P(\theta_s + T) - t\theta_s}{\underline{b}(\theta_s + T) - t\theta_s}\right) - \frac{t\theta_s}{P(\theta_s + T) - t\theta_s} + \frac{t\theta_s}{\underline{b}(\theta_s + T) - t\theta_s}\right] + \\
& \frac{(\theta_n + \theta_s)(\underline{b}(\theta_s + T) - tT)}{(\theta_s + T)^2} \\
& \left[\frac{(\underline{b}(\theta_s + T) - t\theta_s)}{P(\theta_s + T) - t\theta_s}\right] \\
& \left[\frac{P(\underline{b}(\theta_s + T) - t\theta_s) - \left(\frac{\partial \underline{b}}{\partial T}(\theta_s + T) + \underline{b}\right)(P(\theta_s + T) - t\theta_s)}{(\underline{b}(\theta_s + T) - t\theta_s)^2}\right] + \\
& \frac{(\theta_n + \theta_s)(\underline{b}(\theta_s + T) - t\theta_s)}{(\theta_s + T)^2}\left[-\frac{Pt\theta_s}{(P(\theta_s + T) - t\theta_s)^2}\right] + \\
& \frac{(\theta_n + \theta_s)(\underline{b}(\theta_s + T) - t\theta_s)}{\theta_s + T}\left[\frac{-\underline{b}t\theta_s - \left(\frac{\partial \underline{b}}{\partial T}(\theta_s + T)t\theta_s\right)}{(\underline{b}(\theta_s + T) - t\theta_s)^2}\right]
\end{aligned}
$$

$$
\frac{\partial \pi_n}{\partial T} = -P < 0
$$

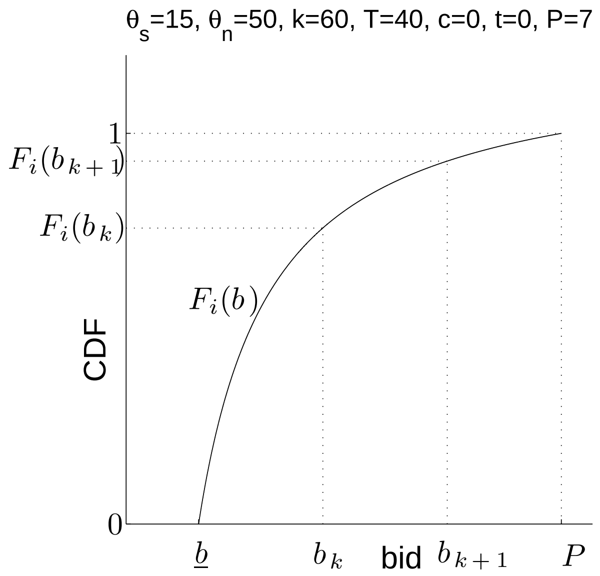
$$
\begin{aligned}
\frac{\partial \overline{\pi}_s}{\partial T} &= \frac{-P}{(\theta_s+\theta_n)}(\theta_s+T)+\frac{P(\theta_n-T)+t\theta_s}{(\theta_s+\theta_n)}-t \\
&= \frac{P(\theta_n-2T-\theta_s)-t\theta_n}{(\theta_s+\theta_n)}
\end{aligned}
$$

Annex 3. Expected equilibrium price: Simulation

Propositions one and three fully characterize the equilibrium. However, due to the complexity of calculations and to ensure that I did not make any algebra mistake, I work out the expected bid for both firms using the algorithm presented in this annex. The algorithm is based on the cumulative distribution function that is the mixed strategies equilibrium from which the rest of the variables of the model are derived.

The differences between the expected bid using the analytical formulas from propositions one and three and using the algorithm proposed here are almost null.[18]

Figure 10: Expected bid. Simulation.



Algorithm: (figure 10)

1. I split the support of the mixed strategies equilibrium into K grid values (where K is a large number e.g., 5000 or 10000). I call each of these values $b_i(k)$ $\forall i = s, n$.

2. For each $b_i(k)$, I work out $F_i(b_i(k))$ using the formulas obtained in propositions one and three.

[18] I have applied this algorithm to work out the expected value for any realization of demand (all areas) and I have compared this with the analytical values and the results are almost identical.

3. The probability assigned to $p_i(b_i(k))$ equals the difference in the cumulative distribution function between two consecutive values $F_i(b_i(k+1)) - F_i(b_i(k))$. Therefore, $p(b_i(k)) = F_i(b_i(k+1)) - F_i(b_i(k))$. It is important to remark that one observation is lost during the process to work out the probabilities.

4. The expected value is the sum of each single bid multiplied by its probability: $E_i(b) = \sum_{k=0}^{K-1} b_i(k)p_i(b_i(k))\ \forall i = s, n$

Annex 4. Characterization of the Nash Equilibrium when firms pay a point of connection tariff

In this paper, I assume that suppliers face transmission constraints and that they are charged by a linear transmission tariff for the electricity sold in the other market. Under this assumption, I show that suppliers' strategies are affected by the "size" and the "cost" effects that work in the opposite direction and determine equilibrium outcome allocations. However, when suppliers face transmission constraints and these are charged on basis of the total electricity that they inject into the grid (point of connection tariff), the suppliers pay the same transmission tariff for the electricity sold in their own market and the electricity sold in the other market. Therefore, the competitive advantage (cost effect) derived from the location in the high-demand market disappears and equilibrium market outcomes exclusively depend on the size effect. Moreover, given that electricity demand is very inelastic, an increase in generation costs is passed through to consumers that face an increase in equilibrium prices in both markets. This result is in line with the pass through literature (Marion and Muehlegger 2011; Fabra and Reguant 2014). Hence, a change in the design of transmission tariffs from the design used in the majority of the countries to the one proposed in this article could induce a large improvement in consumer welfare.

The general formulas of the *lower bound of the support*, the *cumulative distribution function*, the *probability distribution function*, the *expected equilibrium price* and the *expected profit* can be worked out using the same approach as those in annexes one and two. In this annex, I only work out the particular formulas associated with each single area (figure 3). Once that I have characterized the equilibrium, I analyze the effect of an increase in transmission capacity on the main variables of the model. Finally, I compare the equilibrium outcome of the three model specifications: transmission constraints and *zero* transmission costs (model I); transmission constraints and *positive* transmission costs for the electricity sold in the other market (model II) and finally transmission constraints and *positive* transmission costs for the entire generation capacity (model III).

Area $A1$.

First, I work out the lower bound of the support. Using the same approach as in annex one, it is straightforward to show that in areas $A1$ and $B1$, $\underline{b}_n > \underline{b}_s$. Hence, $S = [max\{\underline{b}_n, \underline{b}_s\}, P] = [\underline{b}_n, P]$. Therefore, it is enough to work out $\underline{b}_n$. $\underline{b}_n$ can be derived from the next equation $(\underline{b}_n - t)(\theta_n + \theta_s) = (P - t)(\theta_n - T)$. Therefore, in area $A1$, $S = \left[t + \dfrac{(P-t)(\theta_n - T)}{(\theta_n + \theta_s)}, P\right]$.

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{\theta_n + \theta_s}{\theta_s + T}\dfrac{b - \underline{b}}{b - t} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{\theta_s + T}{\theta_s + T}\dfrac{b - \underline{b}}{b - t} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned} F_s(P) &= \frac{\theta_n + \theta_s}{\theta_s + T}\frac{P - t - \dfrac{(P-t)(\theta_n - T)}{\theta_n + \theta_s}}{P - t} = 1 \\ F_n(P) &= \frac{\theta_s + T}{\theta_s + T}\frac{P - t - \dfrac{P - t(\theta_n - T)}{\theta_n + \theta_s}}{P - t} = \frac{(\theta_s + T)}{(\theta_n + \theta_s)} < 1 \end{aligned}$$

Third, the probability distribution function is equal to:

$$\begin{aligned} f_s(b) &= \frac{\partial F_s(b)}{\partial b} = \frac{\theta_n + \theta_s}{\theta_s + T}\frac{\underline{b} - t}{(b-t)^2} \\ f_n(b) &= \frac{\partial F_n(b)}{\partial b} = \frac{\theta_s + T}{\theta_s + T}\frac{\underline{b} - t}{(b-t)^2} \end{aligned}$$

Fourth, the expected bid is determined by:

$$\begin{aligned} E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{\theta_n + \theta_s}{\theta_s + T}\frac{(\underline{b} - t)}{(b-t)^2}\partial b = \\ &\quad \frac{\theta_n + \theta_s}{\theta_s + T}(\underline{b} - t)\left[ln\left(\frac{P - t}{\underline{b} - t}\right) - \frac{t}{P - t} + \frac{t}{\underline{b} - t}\right] \\ E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_n)\partial b = \int_{\underline{b}}^{P} b\frac{\underline{b} - t}{(b-t)^2}\partial b + (1 - F_n(P))\,P = \\ &\quad (\underline{b} - t)\left[ln\left(\frac{P - t}{\underline{b} - t}\right) - \frac{t}{P - t} + \frac{t}{\underline{b} - t}\right] + (1 - F_n(P))\,P \end{aligned} \tag{29}$$

In equation 29, I have solved by substituting variables:

$$\begin{aligned} U &= b - t \Rightarrow b = U + t \\ \frac{\partial U}{\partial b} &= 1 \Rightarrow \partial b = \partial U \end{aligned}$$

Fifth, the expected profit is defined by $\overline{\pi}_n = (\underline{b} - t)(\theta_s + \theta_n)$ and $\overline{\pi}_s = (\underline{b} - t)(\theta_s + T)$.

*Area B*1.

First, the lower bound of the support is $S = \left[t + \frac{(P-t)(\theta_n - T)}{k}, P\right]$.

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{k}{T+k-\theta_n}\dfrac{b-\underline{b}}{b-t} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{\theta_s+T}{T+k-\theta_n}\dfrac{b-\underline{b}}{b-t} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned} F_s(P) &= \frac{k}{T+k-\theta_n}\frac{(P-t) - \dfrac{(P-t)(\theta_n - T)}{k}}{P-t} = 1 \\ F_n(P) &= \frac{\theta_s+T}{T+k-\theta_n}\frac{(P-t) - \dfrac{(P-t)(\theta_n - T)}{k}}{P-t} = \frac{\theta_s+T}{k} < 1 \end{aligned}$$

Third, the probability distribution function is equal to:

$$\begin{aligned} f_s(b) &= \frac{\partial F_s(b)}{\partial b} = \frac{k}{T+k-\theta_n}\frac{\underline{b}-t}{(b-t)^2} \\ f_n(b) &= \frac{\partial F_n(b)}{\partial b} = \frac{\theta_s+T}{T+k-\theta_n}\frac{\underline{b}-t}{(b-t)^2} \end{aligned}$$

Fourth, the expected bid is determined by:

$$\begin{aligned} E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{k}{T+k-\theta_n}\frac{\underline{b}-t}{(b-t)^2}\partial b = \\ &\quad \frac{k}{T+k-\theta_n}(\underline{b}-t)\left[ln\left(\frac{P-t}{\underline{b}-t}\right) - \frac{t}{P-t} + \frac{t}{\underline{b}-t}\right] \\ E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_n)\partial b = \int_{\underline{b}}^{P} b\frac{\theta_s+T}{T+k-\theta_n}\frac{\underline{b}-t}{(b-t)^2}\partial b + (1 - F_n(P))\,P \\ &= \frac{\theta_s+T}{T+k-\theta_n}(\underline{b}-t)\left[ln\left(\frac{P-t}{\underline{b}-t}\right) - \frac{t}{P-t} + \frac{t}{\underline{b}-t}\right] + (1 - F_n(P))\,P \qquad (30) \end{aligned}$$

In equation 30, I have solved by substituting the variables:

$$\begin{aligned} U &= b - t \Rightarrow b = U + t \\ \frac{\partial U}{\partial b} &= 1 \Rightarrow \partial b = \partial U \end{aligned}$$

Table 4: The effect of the transmission constraint and the transmission costs on the equilibrium outcome ($\theta_s = 5$, $\theta_n = 55$, $k = 60$, $c = 0$, $P = 7$)

	T	$\underline{b}$	$\overline{\pi}_n$	$\overline{\pi}_s$	$\overline{\pi}$	$E_n(b)$	$E_s(b)$	$\theta_n E_n(b) + \theta_s E_s(b)$
Model I	40	1.75	105	184	289	4.2	3.2	247
Model II	40	1.87	105	24	129	3.1	3.3	187
Model III	40	2.87	82.5	62	144.5	4.8	4	284

Fifth, the expected profit is defined by $\overline{\pi}_n = (\underline{b} - t)k$ and $\overline{\pi}_s = (\underline{b} - t)(\theta_s + T)$.

Area B2.

First, the lower bound of the support is $S = [max\left\{\underline{b}_n, \underline{b}_s\right\}, P] = \left[t + \frac{(P-t)(\theta_s + \theta_n - k)}{k}, P\right]$.

Second, I work out the cumulative distribution function.

$$F_i(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \frac{k}{2k - \theta_i - \theta_j} \frac{b - \underline{b}}{b - t} & \text{if } b \in (\underline{b}, P) \quad \forall i = s, n \\ 1 & \text{if } b = P \end{cases}$$

Third, the probability distribution function is equal to:

$$f_i(b) = \frac{\partial F_i(b)}{\partial b} = \frac{k}{2k - \theta_i - \theta_j} \frac{\underline{b} - t}{(b-t)^2} \quad \forall i = s, n$$

Fourth, the expected bid is determined by:

$$\begin{aligned} E_i(b) &= \int_{\underline{b}}^{P} b f_i(b_i) \partial b = \int_{\underline{b}}^{P} b \frac{k}{2k - \theta_n - \theta_s} \frac{\underline{b} - t}{(b-t)^2} \partial b = \\ & \frac{k}{2k - \theta_n - \theta_s} (\underline{b} - t) \left[ln\left(\frac{P-t}{\underline{b} - t}\right) - \frac{t}{P-t} + \frac{t}{\underline{b} - t}\right] + (1 - F_n(P))\, P \quad (31) \end{aligned}$$

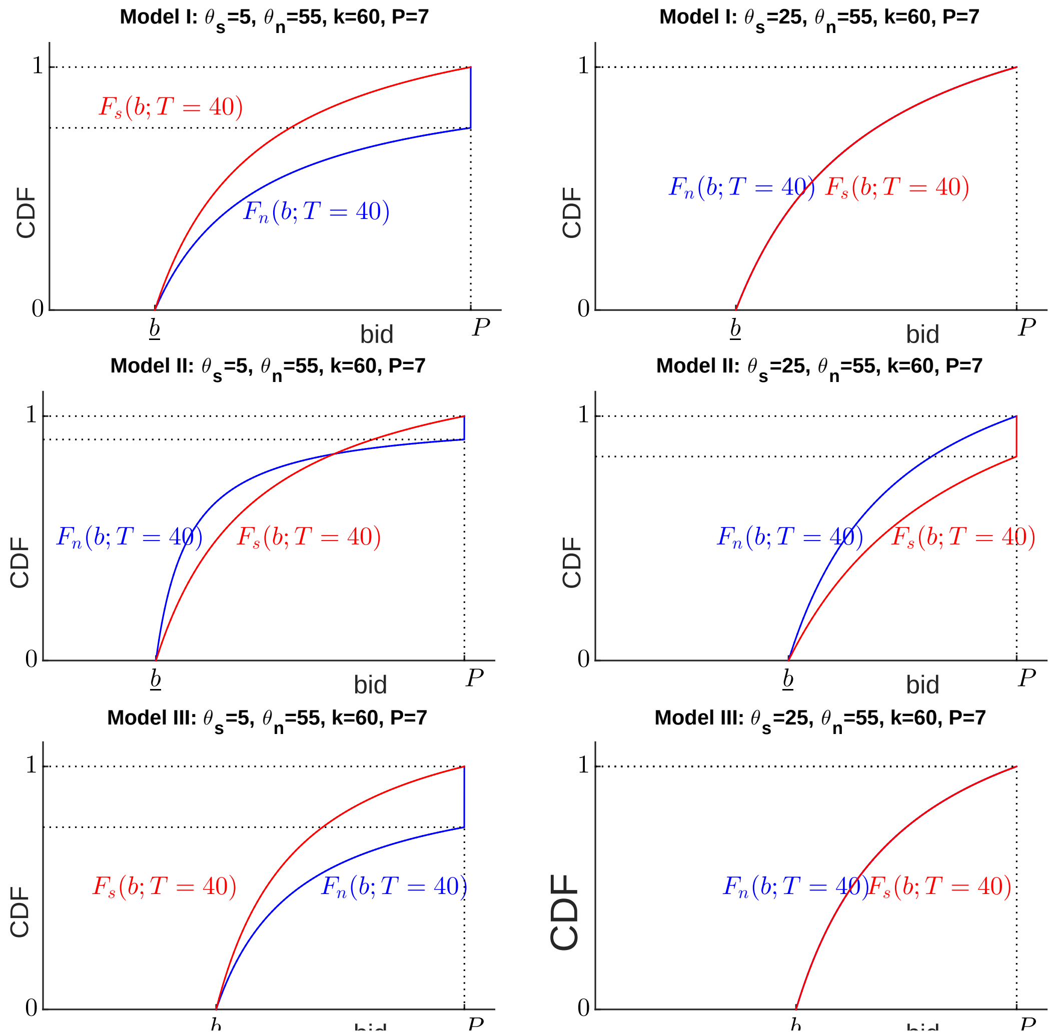
In equation 31, I have solved by substituting variables:

$$\begin{aligned} U &= b - t \Rightarrow b = U + t \\ \frac{\partial U}{\partial b} &= 1 \Rightarrow \partial b = \partial U \end{aligned}$$

Fifth, the expected profit is defined by $\overline{\pi}_n = \overline{\pi}_s = (\underline{b} - t)k$.

It is straightforward to show that an increase in transmission capacity induces the same changes in equilibrium outcome as when the transmission costs are *zero* (proposition two).

Figure 11: Cumulative Distribution Functions of models I, II and III.



Model comparison

In the last part of the annex, I compare the equilibrium outcome of the three different model specifications: transmission constraints and *zero* transmission costs (model I); transmission constraints and *positive* transmission costs for the electricity sold in the other market (model II) and, finally, transmission constraints and *positive* transmission costs for the entire generation capacity (model III).

The three different model specifications affect suppliers' strategies in very different ways as can be observed in figure 11. The diversity of strategies induces important changes on the most relevant variables of the model (table 4).

I have discussed the three models in detail in section four (pages. 17-18). I refer the reader to those pages to follow the analysis.

References

Blázquez, M., 2014, "Effects of Transmission Constraints on Electricity Auctions," PhD dissertation, University of Bologna.

Borenstein, S., Bushnell J. and Stoft S., 2000, "The Competitive effects of transmission capacity in a deregulated electricity industry," *Rand Journal of Economics*, 31, 294-325.

Brezis, E., Krugman P. R. and Tsiddon D., 1993, "Leapfrogging in International Competition: A Theory of Cycles in National Technological Leadership," *American Economic Review*, 83, 1211-219.

Chao, H-P., and Peck S., 1996, "A Market Mechanism for Electric Power Transmission", *Journal of Regulatory Economics*, 10, 25-59.

Dasgupta, P., and Maskin E., 1986, "The Existence of Equilibrium in Discontinuous Economic Games, II: Applications," *Review of Economic Studies*, 53, 27-41.

Deneckere, R., and Kovenock D., 1996, "Bertrand-Edgeworth Duopoly with Unit Cost Asymmetry," *Economic Theory*, 8, 1-25.

Dixon, H., 1984, "The existence of mixed-strategy equilibria in a price-setting oligopoly with convex costs," *Economics Letters*, 16, 205-212.

Downward, A., Philpott A. and Ruddell K., 2015, "Supply Function Equilibrium with Taxed Benefits," *EPOC Working paper.*

ENTSO-E, 2013, "ENTSO-E ITC Transit Losses Data Report."

ENTSO-E, 2014, "ENTSO-E ITC Overview of Transmission Tariffs in Europe: Synthesis 2014."

Escobar, J.F., and Jofré A., 2010, "Monopolistic Competition in Electricity Networks with Resistance Losses," *Economic Theory*, 44, 101-121.

Fabra, N., von der Fehr N. H. and Harbord D., 2006, "Designing Electricity Auctions," *Rand Journal of Economics*, 37, 23-46.

Fabra, N., and Reguant M., 2014, "Pass-Through of Emissions Costs in Electricity Markets," *American Economic Review*, 104, 2872-2899.

von der Fehr, N.H., and Harbord D., 1993, "Spot Market Competition in the UK Eletricity Industry," *Economic Journal*, 103, 531-46.

Flamm, H., and Helpman E., 1987, "Vertical Product Differentiation and North-South Trade," *American Economic Review*, 77, 810-822.

Hogan, W., 1992, "Contract Networks for Electric Power Transmission", *Journal of Regulatory Economics*, 4, 211-242.

Holmberg, P., and Philpott A.B., 2012, "Supply Function Equilibria in Transportation Networks," *IFN Working Paper 945*.

Hu, S., Kapuscinski R. and Lovejoy W. S., 2010, "Bertrand-Edgeworth Auction with Multiple Asymmetric Bidders: The Case with Demand Elasticity," *SSRN Working Paper*.

Janssen, M. C. W., and Moraga-González J. L., 2004, "Strategic Pricing, Consumer Search and the Number of Firms," *Review of Economic Studies*, 71, 1089-1118.

Joskow, P. L., and Tirole J., 2000, "Transmission Rights and Market Power on Electric Power Networks," *RAND Journal of Economics*, 31, 450-487.

Karlin, S., 1959, "Mathematical Methods and Theory in Games, Programming and Economic," London: Pergamon Press.

Kreps, D. M., and Scheinkman J. A., 1984 "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes," *RAND Journal of Economics*, 14, 326-337.

Krugman, P., 1980, "Scale Economies, Product Differentiation, and the Pattern of Trade," *American Economic Review*, 70, 950-959.

Marion, J., and Muehlegger E., 2011, "Fuel Tax Incidence and Supply Conditions," *Journal of Public Economics*, 95, 1202-12.

Osborne, M., and Pitchik C., 1986, "Price Competition in a capacity-constrained duopoly," *Journal of Economic Theory*, 38, 283-260.

Rosenthal, R.W., 1980, "A Model in which an Increase in the Number of Sellers Leads to a Higher Price," *Econometrica*, 48, 1575-1579.

Shapley, L. S., 1957, "A Duopoly Model with Price Competition," *Econometrica*, 25, 354-355.

Shilony, Y., 1977, "Mixed Pricing in Oligopoly," *Journal of Economic Theory*, 14, 373-388.

Shitovitz, B., 1973, "Oligopoly in Markets with Continuum of Traders," *Econometrica*, 41, 467-501.

Svenska Kraftnät, 2012, "Transmission Tariff" *http://www.svk.se/Start/English/Operations-and-market/Transmission-tariff/*.

Varian, H., 1980, "A Model of Sales," *American Economic Review*, 70, 651-659.