

# Legal Protection and Timing of Payment\*

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## Abstract

This paper provides a simple model to analyze the timing of payment with legal consideration. In case of dispute, the conflict will be resolved by court. I showed the efficiency of the legal system and the fixed cost of litigation are the keys for the choice of payment timing. Ex-ante payment contract is more prevalent in region with low fixed cost of litigation. But under this situation, ex-ante payment contract is inefficient, full efficiency can be attained by using certain mixed payment contract. When the fixed cost of litigation is within certain range, ex-ante and ex-post payment contract are identical and perfectly efficient. The model also has some implications for the proposer of the contract. When the seller proposes, ex-ante payment contract dominates ex-post payment and mixed payment contract, which may explain why most of offers in everyday life are made by sellers in a simple ex-ante payment form. And it suggests that primarily mixed payment is a contractual device used by the buyer to mitigate the two sided moral hazard problem.

## 1 Introduction

In textbook discussions of voluntary exchange, agents trade until mutually beneficial terms of trade are found. Goods then exchange hands and the agents go their respective ways. Similarly, when money is used as a medium of exchange, the emphasis is on the market clearing price, under which seller passes goods to the buyer and buyer passes money to the seller. The implicit assumption is that seller receives his payment at the same time the buyer gets his goods. This standard treatment is certainly a convenient abstraction, which allow economists focus on development of various concept of marginal values. However it is a conception which ignores the fact that agreement on a price is a different thing from the revenue collection of sellers and goods acquisition of buyers.

In general, revenue collection vary from payment before to payment after the supply and consumption of the goods or service. Most merchandise, for example, is purchased under what I will call *ex-ante payment schemes*, where buyers must pay (or obtain credit) and acquire ownership of the merchandise before the consumption take place. But in the case of many personal services, supply and consumption precede payment. This type of payment I will call *ex-post payment schemes*. Moreover payment scheme may differ within the same industry. At fast food restaurants customers must pay before eating, but for most other restaurants, the reverse is true.<sup>1</sup> Other examples of payment schemes

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<sup>1</sup>Recently there is a nice discussion on Quora ([www.quora.com](http://www.quora.com)) about the optimal timing of the payment in restaurant. One restaurant owner shared his experience, saying that upfront payment increased table turnover by over 80%. The argument is that customers who have not paid can justify their occupation of a table, by the mere possibility of further ordering. On the other hand, those who have pay will have no moral justification for staying after their meals are finished. However, a higher table turnover does not necessarily associated with higher profit. Another point being raised, is that tips must be pay afterward in order to grade the service, which is one of the key reason why there should be ex-post payment.

are practically as abundant as the number of transactions in the economy. Labor markets typically involve ex-post payments, while many professional services require that a portion of total payment pay in advance, with the balance due after the service is supplied, which I will call *mixed payment schemes*. The goal of this paper is to derive some consistent economic principles which underlies the choice of ex-ante or ex-post payment. Although there are a lot of works done dealing with optimal payment in the principal agent models, little work are found on how two parties will devise an efficient institutional arrangements for the timing of payment.

One clear risk of using the ex-post payment scheme is buyer default. For example, in the web design industry, the most common problem is that after presenting the work, clients take it without paying. From the blog of Ben Hunt, who was one of the most influential figures on the subject of effective web design.<sup>2</sup> “While clients not paying has been a rarity in our business experience (thankfully!), we’ve had 3 such cases in the last 6 months, which has caused significant consequences for an agency of our size.”<sup>3</sup> From his experience, he suggested that it is actually mutually beneficial, to have the payments scheduled over time. On one hand, the risk is managed, on the other hand, this provides you incentive to finish the work at the end. So the problem he tries to solve is actually the two sided moral hazard problem.

If we take the contractual view, what really protect the trading parties is the legal system. If there is any party breaches the contract, the innocent party can seek compensation through litigation. Hence the quality of legal system is an important factor to be considered by the trading parties. The difference of the legal environment can affect the attractiveness of a market. A country’s law regulate business practices, defines business policies, rights and obligations involved in business transactions. For example, China has Communists government where business laws are strictly controlled by government in order to control business sectors. Whereas India has democratic government and business laws are made to protect small businesses and consumers. Although different countries have different laws and regulations, knowledge of common law, civil law, contract laws, laws governing property rights, product safety and liability for a country helps business people to make business decisions.

The common law system is commonly found in former Great Britain’s colonies and is based on country’s legal history, past court rulings on cases and ways in which laws are applied in specific situations. Judges in a common law system have power to interpret the law under unique circumstances for an individual case. Countries like United States, Australia, India uses common law systems. In civil law system, laws are based on detailed set of written rules and codes. Judges have less flexibility and have power only to apply the law. France, Germany, Russia operate with a civil law system. Some countries have legal system, which is based on religious teachings. Countries like Pakistan, Saudi Arabia, Iran and Middle Eastern nations follow Islamic laws, which is based on holy principles of Koran. It is very important to interpret law according to country and its impact on their commercial activities. Many business transactions are regulated by contract, and contract law governs contract enforcement. Contracts drafted under common law system are tend to be very detailed, where as contracts are much shorter and less specific in civil law system due to already drafted civil codes. Therefore common law system has long and expensive jurisdiction process. However it has advantage of greater flexibility and allows judges to interpret a contract dispute in particular situation as compare to civil law system. Considering impact of various aspects of legal system in business, it is very important for business people to have good understanding of the legal system where they do business with.

Alongside the formal legal protection, the timing of payment is also an institutional device being used to protect both parties from renegeing on a promise. By having part of the payment paid in advance, this reduces the loss to the seller in the case of buyer’s default of payment. Even if the seller

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<sup>2</sup> He has written three books and spoken at multiple conferences internationally.

<sup>3</sup><http://webdesignfromscratch.com/business/payment-timing-structure-tips/>

at the end chooses not to sue the buyer, the seller receives at least partly of the total payment. On the other hand, by having part of the payment paid ex-post, this protects the buyer from the seller's malpractice of providing low quality of goods and service, because he can refuse to pay the remaining part of payment. From this perspective, a mixed payment actually mitigates the problem of the two sided moral problem. A interesting point is that both parties want the timing of the payment shifts to their side, the buyer want to pay ex-post, while the seller want to get paid ex-ante. Although this paper will not provide a solution for the optimal mix of the payments, which is most likely done by bargaining, I show that mixed payment can improve efficiency.

In this paper, I consider a trading of goods or service which quality is buyer-specific, and communicates to the seller during the transaction. When the quality of the goods is of different level, firms are disciplined by competition. Low quality goods or goods with highly variable quality, sell for lower prices. But this requires that some sort of quality standard be agreed upon, and that the quality be easily discernible. When the service rendered is highly personalized, it will not be homogenous among sellers. In this case, the cost of renegotiating with alternative sellers in order to avoid shirking is high and the discipline of competition is less relevant. Examples include photographers, construction work, professional services such as legal advice and management consultancy, and creative work such as architectural design, the writing of software, preparation of advertisements, and research and development.

Consider a simple setting, where there are one principal and one agent. The agent is paid to provide good or service, whose quality depends on his effort. So here we have a two sided moral hazard problem. On one side, as long studied, principal worries agent shirks. While on the other side, agent worries that principal not paying the agreed amount. To complete the story, we need a third party, which is a court. In the case of dispute, either the seller complains that amount paid is less than the agreed amount, or the buyer finds the quality is lower than asked for, they can go to the court to seek a judgment. Surely there is no such thing as free lunch, and using the court service is costly. The simplest way to represent this, is to consider both party enter into a contest — completing to provide evidence that the other party commits fraud.<sup>4</sup> Apart from the variable cost, there are fixed costs. The fixed cost plays a key role in the model, not only it is an important part of the legal environment which will be defined in the model setup, but also one of the results shows that there is an optimal range of fixed cost to allow efficient outcome. This hints that reducing fixed cost of litigation need not be beneficial to trade. Also the court efficiency will determine the feasible trading opportunity. A more efficient legal system, as will be defined in the model setup section, will enlarge the feasible set of contracting choice.

An example of the moral hazard problem which is commonly seen in construction industry. Very often contracts involves Milestone Payments where payments are not scheduled by time but by a 'milestone'. This means that payments are due when a certain event or section of work is reached. Typically contractors throwing extra staff and resources at the work with the aim of reaching each milestone extra fast and being able to bank the cash fast. But very often there is no payment at the end, because clients could argue about tiny details or that some particular standard is not met. Clients will then simply terminate the contract. Now a part of the work has been done in record time but with no cost, and clients will simply appoint someone else to finish the work. So there is an advice that contractors should try their best to avoid this type of payment scheme at all.<sup>5</sup>

This example seems to suggests that ex-post payment should not be welcomed by sellers. One of the results in this paper seems to confirm this idea, where ex-ante payment dominates ex-post payment. But it comes with an important condition, which is when the contract is proposed by

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<sup>4</sup>See Corchón, L. C. (2007).

<sup>5</sup>Contractor debt recovery, <http://www.contractorsdebtrecovery.com.au/articles/Milestones.pdf>

the seller. However, it seems that in the example above, it is the buyer who propose the contract. Surprisingly, when the contract is proposed by the buyer, we have an equivalent result, ex-ante and ex-post payment gives the same outcome, apart from the case where litigation involves very low fixed cost. Nevertheless, the outcome may not be efficient. I showed that there exists two kind of mixed payment scheme provides efficient outcomes when other schemes could not.

There may be a concern for the alternative solution for this problem, like seller offer guarantees. Reputation based approaches are mostly studied for the sellers, but not for individual consumers.<sup>6</sup> Hence they are not complete solutions, particularly for trading parties who are new, small or distant from each other. Since payment terms can be selected to mitigate these concerns, I expect systematic relations between terms of payment and variable that capture the potential for opportunism. In the next section, I will present the model. Then followed by the analysis of litigation under different legal system. In section 4 and 5, ex-ante and ex-post payment contract is examined in detail. A comparison between these two payment contract is provided in section 6. Mixed payment contract is then introduced to the analysis in section 7. The role of contract proposer will be shifted to the seller in section 8. Literature review is provided in section 9. After discussion on some alternative explanation for different payment contract is in section 10, section 11 concludes.

## 2 Model

I consider a simple trading relationship, one buyer trades with one seller. Both parties are risk neutral. Production technology is linear,  $q = e$  where  $q$  is quality of the good or service and  $e$  is effort. The buyer pays the price  $P$  for the good or service which seller incurs effort cost  $e^2/2 + E$  to produce.  $E$  is the fixed cost to finish the basics, and  $e^2/2$  is the variable cost due to the specific requirement of the buyer. The preferences are represented by the following utility functions,

$$\begin{aligned} EU_B &= q - P + V, \\ EU_S &= P - \frac{e^2}{2} - E + V, \end{aligned}$$

where  $V$  is the expected values from litigation process which will be analyzed in the next section. Consider a benchmark case which both parties cannot cheat, then there is no need to have any litigation, hence  $V = 0$ . Clearly we have the efficient effort  $\tilde{e} = 1$ . As in the standard model, the equilibrium quality is always below this first best level, but we will see that Principal would always minimize the deadweight loss by choosing the more efficient scheme.

The timing of the model will be as follows: First, the buyer offers a contract to the seller, which lists out ex-ante price  $P_{\text{ante}}$ , required quality  $q$ , and ex-post price  $P_{\text{post}}$ . Second, the seller accepts or rejects the offer. Third, if the seller accepts, the seller receives  $P_{\text{ante}}$  and chooses his effort  $e$ ; otherwise, the game ends. Fourth, the buyer chooses whether to fulfill the contract, by deciding whether to pay  $P_{\text{post}}$ . Fifth, if there is a dispute, either party can bring the case to court. There is no private information and everything above is common knowledge to both buyer and seller. For the contract which contains a zero ex-ante price or zero ex-post price, I will call that as pure payment contract. For contracts contain positive ex-ante price and ex-post price, I refer them as mixed payment contract. The time line of the model can be represented as follows:

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<sup>6</sup>Some firms are now spending effort to bulid up profiles for consumers. Good examples are Uber and Airbnb, where consumers are rated by drivers after each ride and hosts after each stay, which will add up to their overall ratings. However these data are not available to other companies and consumers cannot, at least for now, request it as proof of being a “nice consumer” to other companies.

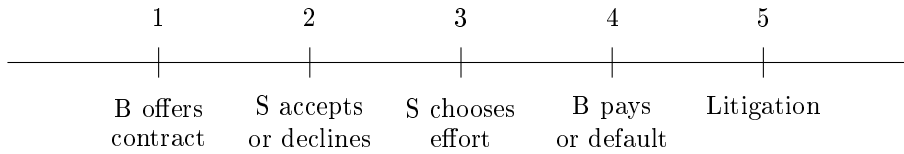


Figure 1: Time line

### 3 Litigation

Now let us make clear the details for the litigation. In the case of dispute, there must be a party which breaks the contract (first), which I call the defaulting party and the other one as the innocent party. Denote the resource spend by  $i$  on litigation  $x_i$ , where  $i, j = B, S$  which is a shorthand notation for the buyer and seller. Party  $i$  is the innocent party, while party  $j$  is the defaulting party. Let  $p(x_i, x_j)$  to be the probability that agent  $i$  wins, so  $1 - p(x_i, x_j)$  is the probability that agent  $j$  wins,<sup>7</sup>

$$p(x_i, x_j) = \frac{\alpha x_i}{\alpha x_i + x_j}.$$

Noted that there is a term  $\alpha$  attached to  $x_i$ . I assume  $\alpha > 1$ , to represent the advantage enjoyed by the innocent party. Another way to understand  $\alpha$  is to consider it represents the efficiency of the legal system. Under this interpretation, a more efficient legal system costs less for the innocent party to achieve the same probability of winning the case. Apart from the variable cost  $x_i$ , there are fixed cost of litigation, which is denoted by  $K$ . Expert fees, document preparation fees, and investigator fees add to the cost. In arbitration, filing fees can cost thousands and the fee for arbitrators may easily cost thousands for each day of a hearing. In advance of trial, mediation will cost \$500 per half day, not including the attorney’s fees.

One may notice that under this simple litigation structure, neither the effort exerted nor the amount paid has effect on the probability of winning the litigation. As a result, for any given contract, if the seller accepts, the optimal effort to be exerted is either  $q$  or 0, and optimal ex-post payment is either the specified amount or nothing. This simplifies the analysis, and provides some clean and clear cut results. Relaxing this assumption is part of the plan for future work.

Certainly there are other indirect costs associated with the litigation. These indirect costs stem from the uncertainty created by litigation, which may deter investment in high-cost jurisdictions. They also may affect companies’ borrowing costs and hence their ability to invest, grow, and create jobs. Concerns surrounding litigation can also occupy management time, which may distort or hinder effective business decision making. Apart from that there are hidden costs, like the personal cost and business stress. The business is stressed by occupying key personnel time with the duties of litigation instead of their job responsibilities. Each day of trial will occupy three days of each witness’s non-trial time. In addition, employee suffers from personal stress over being a witness or being involved in the litigation or, in the worst case scenario, being the object of a key portion of the litigation – either from their decisions or their actions. All three of these initiators of stress carry direct costs – the loss of the employee’s services – and indirect costs – the minimization of these employees. It is a rare employee indeed, who, once accused of “causing” litigation whether by contract decisions, omissions,

<sup>7</sup>The literature has developed from the seminal contributions by Tullock (1967). The contest success function in this paper is similar to the one in Gradstein (1995), with the exception that there is no restriction on  $\alpha$ . For more recent development about the contest success function, see Corchón, L., & Dahm, M. (2010).

or direct action is not chastened and therefore hesitant to act. This hesitation leads to indecision, inefficiency, and loss. These costs are difficult to measure but are a significant expense nonetheless. On the personal side, stress manifested as anger and anxiety are common occurrences. Add to that the cloud of uncertainty that hangs over the individual as long as the litigation is unresolved. Combined this equals an emotional, and physical toll that must be added to the financial cost of litigation.

All these factors are too complicated to put into one single model, although they are highly important and relevant, I will not include them as this will certainly make the model too messy to analyze. Certainly one put some of them into analysis, but before that, I believe an understanding of the basic mechanism behind all these other factors is essential for us to know how contracts with different timing of payment will affect the decision makers.

### 3.1 US system

Throughout this paper, we will focus on the US legal system, where each party is responsible for its own spending in court.<sup>8 9</sup> The expected value from litigation is<sup>10</sup>

$$\begin{aligned} V_i &= C \cdot p(x_i, x_j) - x_i - K, \\ V_j &= -C \cdot p(x_i, x_j) - x_j - K, \end{aligned}$$

where  $K$  is the fixed cost of litigation, and  $C$  denotes the compensation paid by the losing party to the winning party.<sup>11</sup> In the case of buyer default, the compensation would be  $P_{\text{post}}$ .<sup>12</sup> In the case of seller default, the compensation would be  $P_{\text{ante}}$ , which leaves buyer not damaged.<sup>13</sup> This specification does not incorporate any relationship between the damage being made and the corresponding compensation. A slight or big discrepancy results in the same amount of compensation. Hence this creates no incentive for partial defaulting, which is consistent with the previous setting. Relaxing this assumption is part of the plan for future work.

Simple differentiation give us the following first order conditions,

$$\begin{aligned} C \frac{\alpha x_j}{(\alpha x_i + x_j)^2} &= 1, \\ C \frac{\alpha x_i}{(\alpha x_i + x_j)^2} &= 1. \end{aligned}$$

This implies the following lemma, define  $x_i^*, x_j^*$  as the equilibrium litigation spending of party  $i$  and  $j$  respectively:

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<sup>8</sup>For reader interested in seeing different legal system, there is an analysis of the UK system in the appendix.

<sup>9</sup>The contest literature has developed from the seminal contributions by Tullock (1967). Corchón, L (2007) provides a good survey about theory of contest.

<sup>10</sup>Under this formulation, it is always the innocent party to initiate a law suit to claim for compensation, which precludes opportunistic suing.

<sup>11</sup>One may conceive that the compensation can be stipulated in the contract, preferably with an exceedingly high amount to deter breaching from either sides. Formally, such kind of clause is called as liquidated damages. However under common law, a liquidated damages clause will not be enforced if its purpose is to deter some party from breaching it rather than to compensate the innocent party. Because courts seek to achieve a fair result and will not enforce a term that will lead to the unjust enrichment of the enforcing party. Under civil law, for example in France, such clauses are not generally void, but judges may still adjust excessive contract penalties.

<sup>12</sup>This compensation is according to the expectation damage principle. Under the expectation damage measure, the defaulting party pays an amount that puts the other party in the position he would have been in had the contract been performed. For details, see Shavell (1980).

<sup>13</sup>This compensation is according to the restitution damage principle. Under the restitution damage measure the defaulting party returns only the payments made to him. See Shavell (1980).

**Lemma 1.** *In equilibrium, the two parties spend the same amount on litigation.*

$$x_i^* = x_j^* = \frac{\alpha C}{(1 + \alpha)^2}.$$

Both parties will spend more to compete for a higher compensation (when  $C$  is higher), and less when the legal system is more efficient (when  $\alpha$  is higher). Thus in equilibrium, the probability of  $i$  winning is

$$p^*(x_i^*, x_j^*) = \frac{\alpha}{1 + \alpha}.$$

The equilibrium expected value from litigation is

$$\begin{aligned} V_i^* &= \frac{\alpha}{1 + \alpha} C - \frac{\alpha}{(1 + \alpha)^2} C - K \\ &= C p^{*2} - K. \end{aligned}$$

$$\begin{aligned} V_j^* &= -C \frac{\alpha}{1 + \alpha} - \frac{\alpha}{(1 + \alpha)^2} C - K \\ &= -C \frac{\alpha(2 + \alpha)}{(1 + \alpha)^2} - K. \end{aligned}$$

For the innocent party, there is a cutoff for litigation decision, he will sue if and only if  $K \leq C p^{*2}$ . For the defaulting party, if he admits, he need to pay  $C$  to the innocent party, so the cutoff for fighting a legal battle is  $K \leq C/(1 + \alpha)^2$ . Hence, when the innocent party sues, it is always in the interest of the defaulting party to fight the legal battle.

### 3.1.1 Comparative Statics

We can see how changes in  $\alpha$  and  $C$  affect the litigation,

$$\frac{\partial p^*}{\partial \alpha} = \frac{1}{(1 + \alpha)^2} > 0.$$

The innocent party winning probability increases with the efficiency of the legal system. Notice that  $V_i^* = p^{*2} C$ , it is obvious that  $V_i^*$  increases in both  $\alpha$  and  $C$ . With the previous calculation, it is easy to get,

$$\begin{aligned} \frac{\partial V_j^*}{\partial \alpha} &= -C \frac{2}{(1 + \alpha)^3} < 0, \\ \frac{\partial V_j^*}{\partial C} &= -\frac{\alpha(\alpha + 2)}{(1 + \alpha)^2} < 0. \end{aligned}$$

**Lemma 2.** *In terms of absolute value, the innocent party has a stronger marginal effect in changes of  $\alpha$ , but weaker in changes of  $C$ .*

*Proof.*

$$\begin{aligned}\frac{\partial V_i^*}{\partial \alpha} &= \frac{2\alpha C}{(1+\alpha)^3} > \frac{2C}{(1+\alpha)^3} = \left| \frac{\partial V_j^*}{\partial \alpha} \right|, \\ \frac{\partial V_i^*}{\partial C} &= \left( \frac{\alpha}{(1+\alpha)} \right)^2 < \frac{\alpha(\alpha+2)}{(1+\alpha)^2} = \left| \frac{\partial V_j^*}{\partial C} \right|.\end{aligned}$$

□

## 4 Ex-ante Payment

Let us firstly consider the case where the contract only involves ex-ante payment. After the buyer makes the payment, as explained earlier, the seller has only 2 potential optimal effort choices  $e = 0$  or  $q$ , essentially choosing to shirk or not. If the seller shirks, the buyer chooses to sue or not. The buyer will sue if

$$K \leq P_{\text{ante}} \cdot p^{*2} = P_{\text{ante}} \left( \frac{\alpha}{1+\alpha} \right)^2.$$

As a consequence,  $P_{\text{ante}}$  need to be sufficiently large to induce buyer to sue. When forming the optimal ex-ante contract, this condition must be met. Otherwise, any  $P_{\text{ante}}$  lower than  $K/p^{*2}$  is like a gift to the seller, because the buyer will not find it beneficiary to sue. The expected utility of the seller is

$$EU_S = \begin{cases} P_{\text{ante}} - \frac{\tilde{e}^2}{2}, & \text{if the seller works;} \\ P_{\text{ante}} - \frac{\alpha(2+\alpha)}{(1+\alpha)^2} P_{\text{ante}} - K, & \text{if the seller shirks.} \end{cases}$$

To induce the seller to exert required effort ( $\tilde{e}$ ), we need

$$\frac{\alpha(2+\alpha)}{(1+\alpha)^2} P_{\text{ante}} + K \geq \frac{\tilde{e}^2}{2}.$$

The whole problem can be formulated as follows,

$$\begin{aligned}\max_{\tilde{e}, P_{\text{ante}}} & \tilde{e} - P_{\text{ante}} \\ \text{s.t.} & \frac{\alpha(2+\alpha)}{(1+\alpha)^2} P_{\text{ante}} + K \geq \frac{\tilde{e}^2}{2}, & \text{(IC)} \\ & P_{\text{ante}} - \frac{\tilde{e}^2}{2} \geq 0, & \text{(PC)} \\ & P_{\text{ante}} \geq K \left( \frac{1+\alpha}{\alpha} \right)^2, & \text{(LC)}\end{aligned}$$

where equation (IC) is the incentive compatibility constraint of the seller, equation (PC) is the participation constraint of the seller, and equation (LC) is the legal constraint for buyer to sue seller, if seller shirks.

There are four types of contract where either IC or PC binds, or both and when the LC and PC binds. It is impossible for none of constraints to be binding, due to the maximization behavior, as shown clearly in figure 2. The indifference curves of the buyer is the 45 degree line. The optimal solution depends on the intersection point of the three curves IC, PC and LC. Consider the extreme



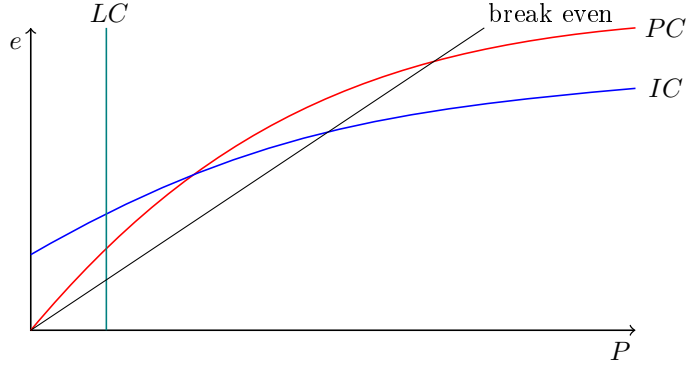


Figure 2: Constraints for ex-ante payment contract

For the region below the red curve,  $PC$  is satisfied. For the region below the blue curve,  $IC$  is satisfied. For the region to the right of the teal curve,  $LC$  is satisfied. The buyer will not offer any  $(P, e)$  below the breakeven line.

case when  $K = 0$ , under this case,  $IC$  will be underneath  $PC$ ,  $LC$  is on the vertical axis, the only possible tangent point is on the  $IC$ . Thus we should expect when  $K$  is low, we will only have  $IC$  binding, which is case I in the following analysis. When  $K$  increases,  $IC$  shifts up vertically, and  $LC$  shifts right horizontally, leaving  $PC$  unchanged. If the tangent point is still on the right of the intersection point of  $IC$  and  $PC$ , case I follows. If it happens to be at the intersection point of  $IC$  and  $PC$ , where both  $IC$  and  $PC$  is binding, then we have case II. When it happens to be the left of the intersection point, where  $PC$  is binding, we have case III. The last case, is when  $K$  becomes really high, such that the solution is determined by the intersection point of  $PC$  and  $LC$ .

The optimal ex-ante payment contracts is summarized by the following proposition.<sup>14</sup>

**Proposition 1.** *The optimal choice of the ex-ante payment contract depends on the specific legal environment,  $(\alpha, K)$ , which is summarized in the following table.*

Case	$(\alpha, K)$	$e^*$	$P_{\text{ante}}^*$
I	$0 \leq K \leq \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6}$	$\frac{\alpha(2+\alpha)}{(1+\alpha)^2}$	$\frac{\alpha(2+\alpha)}{2(1+\alpha)^2} - K \frac{(1+\alpha)^2}{\alpha(2+\alpha)}$
II	$\frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6} \leq K \leq \frac{1}{2(1+\alpha)^2}$	$\sqrt{2K}(1+\alpha)$	$K(1+\alpha)^2$
III	$\frac{1}{2(1+\alpha)^2} \leq K \leq \frac{\alpha^2}{2(1+\alpha)^2}$	1	1/2
IV	$\frac{\alpha^2}{2(1+\alpha)^2} \leq K \leq \frac{2\alpha^2}{(1+\alpha)^2}$	$\sqrt{2K} \left( \frac{1+\alpha}{\alpha} \right)$	$K \left( \frac{1+\alpha}{\alpha} \right)^2$

To sum up, these four type of contracts correspond with different fixed litigation cost, from low to high, it starts with case I, and finally case IV. This also gives an idea of optimal range  $K$  for efficient contracting, with a given  $\alpha$ .

**Corollary 1.** *When the fixed cost of litigation is either too high or too low, the efficient contract (case III) will not be offered.*

*Proof.* See Lemma 21. □

<sup>14</sup>The details of the analysis is in the appendix 12.2.

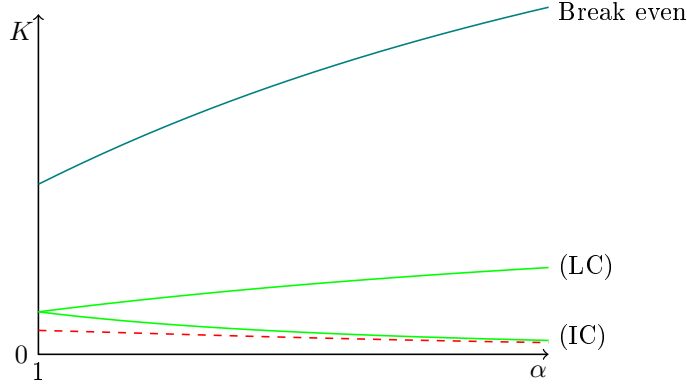


Figure 3: Optimal choice of ex-ante payment contract

For the region below the red dashed curve, the case I contract will be chosen; For the region between the red dashed curve and the lower green curve (21), case II contract will be offered. For the region between the two green curves (21 and 22), the case III contract will be chosen; For the region between the upper green curve (22) and the teal curve (23), the case IV contract will be offered.

## 5 Ex-post Payment

Let us go through the timing of ex-post payment again. After receiving the contract from the buyer, the seller chooses to shirk or not. If yes, the game ends, otherwise the buyer chooses to pay or not. If yes, the game ends, otherwise the seller chooses to sue or not. If the seller is not suing buyer, the buyer will not pay for sure. So let us consider the contract that the seller will sue the buyer if he is not paying:

$$P_{\text{post}} \geq \frac{K}{p^{*2}} = \left(\frac{1+\alpha}{\alpha}\right)^2 K. \quad (1)$$

Equation (1) is the legal constraint in the ex-post payment case, which defines the minimum feasible payment.

For the buyer,

$$EU_B = \begin{cases} \tilde{e} - P_{\text{post}}, & \text{if pay,} \\ \tilde{e} - P_{\text{post}} \left(\frac{\alpha(2+\alpha)}{(1+\alpha)^2}\right) - K, & \text{otherwise.} \end{cases}$$

So the buyer will pay if  $P_{\text{post}} \leq (1+\alpha)^2 K$ , which is the condition for the buyer to pay.

The whole problem can be formulated as follows,

$$\begin{aligned} & \max_{\tilde{e}, P_{\text{post}}} \tilde{e} - P_{\text{post}} \\ & s.t. \quad P_{\text{post}} - \frac{\tilde{e}^2}{2} \geq 0, & (\text{PC}) \\ & \quad \quad P_{\text{post}} \leq (1+\alpha)^2 K & (\text{IC}_B) \\ & \quad \quad P_{\text{post}} \geq K \left(\frac{1+\alpha}{\alpha}\right)^2 & (\text{LC}_S) \end{aligned}$$

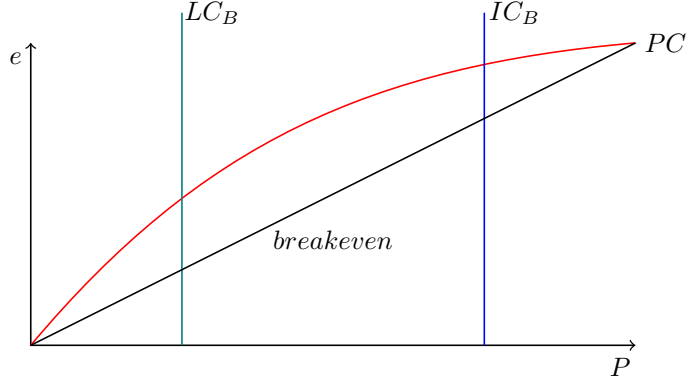


Figure 4: Constraints for ex-post payment contract

For the region below the red curve,  $PC$  satisfies. For the region to the left of the blue curve,  $IC_B$  satisfies. For the region to the right of the teal curve,  $LC_S$  satisfies. All the region above the black line give the buyer positive payoff, and hence itself is the breakeven line.

where equation ( $PC$ ) is the participation constraint of the seller, equation ( $IC_B$ ) is the incentive constraint of the buyer, and equation ( $LC_S$ ) is the legal constraint for the seller to sue the buyer, if the buyer do not pay.

When  $K = 0$ , both  $IC_B$  and  $LC_S$  lies on the vertical axis, see figure (4). When  $K$  increases, both  $IC_B$  and  $LC_S$  shifts to the right and there is a gap between them. The higher the  $K$ , the larger is the gap. The indifference curves of the buyer are any 45 degree lines. The optimal solution depends on the intersection point of the three curves,  $PC$ ,  $IC_B$ ,  $LC_S$ . When  $K$  is low, the solution is at the intersection point of  $PC$  and  $IC_B$ , where both  $PC$  and  $IC_B$  are binding, then that is the case I in the following analysis. When we have medium  $K$ , tangent point happens on the  $PC$  which lies in the gap between between the  $IC_B$  and  $LC_S$ , that is case II in the following subsection. If the solution is at the intersection point of the  $PC$  and  $LC_S$ , where both  $PC$  and  $LC_S$  are binding, and that is case III in the following subsection, which happens when  $K$  is high.

The optimal ex-post payment contracts is summarized by the following proposition.<sup>15</sup>

**Proposition 2.** *Optimal ex-post payment contract depends on specific legal environment,  $(\alpha, K)$ , which is summarized in the following table.*

Case	$(\alpha, K)$	$e^*$	$P_{\text{ante}}^*$
I	$K \leq -\frac{\alpha^2(2\sqrt{2(1+\alpha)}-\alpha-3)}{2(\alpha-1)(\alpha^4+4\alpha^3+3\alpha^2-4\alpha-4)}$	$\sqrt{2K}(1+\alpha)$	$(1+\alpha)^2 K$
II	$\hat{K}$	$\frac{\alpha}{2+\alpha}$	$\frac{(1+\alpha)^2}{2(2+\alpha)^2} + \left(\frac{1+\alpha}{\alpha}\right)^2 K$
III	$\frac{1}{2(1+\alpha)^2} \leq K \leq \frac{\alpha^2}{2(1+\alpha)^2}$	1	$1/2$
IV	$\frac{\alpha^2}{2(1+\alpha)^2} \leq K \leq \frac{2\alpha^2}{(1+\alpha)^2}$	$\sqrt{2K} \left(\frac{1+\alpha}{\alpha}\right)$	$K \left(\frac{1+\alpha}{\alpha}\right)^2$

where  $\hat{K}$  stands for  $K \geq -\frac{\alpha^2(2\sqrt{2(1+\alpha)}-\alpha-3)}{2(\alpha-1)(\alpha^4+4\alpha^3+3\alpha^2-4\alpha-4)}$ ,  
 $K \leq \frac{\alpha^2}{2(2+\alpha)^2(\alpha^2-1)}$  and  $K \leq \frac{\alpha^2}{4(1+\alpha)(2+\alpha)}$

*Proof.* By lemma 23 - 27. □

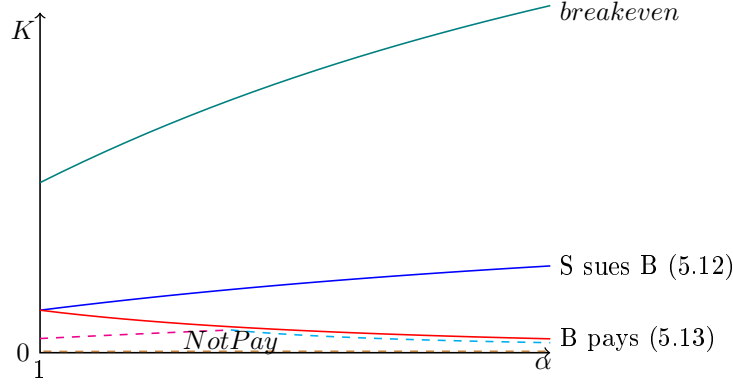


Figure 5: Optimal ex-post payment contract

For the region below the red curve (26), excluding the area bounded by the dashed curves, the case I ex-post payment contract will be offered. For the region between blue (25) and red curve (26), the case II ex-post payment contract will be offered. For the region bounded by the dashed curves, the buyer-default ex-post payment contract will be offered. For the region between the teal curve (28) and the blue curve (25), the case III ex-post payment contract will be offered.

## 6 Optimal Contract: Ex-ante Vs Ex-post

We have seen various ex-ante and ex-post payment contract in section 4 and 5, a natural question follows: under a specific legal environment, which contract is going to be offered? The optimal payment scheme depends on the following comparison,

$$\begin{aligned}
& EU_B^{\text{ante}}(\text{case I}) - EU_B^{\text{post}}(\text{non pay}) \\
&= \frac{\alpha(2+\alpha)}{2(1+\alpha)^2} + K \frac{(1+\alpha)^2}{\alpha(2+\alpha)} - \frac{\alpha}{2(2+\alpha)} + \frac{2(1+\alpha)}{\alpha} K \\
&= \frac{\alpha((2+\alpha)^2 - (1+\alpha)^2)}{2(2+\alpha)(1+\alpha)^2} + K \frac{(1+\alpha)^2 + 2(1+\alpha)(2+\alpha)}{\alpha(2+\alpha)} \\
&> 0
\end{aligned}$$

So the case IV buyer default ex-post payment contract is dominated by the case I ex-ante payment contract. In total the optimal payment scheme will be as follows, where  $P$  without subscript stands for either  $P_{\text{ante}}$  or  $P_{\text{post}}$ ,

**Proposition 3.** *Optimal pure payment contract depends on specific legal environment,  $(\alpha, K)$ , which is summarized in the following table.*

<sup>15</sup>The details of the analysis is in the appendix 12.3.

Case	$(\alpha, K)$	$e^*$	$P_{\text{ante}}^*$
I	$0 \leq K \leq \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6}$	$\frac{\alpha(2+\alpha)}{(1+\alpha)^2}$	$\frac{\alpha(2+\alpha)}{2(1+\alpha)^2} - K \frac{(1+\alpha)^2}{\alpha(2+\alpha)}$
II	$\frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6} \leq K \leq \frac{1}{2(1+\alpha)^2}$	$\sqrt{2K}(1+\alpha)$	$(1+\alpha)^2 K$
III	$\frac{1}{2(1+\alpha)^2} \leq K \leq \frac{\alpha^2}{2(1+\alpha)^2}$	1	1/2
IV	$\frac{\alpha^2}{2(1+\alpha)^2} \leq K \leq \frac{2\alpha^2}{(1+\alpha)^2}$	$\sqrt{2K} \left(\frac{1+\alpha}{\alpha}\right)$	$K \left(\frac{1+\alpha}{\alpha}\right)^2$

In diagram, see figure (6).

**Proposition 4.** *The choice of ex-ante or ex-post payment contract is irrelevant, only when the fixed cost of litigation is relatively low, ex-ante payment contract is certainly chosen.*

*Proof.* By lemma 3. □

To see what level of the fixed cost is low enough to have ex-ante payment contract, consider the legal environment where  $\alpha = 1$ . Under this legal environment, the ratio  $K/e^* = 3/32 \approx 9\%$ . This suggests that if the fixed cost of the litigation is less than approximately 9% of the total value of the transaction, ex-ante payment contract is better than ex-post payment contract. If we use the efficient level of quality as a benchmark, the ratio  $K/e^* \approx 7\%$ , which is not too far from the previous one.

We can understand this result by looking the ex-post payment contract when  $K$  is sufficiently low. Consider the extreme case, when  $K = 0$ . Under this case, no ex-post payment contract can be offered, because  $IC_B$  and  $LC_S$  requires  $P_{\text{post}} = 0$ . Thus when  $K$  is relatively low, the possible range of  $P_{\text{post}}$  is severely restricted. If  $P_{\text{post}}$  goes too high, the buyer has no incentive to pay, but when  $P_{\text{post}}$  goes too low, the seller will have no incentive to sue even if the buyer defaults. Hence only highly inefficient ex-post payment contract can be offered, whereas ex-ante payment contract do not have this problem. There are no restrictions on the possible range of  $P_{\text{ante}}$  when  $K = 0$ , apart from  $P_{\text{ante}} \geq 0$ . Note that there are still other inefficient contracts offered, with  $\tilde{e} < 1$ . The following result points out the reason.

**Proposition 5.** *Inefficient contracts are offered due to either too high or too low fixed cost of litigation charged.*

*Proof.* i) When  $\frac{\alpha^2}{2(1+\alpha)^2} \leq K \leq \frac{2\alpha^2}{(1+\alpha)^2}$ , the contract with  $\tilde{e}^* = \sqrt{2K} \left(\frac{1+\alpha}{\alpha}\right)$ ,  $P^* = K \left(\frac{1+\alpha}{\alpha}\right)^2$  will be offered. If instead the efficient contract is offered, either ex-ante or ex-post payment contract, one party will breach the contract, the other party will be better off not suing the defaulting party, because  $K$  is too high. The breaching party will be the seller if the efficient ex-ante payment contract is offered, see equation (22). While the breaching party will be the buyer if the efficient ex-post payment contract is offered, see equation (25).

ii) When  $\frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6} \leq K \leq \frac{1}{2(1+\alpha)^2}$ ,  $\tilde{e}^* = \sqrt{2K}(1+\alpha)$ ,  $P^* = (1+\alpha)^2 K$  is offered. And when  $K \leq \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6}$ , the case I ex-ante payment contract is offered. These two cases share the same reasoning, the problem is not because  $K$  is too high, instead the opposite is true. If instead an efficient contract is offered, either ex-ante or ex-post payment contract, one party will find it better off not to follow the contract and seeking litigation to resolve the conflict, because  $K$  is too low. The breaching party will be the seller if the efficient ex-ante payment contract is offered, see equation (21). While the breaching party will be the buyer if the efficient ex-post payment contract is offered, see equation (26). □

We understand that, from the proposition above, the cause of inefficient contract is the wrong level of fixed cost of litigation being charged. So naturally that points out a simple solution to the problem.

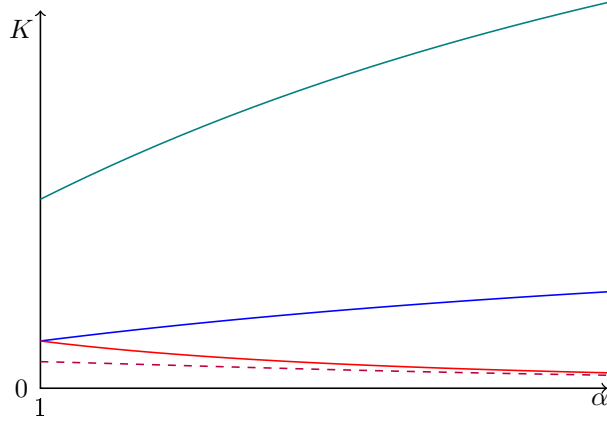


Figure 6: Optimal contract: ex-ante Vs ex-post

For the region below the purple dashed curve, the case I ex-ante payment contract will be offered. For the region between the purple dashed curve and the red curve (21) or (26), the case II ex-ante or case I ex-post payment contract will be offered. For the region between blue (22) or (25) and red curve (21) or (26), the case III ex-ante or case II ex-post payment contract will be offered. For the region between the teal curve (23) or (28) and the blue curve (22) or (25), the case IV ex-ante or case III ex-post payment contract will be offered.

**Proposition 6.** *Inefficient contracts can be replaced by efficient one when the the fixed cost of litigation is within the range of  $\frac{1}{2(1+\alpha)^2} \leq K \leq \frac{\alpha^2}{2(1+\alpha)^2}$ .*

*Proof.* By lemma 3, for any given  $\alpha$ , if the fixed cost of litigation is within the following bounds,  $\frac{1}{2(1+\alpha)^2} \leq K \leq \frac{\alpha^2}{2(1+\alpha)^2}$ , efficient contract will be offered, either ex-ante, ex-post or mixed payment.  $\square$

This result sheds some lights on legal reforms. One particular question is whether should the legal sector shares some part of the cost. In the model we presented, a legal reforms can be represented by a higher  $\alpha$ , where the funding burden can be represented by a higher  $K$ . If one of the purpose of the legal reform is to enhance market efficiency, then there are two general lessons to be taken from our model. If the status quo  $K$  is too high, as described in part i) of the proof of proposition 5, the best thing to do is to seek external funding. Otherwise, by raising  $K$ , it will just make it harder to move into the “efficient region” for a given  $\alpha$ . On the other hand, when the status quo  $K$  is too low, as described in part ii) of the proof of proposition 5, the legal reforms better be funded by legal sector itself. The only exception, is that if the status quo is already close to the “efficient region”, then partial external funding should be considered.

A question follows naturally is that how to we know the status quo  $K$  is too high or too low, given the efficiency of the litigation system  $\alpha$ . One defining feature is to look at the output level. There will be overproduction when  $K$  is too high, as shown in part iv) of lemma 3. And underproduction when  $K$  is too low, as shown in part i) and ii) of lemma 3. Another way to discern it to look at the price, price will be higher than efficient level when  $K$  is too high, and price will be lower than the efficient level when  $K$  is too low. By looking at either price or output, and compare to the efficient level, we can get an idea which direction should the legal reform goes.

## 7 Mixed Payment

Up till now, we have only considered the pure form of payment, which is either ex-ante or ex-post, but not both. There is still one type of payment we have not consider yet, which is the mixed payment. There are many possible way to mix the payment, and I will focus on those contract which satisfies the following conditions:

$$P_{\text{ante}} \geq \left(\frac{1+\alpha}{\alpha}\right)^2 K, \quad (LC_B)$$

$$P_{\text{post}} \geq \left(\frac{1+\alpha}{\alpha}\right)^2 K, \quad (LC_S)$$

$$P_{\text{post}} \leq (1+\alpha)^2 K, \quad (IC_B)$$

$$\frac{\alpha(2+\alpha)}{(1+\alpha)^2} P_{\text{ante}} + P_{\text{post}} \geq \frac{\tilde{e}^2}{2} - K, \quad (IC_S)$$

$$P_{\text{ante}} + P_{\text{post}} \geq \frac{\tilde{e}^2}{2}. \quad (PC_S)$$

Equation  $(LC_B)$  is the legal condition for the buyer will sue the seller if the seller shirks. Equation  $(LC_S)$  is the legal constraint for the seller to sue the buyer if the buyer defaults on the ex-post price. Equation  $(IC_B)$  is the incentive compatibility condition for the buyer to pay the ex-post price. Equation  $(IC_S)$  is the familiar incentive constraint to motivate the seller to pay effort. Finally equation  $(PC_S)$  is the participation constraint of the seller. Given the number of the constraints we face and the possibility of corner solutions, we are not going to study in details the optimal mixed payment contract, but instead we will show that, when the pure payment contract is inefficient, there are efficient mixed payment contract can be offered and it is in the interest of the buyer to offer that.

### 7.1 Case I

Firstly we consider the solution with binding  $LC_B$ :  $P_{\text{ante}} = K \left(\frac{1+\alpha}{\alpha}\right)^2$ . Then by the binding  $PC_S$ ,

$$P_{\text{post}} = \frac{\tilde{e}^2}{2} - K \left(\frac{1+\alpha}{\alpha}\right)^2.$$

Then we have  $EU_S = 0$ ,  $EU_B = \tilde{e} - \frac{\tilde{e}^2}{2}$ . Thus  $\tilde{e}^* = 1$ , and hence  $EU_B^* = 1/2$ . Since IC will always be satisfied, as long as  $\alpha > 1$ , the remaining two constraints delineate the effective legal environment for this contract. The first inequality is derived from  $LC_S$ , while the second inequality is derived from  $IC_B$ .

$$K \leq \frac{\alpha^2}{4(1+\alpha)^2}, \quad (2)$$

$$K \geq \frac{\alpha^2}{2(1+\alpha)^2(1+\alpha^2)}. \quad (3)$$

**Lemma 3.** *When the litigation fixed cost is within certain bound, where  $\frac{\alpha^2}{2(1+\alpha)^2(1+\alpha^2)} \leq K \leq \frac{\alpha^2}{4(1+\alpha)^2}$ , the following mixed payment contract is feasible,*

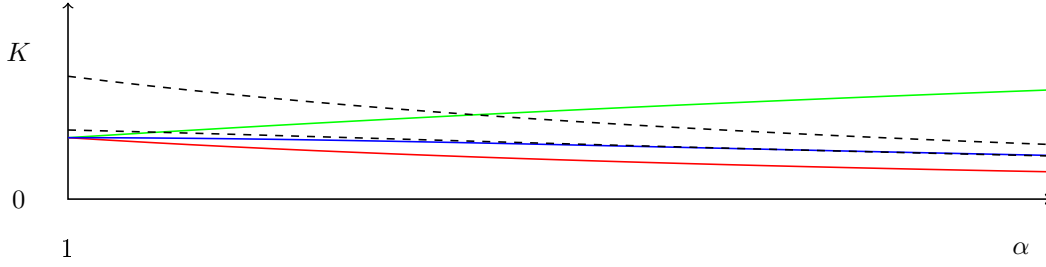


Figure 7: Feasible legal environment for the mixed payment contracts

The case I mixed payment contract is feasible in the region between the green (2) and blue curves (3 or 4). The case II mixed payment contract is feasible in the region between the red (5) and blue curves (3 or 4). The region between the dashed curve is the region for the inefficient case II ex-ante payment contract or case I ex-post payment contract.

$$P_{ante} = K \left( \frac{1+\alpha}{\alpha} \right)^2, \quad P_{post} = \frac{1}{2} - K \left( \frac{1+\alpha}{\alpha} \right)^2.$$

In order to have non-negative ex-post price, we need

$$K \leq \frac{\alpha^2}{2(1+\alpha)^2},$$

where this is satisfied by the  $LC_S$ .

An natural question follows is that will  $IC_S$  be binding? The answer is if so, the  $PC_S$  will not satisfy. By binding  $IC_S$ , substitute in  $P_{ante} = K \left( \frac{1+\alpha}{\alpha} \right)^2$  and  $e = 1$ , we have

$$P_{post} = \frac{1}{2} - 2K \left( \frac{1+\alpha}{\alpha} \right).$$

This is the  $P_{post}$  that satisfies the  $IC_S$ , and it can be shown that this  $P_{post}$  cannot satisfy the  $PC_S$  as follows.

$$\begin{aligned} P_{ante} + P_{post} &= K \left( \frac{1+\alpha}{\alpha} \right)^2 + \frac{1}{2} - 2K \left( \frac{1+\alpha}{\alpha} \right) \\ &= \frac{1}{2} - K \frac{(1+\alpha)(\alpha-1)}{\alpha^2} < \frac{1}{2}. \end{aligned}$$

As a result, it is impossible for both  $IC_S$  and  $PC_S$  to be binding in this case.

## 7.2 Case II

Another possible mixed payment contract is to set  $P_{ante} = K(1+\alpha)^2$ . By the binding  $PC_S$ , we have

$$P_{post} = \frac{\tilde{e}^2}{2} - K(1+\alpha)^2$$



Again we have  $EU_S = 0$ ,  $EU_B = \tilde{e} - \frac{\tilde{e}^2}{2}$ . Thus  $\tilde{e}^* = 1$ , and hence  $EU_B^* = 1/2$ . Since  $IC_S$  will always be binding, the remaining two constraints delineate the effective legal environment for this contract. The first inequality is derived from  $LC_S$ , while the second inequality is derived from  $IC_B$ .

$$K \leq \frac{(\alpha\tilde{e})^2}{2(1+\alpha)^2(1+\alpha^2)}, \quad (4)$$

$$K \geq \frac{\tilde{e}^2}{4(1+\alpha)^2}. \quad (5)$$

**Lemma 4.** *When the litigation fixed cost is between,  $\frac{\tilde{e}^2}{4(1+\alpha)^2} \leq K \leq \frac{(\alpha\tilde{e})^2}{2(1+\alpha)^2(1+\alpha^2)}$ , the following mixed payment contract is feasible,*

$$P_{ante} = K(1+\alpha)^2, \quad P_{post} = \frac{\tilde{e}^2}{2} - K(1+\alpha)^2.$$

It can be shown easily that the remaining constraint  $IC_S$  is binding. To consider the possible range of this two contracts, see figure 7.

From equation (??) and (27), because  $EU_B^* = 1/2$ , we know that the mixed payment contracts are preferred to the case II ex-ante payment contract or case I ex-post payment contract. And hence we have the following proposition.

**Proposition 7.** *Mixed payment contracts improve efficiency.*

*Proof.* Consider the region where  $\frac{1}{4(1+\alpha)^2} \leq K \leq \frac{\alpha^2}{4(1+\alpha)^2}$ , which is within the feasible region for the inefficient case II ex-ante payment contract and case I ex-post payment contract. Now instead with the mixed payment contract, full efficiency can be attained. See Figure 7.  $\square$

## 8 Who should offer the contract

Up till now, we assume the buyer offers the contract to the seller. In real life, there are a lot of cases where the reverse is true, which sellers decides the price, payment timing and the product quality. Typically it is also a take it or leave it offer. So in the following, the role of contracting party will swap. Similar to the previous analysis, all possible cases of ex-ante and ex-post payment contract will be studied in details. The first result is that, if ex-ante payment contract is used, it is more efficient for have the seller as the contract proposer. The second result is that, when the seller proposes, ex-ante payment contract weakly dominates ex-post payment contract. Finally, when the mixed payment contract is taken into account, in particular the two specific mixed payment contracts we have analyzed previously, ex-ante payment contract still weakly dominates.

### 8.1 Ex-ante Payment

Consider firstly with pure ex-ante payment contract. The problem now can be formulated as follows,

$$\begin{aligned}
& \max_{\tilde{e}, P_{\text{ante}}} P_{\text{ante}} - \frac{\tilde{e}^2}{2} \\
& \text{s.t. } \frac{\alpha(2+\alpha)}{(1+\alpha)^2} P_{\text{ante}} + K \geq \frac{\tilde{e}^2}{2}, & (IC_S) \\
& \tilde{e} - P_{\text{ante}} \geq 0, & (PC_B) \\
& P_{\text{ante}} \geq K \left( \frac{1+\alpha}{\alpha} \right)^2, & (LC_B)
\end{aligned}$$

Equation  $(IC_S)$  is the incentive constraint for the the seller to discourage shirking, while equation  $(PC_B)$  is the participation constraint for the buyer, lastly equation  $(LC_B)$  is the legal constraint for the buyer such that buyer will sue the seller in the case of default. Similar to the previous analysis, to envisage the solutions, we can start from  $K = 0$ , when  $LC_B$  becomes  $P_{\text{ante}} \geq 0$ . In this case, the solution is the tangent point on the  $PC_B$ , as the  $IC_S$  intersects with the  $PC_B$  only at points where  $\tilde{e} \geq 1$ .<sup>16</sup> When  $K$  increases up to some threshold, the  $LC_B$  will becomes binding, limiting the offer  $\tilde{e} \geq 1$ . Under this case, the solution is at the intersection point of  $LC_B$  and  $PC_B$ . We are now going to analyze these two cases formally.

### 8.1.1 Case I (Buyer's Participation Constraint binds)

When  $K$  is small, the only binding constraint is the  $PC_B$ . The incentive constraint of the seller must be slack, as shown in footnote 16. The seller's problem is

$$\max P_{\text{ante}} - \frac{\tilde{e}^2}{2}, \quad \text{s.t. } \tilde{e} - P_{\text{ante}} \geq 0.$$

The solution to the problem is

$$\tilde{e}^* = 1 \quad P_{\text{ante}}^* = 1.$$

There are two corresponding conditions for this contract to be effective. First, it is the  $IC_S$ , which requires

$$K \geq \frac{1 - 2\alpha - \alpha^2}{2(1+\alpha)^2}.$$

Since  $\alpha \geq 1$  and  $K \geq 0$ , this condition must be satisfied. In other words, we only need to consider  $LC_B$ , which requires

$$K \leq \left( \frac{\alpha}{1+\alpha} \right)^2. \tag{6}$$

**Lemma 5.** *When the fixed cost of litigation is low enough,  $K \leq \left( \frac{\alpha}{1+\alpha} \right)^2$ , the optimal seller-proposed ex-ante payment contract is  $\tilde{e}^* = P_{\text{ante}}^* = 1$ .*

<sup>16</sup>When  $K = 0$ , the intersection point of  $PC_B$  and  $LC_B$  is  $P_{\text{ante}} = \tilde{e} = \frac{2\alpha(2+\alpha)}{(1+\alpha)^2}$ , it will be greater than 1 if  $2\alpha(2+\alpha) \geq (1+\alpha)^2 \implies \alpha^2 + 2\alpha - 1 \geq 0$ , which is satisfied by the assumption  $\alpha \geq 1$ .

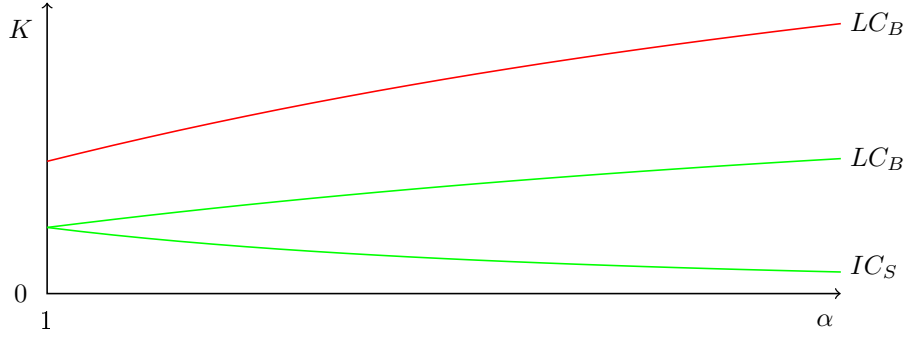


Figure 8: Legal environment for ex-ante payment contract

For the region below the red curve (6), the seller-proposed ex-ante payment contract is feasible. For the region between the two green curves (21 and 22), the case III buyer-proposed ex-ante payment contract is feasible.

### 8.1.2 Case II (Buyer's legal constraint and participation constraint binds)

When  $K$  increases to certain threshold, the  $LC_B$  will become binding. The solution will be at the intersection point of the  $LC_B$  and  $PC_B$ , where

$$\tilde{e} = P_{ante} = K \left( \frac{1 + \alpha}{\alpha} \right)^2$$

To ensure the  $IC_S$  is satisfied, we have condition:

$$K \leq \frac{4\alpha^3}{(1 + \alpha)^3}.$$

Because case II is a corner solution due to the binding  $LC_B$ , the  $EU_S$  must be lower than case I. Hence we have the optimal ex-ante payment contract when seller is the proposer:

**Lemma 6.** When  $0 \leq K \leq \left( \frac{\alpha}{1 + \alpha} \right)^2$ , the optimal seller-proposed ex-ante payment contract is  $\tilde{e}^* = 1$ ,  $P_{ante}^* = 1$ . When  $\left( \frac{\alpha}{1 + \alpha} \right)^2 \leq K \leq \frac{4\alpha^3}{(1 + \alpha)^3}$ , the optimal seller-proposed ex-ante payment contract is  $\tilde{e} = P_{ante} = K \left( \frac{1 + \alpha}{\alpha} \right)^2$ .

From figure 8, we could notice that the region of efficient contracting is much wider than the previous case when buyer propose the contract. So we have the following proposition.

**Proposition 8.** For ex-ante payment contract, it is more efficient to have seller as the proposer.

The main intuition lies in that  $P_{ante}$  is higher in this case, which can be viewed as a larger pie to be competed in the litigation, thus allows a wider range of  $K$ .

## 8.2 Ex-post Payment

When seller propose ex-post payment contract, the whole problem can be formulated as follows,

$$\begin{aligned}
& \max_{\tilde{e}, P_{\text{post}}} P_{\text{post}} - \frac{\tilde{e}^2}{2} \\
& s.t. \quad \tilde{e} - P_{\text{post}} \geq 0, & (PC_B) \\
& \quad P_{\text{post}} \leq K(1 + \alpha)^2 & (IC_B) \\
& \quad P_{\text{post}} \geq K \left( \frac{1 + \alpha}{\alpha} \right)^2 & (LC_S)
\end{aligned}$$

where equation  $(PC_B)$  is the participation constraint of the buyer, equation  $(IC_B)$  is the incentive constraint of the buyer, and equation  $(LC_S)$  is the legal constraint for the seller to sue the buyer, if the buyer do not pay. Again we can envisage the solution by considering first an extreme case. When  $K = 0$ , virtually no ex-post payment contract can be offered, except the one with  $\tilde{e} = 0$ ,  $P_{\text{post}} = 0$ . When  $K$  goes positive, we expect the  $IC_B$  will be binding, the solution will be the intersection point of  $IC_B$  and  $PC_B$ . After  $K$  moves up to higher level, the  $IC_B$  will no longer be binding, and the solution is the tangency point on the  $PC_B$ . If  $K$  is of a even higher level, the  $LC_S$  kicks in, the solution will now be the intersection point of the  $LC_S$  and the  $PC_B$ . We are going to examine these three cases one by one in the following subsections.

### 8.2.1 Case I (Buyer's incentive compatibility constraint and participation constraint binds)

When  $K$  is very low, we will have the following solution,

$$\tilde{e} = P_{\text{post}} = (1 + \alpha)^2 K$$

The  $LC_S$  is satisfied as  $K(1 + \alpha)^2 \geq K \left( \frac{1 + \alpha}{\alpha} \right)^2$ . The only condition is that the intersection point is lower than  $\tilde{e} \leq 1 \iff K \leq \frac{1}{(1 + \alpha)^2}$ ,

**Lemma 7.** *When  $K \leq \frac{1}{(1 + \alpha)^2}$ , the optimal seller-proposed ex-post payment contract is  $\tilde{e}^* = P_{\text{post}}^* = (1 + \alpha)^2 K$ .*

### 8.2.2 Case II (Buyer's participation constraint binds)

When  $K$  is higher, the  $IC_B$  will be slack, and hence the only binding constraint is the  $PC_B$ , and the solution is:

$$\tilde{e}^* = 1, \quad P_{\text{post}}^* = 1.$$

The two corresponding conditions are, firstly from  $LC_S$ :

$$K \leq \left( \frac{\alpha}{1 + \alpha} \right)^2; \tag{7}$$

Secondly from  $IC_B$ :

$$K \geq \frac{1}{(1 + \alpha)^2}. \tag{8}$$

**Lemma 8.** When  $\frac{1}{(1+\alpha)^2} \leq K \leq \left(\frac{\alpha}{1+\alpha}\right)^2$ , the optimal seller-proposed ex-post payment contract is  $\tilde{e}^* = 1$ ,  $P_{post}^* = 1$ .

### 8.2.3 Case III (Buyer's legal constraint and participation constraint binds)

When  $K$  is even higher, the  $LC_S$  kicks in, the solution will be at the intersection of  $LC_S$  and  $PC_B$ .

$$\tilde{e} = P_{post} = \left(\frac{1+\alpha}{\alpha}\right)^2 K$$

The only condition needs to be consider in this case is the  $PC_S$ , which requires

$$K \leq 2 \left(\frac{\alpha}{1+\alpha}\right)^2$$

**Lemma 9.** When  $\left(\frac{\alpha}{1+\alpha}\right)^2 \leq K \leq 2 \left(\frac{\alpha}{1+\alpha}\right)^2$ , the optimal seller-proposed ex-post payment contract is  $\tilde{e}^* = P_{post}^* = \left(\frac{1+\alpha}{\alpha}\right)^2 K$ .

To combine all these three cases, we have the optimal seller-proposed ex-post payment contract.

**Lemma 10.** The optimal seller-proposed ex-post payment contracts are as follows:

$(\alpha, K)$	$\tilde{e}^* \ \& \ P_{post}^*$
$0 \leq K \leq \frac{1}{(1+\alpha)^2}$	$(1+\alpha)^2 K$
$\frac{1}{(1+\alpha)^2} \leq K \leq \left(\frac{\alpha}{1+\alpha}\right)^2$	1
$\left(\frac{\alpha}{1+\alpha}\right)^2 \leq K \leq 2 \left(\frac{\alpha}{1+\alpha}\right)^2$	$\left(\frac{1+\alpha}{\alpha}\right)^2 K$

From figure 9, again as we have seen, the range of efficient contracting is wider in ex-post also, when seller proposes. But the result is not as clear cut as the ex-ante case, since some region is not covered. Notice that the the upper constraint (7) is the same as the one for ex-ante payment (6), that means, when seller propose, the efficient ex-ante payment contract covers all the efficient ex-post payment contract feasible legal environment, and there is more than that. So we have the following result,

**Proposition 9.** When the seller offers contract, ex-ante payment contract weakly dominates ex-post payment contract.

*Proof.* When  $K \geq \left(\frac{\alpha}{1+\alpha}\right)^2$ , the case II ex-ante payment contract and case III ex-post payment contract is feasible, and having the same  $EU_S = K \left(\frac{1+\alpha}{\alpha}\right)^2 \left(1 - \frac{K}{2} \left(\frac{1+\alpha}{\alpha}\right)^2\right)$ . However, the coverage of the ex-ante payment contract is wider than the ex-post contract, as  $2 \left(\frac{\alpha}{1+\alpha}\right)^2 \leq \frac{4\alpha^3}{(1+\alpha)^3}$ . The inequality holds as  $\alpha \geq 1$ .  $\square$

From this we know that, when seller proposes, the optimal pure payment contracts is the ex-ante payment contracts shown in lemma 6.

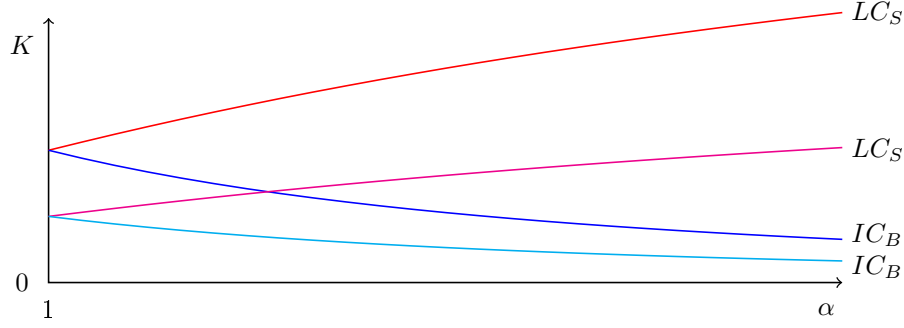


Figure 9: Legal environment for the ex-post payment contract

For the region between the red (7) and blue curves (8), the seller-proposed ex-post payment contract will be offered. For the region between the light red (4.5) and light blue curves (4.6), the case I buyer-proposed ex-post payment contract will be offered.

### 8.3 Mixed Payment

Similar to the buyer-propose mixed payment contract, we have place the same constraints on the seller-propose mixed payment contract,

$$\begin{aligned}
 P_{\text{ante}} &\geq \left(\frac{1+\alpha}{\alpha}\right)^2 K, & (LC_B) \\
 P_{\text{post}} &\geq \left(\frac{1+\alpha}{\alpha}\right)^2 K, & (LC_S) \\
 P_{\text{post}} &\leq (1+\alpha)^2 K, & (IC_B) \\
 \frac{\alpha(2+\alpha)}{(1+\alpha)^2} P_{\text{ante}} + P_{\text{post}} &\geq \frac{\tilde{e}^2}{2} - K, & (IC_S) \\
 P_{\text{ante}} + P_{\text{post}} &\leq \tilde{e}. & (PC_B)
 \end{aligned}$$

Again given the number of the constraints we face and the possibility of corner solutions, we are not going to study in details the optimal mixed payment contract, but instead we will show that, when the pure payment contract is inefficient, there are efficient mixed payment contract can be offered.

#### 8.3.1 Case I

Set  $P_{\text{ante}} = \left(\frac{1+\alpha}{\alpha}\right)^2 K$ , then by binding  $PC_B$ ,

$$P_{\text{post}} = \tilde{e} - \left(\frac{1+\alpha}{\alpha}\right)^2 K.$$

Seller will always choose to pay effort if  $\tilde{e} \leq 2$ , which in order to maximize expected utility,  $EU_S = \tilde{e} - \frac{\tilde{e}}{2}$ , seller will choose  $\tilde{e} = 1$  if feasible. The feasibility depends on the  $K$  which is limited by the following two constraints. The first inequality is derived from  $LC_S$  while the second inequality is derived from  $IC_B$ .

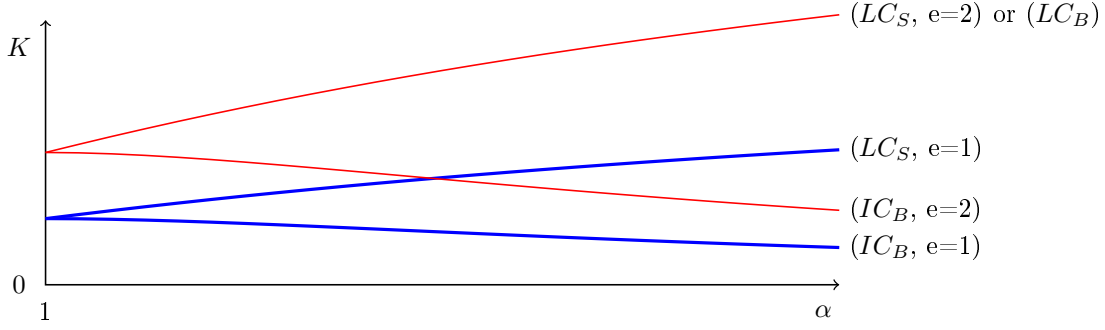


Figure 10: Legal environment for the case I mixed payment contract  
When  $\tilde{e} = 1$ , for the region between the two thick blue curves (8.15 and 8.16), the case I seller-proposed mixed payment contract is feasible.

$$K \leq \frac{\alpha^2 \tilde{e}}{2(1+\alpha)^2}, \quad (9)$$

$$K \geq \frac{\alpha^2 \tilde{e}}{(1+\alpha)^2(1+\alpha^2)}. \quad (10)$$

**Lemma 11.** When the fixed cost of litigation is between  $\frac{\alpha^2}{(1+\alpha)^2(1+\alpha^2)} \leq K \leq \frac{\alpha^2}{2(1+\alpha)^2}$ , the optimal seller-proposed mixed payment contract is  $\tilde{e} = 1$ ,  $P_{ante} = \left(\frac{1+\alpha}{\alpha}\right)^2 K$ ,  $P_{post} = 1 - \left(\frac{1+\alpha}{\alpha}\right)^2 K$ .

In figure 10, the thick blue curves are the two constraints respectively, assuming  $\tilde{e} = 1$ . And if  $\tilde{e} = 2$ , the upper constraint will be the upper red curve (9, e=2), while the lower constraint is the lower red one (10, e=2). Notice that the upper constraint (9) is, when  $\tilde{e} = 2$ , exactly the same as the constraint for the seller-proposed ex-ante payment contract (6). So this mixed payment contract can cover as much as the legal environment as the ex-ante one, but there comes a cost. Which is the reduced returns from this mixed payment contract. Only within the blue curves region will the seller earns  $\frac{1}{2}$ , moving either up or down the  $\tilde{e}$  from 1 to escape from the bounds, will reduce marginal returns by  $1 - \tilde{e}$ .

### 8.3.2 Case II

Just as above, we can set  $P_{ante} = (1+\alpha)^2 K$ , by binding PC,

$$P_{post} = \tilde{e} - (1+\alpha)^2 K.$$

Seller would exert effort so long as  $\tilde{e} \leq 2$  and the remaining two constraints are as follows. The first inequality is derived from  $LC_S$  while the second inequality is derived from  $IC_B$ .

$$K \leq \frac{\alpha^2 \tilde{e}}{(1+\alpha)^2(1+\alpha^2)}, \quad (11)$$

$$K \geq \frac{\tilde{e}}{2(1+\alpha)^2}. \quad (12)$$

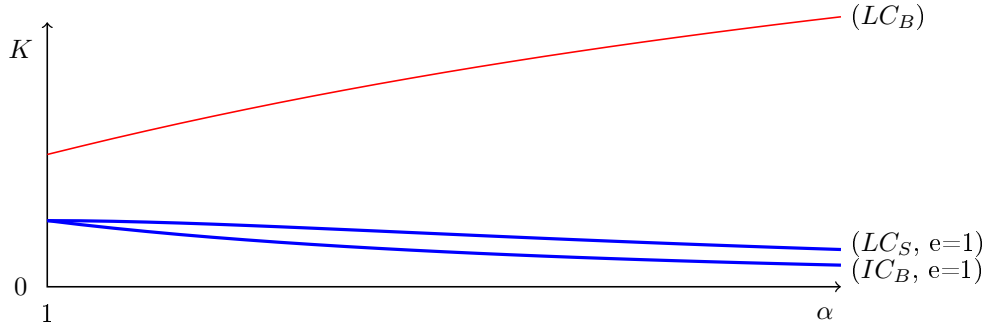


Figure 11: Legal environment for the case II mixed payment contract  
For the region between the two thick blue curves (8.18 and 8.19), the seller-proposed mixed payment contract is feasible.

**Lemma 12.** When the fixed cost of litigation is between,  $\frac{1}{2(1+\alpha)^2} \leq K \leq \frac{\alpha^2}{(1+\alpha)^2(1+\alpha^2)}$ , the optimal seller-proposed mixed payment contract is  $\tilde{e} = 1$ ,  $P_{ante} = (1 + \alpha)^2 K$ ,  $P_{post} = 1 - (1 + \alpha)^2 K$ .

In figure 11, the two constraints are the two thick blue curves respectively, assuming  $\tilde{e} = 1$ . Which we can see again the feasible region is within the covered area of the ex-ante payment contract. This again shows the inferiority of the mixed payment contract, and we have the following result,

**Proposition 10.** When the seller proposes, (purely) ex-ante payment contract weakly dominates mixed payment contract.

*Proof.* Given the condition which the ex-ante payment contract works is  $K \leq \left(\frac{\alpha}{1+\alpha}\right)^2$ . Both conditions, (11) and (12), are less stringent than  $K \leq \left(\frac{\alpha}{1+\alpha}\right)^2$ , when  $\tilde{e} = 1$ .  $\square$

The key intuition is that when the seller proposes, he will try to maximize the price, which encourages suing in case of breaching contract, which provides enough incentive for fully efficient contract to be implemented. However when the buyer proposes, he will try to minimize the price, which makes it more restrictive for damaged party willing to sue, thus inefficient contract are proposed, and that opens a room of improvement for mixed contracts, which is not present in the case of seller as the proposer.

## 9 Literature Review

If we consider both trading parties are firms, then the timing of payment problem is studied under the trade credit literature. The main focus of analysis is place on the optimal credit period to be given. If prepayment is selected, the buyer assumes greater product quality risk since buyer cannot inspect the product before payment. Conversely, if trade credit is extended, the seller assumes responsibility for assessing credit risk, financing, and collecting receivables. To put this into a international trade context, recent cases of product adulteration by foreign suppliers have compelled many manufacturers to rethink approaches to deterring suppliers from cutting corners.<sup>17</sup> Recognizing that product liability

<sup>17</sup>Baxter recalled its Heparin in 2008 (Fairclough, 2008), Mattel toys of unapproved lead paint (Story and Barboza, 2007), Pet food due to harmful ingredients such as melamine (Newman, 2007).



and product warranty with foreign suppliers are rarely enforceable,<sup>18</sup> some manufacturers turn payment into ex-post contingent on no defects discovery.<sup>19</sup> Which is the case where the  $K$  is too high, and outside the bound of any contract with legal protection, and hence the the buyer requests a ex-post payment in order to control the quality of the goods supplied.

An example of ex-post payment in practice is a well-known and widely used financial contract called trade credit.<sup>20</sup> Trade credit is the largest source of external short-term financing for firms both in the US (Petersen and Rajan, 1994) and internationally (Rajan and Zingales, 1995). Ex-post contingent payments (via trade credit) allow the buyers to learn about suppliers' product quality and to withhold contingent payments in case the suppliers produced defective products (Smith, 1987), (Long et al., 1993). Lee and Stowe (1993) argue that ex-post contingent payments (via trade credit) can be thought of as a very strong form of implicit product warranty, because the buyer can return the product to the supplier and refuse to pay without having to prove that the product is of low quality. More recently, Klapper et al. (2010) articulated that the ex-post contingent payments (via trade credit) reduce the buyers' risk because the buyers have more time to investigate product quality before deciding whether or not to make the contingent payments. For a general review of supply risk problems and solutions, see Tang (2006). Theoretical models that explain trade credit's popularity and the corresponding empirical findings are reviewed in Petersen and Rajan (1997), Biais and Gollier (1997), Ng et al. (1999), and Giannetti et al. (2011).

In the trade credit literature, Long et al. (1993) present an empirical model that builds on the idea articulated by Smith (1987) that trade credit can provide product quality guarantees. By analyzing a sample data that contains all industrial firms from 1984 to 1987, Long et al. (1993) provide empirical evidence to show that the trade credit period increases when defects take more time to discover. Emery and Nayar (1998) present a trade credit model in which the supplier knows the exact time at which the buyer can verify the quality of the product and they show that it is optimal for the supplier to demand payment at the instant before the buyer can verify the product quality. Similarly, Lee and Stowe (1993) propose a signaling model where the quality of the product is known to the supplier but not the buyer and find a separating equilibrium, in which trade credit terms reflect product quality. My model differs from Smith (1987); Lee and Stowe (1993); Emery and Nayar (1998). I do not assume that the quality of the product is exogenous. Instead, I consider the case when the seller can optimally decide on whether or not to shirk, so that the seller's decision is endogenous. Thus I am solving a moral hazard problem rather than a signaling problem.

Babich and Tang (2012) examined simple contingent payment mechanisms to deter suppliers from product adulteration under which the contingent payments are fully controlled by the manufacturer: deferred payment, inspection, and combined mechanisms. They established the conditions under which one mechanism dominates the others. Their study primarily focused on the moral hazard on the seller side, but the moral hazard on the buyer side is not discussed at all. Different from their setup,

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<sup>18</sup>Foreign supplier's product liability is rarely enforceable due to different legal systems and inconsistent law enforcement practices in different countries. In some cases, the manufacturer may not even be able to trace the true identity of the fraudulent supplier. And the legal process to claim supplier's product liability can take up to 10 years in foreign countries such as China, the processing cost of a legal case can be very high (Yang, 2007).

Similarly, warranties can be difficult to implement in practice. To get the supplier to pay, the buyer has to prove that a warranty event occurred and that the fault for this event lies with the supplier. And the supplier may be unable to pay, may refuse to pay, or may declare bankruptcy, and courts may need to get involved. For example, as Sherefkin and Armstrong (2003) report, GM demanded \$37.5 million warranty payment from its supplier. But because Oxford Automotive was in bankruptcy, GM and the supplier eventually settled for a smaller amount.

<sup>19</sup>A recent article in the Wall Street Journal (Vandenbosch and Sapp, 2010) supports ex-post payment as a way to "keep the suppliers honest".

<sup>20</sup>For example, trade credit contract "net 30" allows the buyer to delay the payment for 30 days, giving the buyer a 30-day interest-free loan. The trade credit period depends on the industry and on the countries associated with the buyer and the supplier.

inspection carries no uncertainty in my paper. Product quality is known immediately once passed to the buyer, which I abstract away from the exact duration of trade credit period.

Dnes, A. W. (1999) consider licensing or franchising agreement in a country with limited rule of law where the franchiser may abscond. The contract needs to be self-supporting, which may be achieved by the careful structuring of the timing of payments. The main idea can be illustrated by the following inequality,  $P \leq kL + (1 - k)(L - D)$ , where  $P$  = franchiser's profits from completing contract,  $L$  = initial franchise fee,  $k$  = probability of franchiser successfully absconding and  $D$  = penalty applied to absconding franchiser.<sup>21</sup> So this is essentially an incentive constraint. By singling out  $L$ , we have an upper limit of the initial fee (ex-ante payment),  $L \leq P + (1 - k)D$ . So the rest of the payment will pay ex-post to ensure franchiser at least break-even. The probability of run-away,  $k$ , is exogenous, but in my paper, it is endogenous and both parties have this chance. My paper explicitly model the litigation part which each party decides his spending for the lawsuit.

Faith and Tollison (1980) is the most relevant paper which they directly deal with timing of payment problem. Although there are several interesting claims, it contains no formal model. They suggest timing of payment is an informal institutions that have evolved alongside formal contracts to mitigate agency cost. Their basic claim is that ex-post payment is a rational institutional arrangement to control the significant transaction costs inherent in certain types of exchanges characterized by interpersonal differences in information. There are two cases: Case I - Both traders are equally imperfectly informed ex-ante and equally better informed ex-post. Case II - The seller has an informational advantage ex-ante, but buyer and seller are equally informed ex post. The main problem faced by the seller in the product market, when using ex-post payment is that buyer refuse to pay. But since in general, sellers are competing for buyers, while the reverse is not usually the case, so ex-post payment is a more rational arrangement. Faith and Tollison (1980) paper is full of insight, the main drawback is that the lack of formal modeling of their claims. My paper provides a simple formal model to analyze the timing of payment, while there is no interpersonal differences in information.

Smith and Cox (1985) provides an empirical study of pricing routine legal services. Price may be determined either ex ante or ex post supply. A large cross-sectional survey of law firms does suggest significant variation in whether fixed fee pricing or hourly rate pricing is adopted. Contractual provisions may be understood as a market response to differences in the relative potential for opportunistic cheating by buyers (clients) and by sellers (law firms). But there is a key difference between my paper and theirs. They assume that contracts are perfectly enforceable, so the main focus is on the timing of price determination.

Lee and Png (1990) studies the role of installment payments in relationships characterized by moral hazard and sunk costs. They rule out vertical integration and payments contingent on the product of the contractor. Instead, each payment is negotiated as and when made. In such circumstances, an initial payment serves to redress the weakness of the contractor in ex post renegotiation. If higher effort by the contractor in the first stage increases the marginal product of effort in the second stage, a second installment payment induces the contractor to invest greater effort initially. The contract that they consider is renegotiation-proof, which means there is no need for court to get involved at all, which is the key difference between their study and my paper.

Chen (2004) shows the role of an up-front payment to a contract, with a two-state-two-period model, under the reliance damage measure. He find that in most cases efficiency is not achievable even when an up-front payment is employed. To achieve efficiency, we need three conditions. First, a high enough total payment to make the seller unwilling to breach under the efficient reliance level; second, a high enough up-front payment to make the seller unwilling to sue under the efficient reliance level when the buyer breaches; third, a high enough trading price to make the buyer breach when the low

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<sup>21</sup>This is a simplified version by assuming sunk cost to be zero.

state appears. Lacking any one of these conditions, efficiency fails. The up-front payment therefore plays an indispensable role for efficiency. However my findings suggest that efficiency can be attained even with pure ex-post payment, provided that the legal environment is suitable.

## 10 Some alternative explanations of different payment schemes

There are undoubtedly many other possible explanations for the various payment schemes found in the market. It will be useful to examine some competing explanations. The following discussion does not mean to be a complete list of alternative hypotheses, nor is it a denial of the relevance of alternative explanations. The purpose here is simply to point out the existence of some of the more prominent competing explanations.

One conceivable explanation of the timing of payment is that, depends on the good in question, there is a payment scheme which minimizes the time costs incurred in making payment. This explains multiple cash registers, express line, exact-change requirement. Generally speaking, however, the time cost of paying are likely to be the same with respect to ex-ante or ex-post payment.<sup>22</sup> Ex-ante or ex-post payment would not seem to play a very significant role in reducing the time costs of making payment.

The second potential explanation for the economic distinction between ex-ante and ex-post payment system is that the latter involve an extension of short term credit to customers. But according to this reasoning, there should not be any systematic difference among companies in the timing of collecting revenue. If buyers prefer these short-term loans, competition among sellers will drive all payment to ex-post. What can be said along this line of logic is that firms which choose to extend short term credit by the means of ex-post payment, will try to reduce relevant costs of default. For example, restaurants will have restricted and closely monitored exits if payment is made ex-post.

A firm's size affects its decision of payment scheme. As there is fixed cost associated with managing outstanding credit, it spreads over more customers as the firm's customer base expands. Furthermore, the larger the seller's customer base, it is more likely to have more information for some particular type of customers, specifically their credit quality. This led to the prediction that bigger firms would be more likely to charge ex-post. It is not very clear whether this holds in general, as counter examples can be found easily.

Another reason why upfront payment will be preferred is because it increases turnover. Consider any restaurant who has a fixed number of tables, limited operating hours per day, the longer a customer stays after finishing his meal, the less money is made. But why is the staying time related to the timing of payment? The argument is mainly about customer psychology, customers who have not paid can justify their occupation of a table, by the mere possibility of further ordering. On the other hand, those who have paid will have no moral justification for staying after their meals are finished. Although this sounds convincing, a higher table turnover does not necessarily go along with higher profit. A related side issue is that, tips must be paid afterward in order to grade the service, which is one of the key reasons why there is ex-post payment in restaurants who concern for their service quality.

The last alternative explanation is that, from an anthropologist or marketing specialist view, timing of payment may be due to certain social customs. Although there is doubtlessly an element of truth in it, all normal social behavior is to some extent customary. A positive economist cannot rest easy with such irrefutable arguments. Custom itself is often an implicit compromise of fundamental economic conflict, it deserves an economic explanation.

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<sup>22</sup>But not likely to be the same for mixed payment, which requires customer paying twice, in advance and afterward.

## 11 Conclusion

This paper provides a simple model to analyze the timing of payment problem. In case of dispute, either the seller fails to perform or the buyer defaults, the court will get involved to resolve the conflict. The efficiency of the legal system and the fixed cost of litigation are the keys for the choice of payment timing. My model predicts, generally speaking, ex-ante payment contract is more prevalent in region with low fixed cost of using the legal system. But since under this situation, ex-ante payment contract is inefficient, by using certain mixed payment contract, full efficiency can be attained. The choice of payment timing becomes unimportant, as the fixed cost goes up to certain range, with respect to the efficiency level of the legal system, both ex-ante and ex-post payment contract are the same and perfectly efficient. Apart from the timing of the payment, the model also have some implications for the proposer of the contract. For ex-ante payment contract, it is more efficient to have the seller as the proposer of the contract, in the sense that it is enforceable in a much wider legal environment. An interesting finding is that, when seller proposes, a simple ex-ante payment contract dominates ex-post payment, which may explain why most of the offers in everyday life are made by the sellers in a simple ex-ante payment form. This suggests that primarily mixed payment is a contractual device used by the buyer to mitigate the two sided moral hazard problem. Besides, this paper also sheds light on the self-funding issue of legal reform.

## 12 Appendices

### 12.1 UK system

Under the UK system, the losing party is responsible for paying all the litigation expenses. In the litigation stage, if  $i$  is the innocent party and  $j$  is the faulty one, the expected value of litigation will be:

$$\begin{aligned} V_i &= p(x_i, x_j)C - (1 - p(x_i, x_j))(x_i + x_j + 2K) \\ &= \frac{\alpha x_i}{\alpha x_i + x_j}C - \frac{x_j}{\alpha x_i + x_j}(x_i + x_j + 2K) \end{aligned} \quad (13)$$

$$\begin{aligned} V_j &= -(C + x_i + x_j + 2K)p(x_i, x_j) \\ &= -(C + x_i + x_j + 2K)\frac{\alpha x_i}{\alpha x_i + x_j} \end{aligned} \quad (14)$$

where  $C$  denotes the compensation paid by the losing party to the winning party,  $K$  is the fixed cost of litigation. In the case of buyer default, the compensation would be  $P_{\text{post}}$ .<sup>23</sup> In the case of seller default, the compensation would be  $P_{\text{ante}}$ , which would leaves buyer not damaged.<sup>24</sup>

<sup>23</sup>This compensation is according to the expectation damage principle. Under the expectation damage measure, the defaulting party pays an amount that puts the other party in the position he would have been in had the contract been performed. For details, see Shavell (1980).

<sup>24</sup>This compensation is according to the restitution damage principle. Under the restitution damage measure the defaulting party returns only the payments made to him. See Shavell (1980).

The respective first order conditions of the expected value maximization are:

$$\begin{aligned}
\frac{dV_i}{dx_i} &= \frac{\alpha x_j}{(\alpha x_i + x_j)^2} C + \frac{\alpha x_j}{(\alpha x_i + x_j)^2} (x_i + x_j + 2K) - \frac{x_j}{\alpha x_i + x_j} = 0 \\
&= \frac{x_j}{(\alpha x_i + x_j)^2} (C + (\alpha - 1)x_j + 2\alpha K) = 0 \\
\frac{dV_j}{dx_j} &= -\frac{\alpha x_i}{(\alpha x_i + x_j)^2} (C + x_i + x_j + 2K) - \frac{\alpha x_i}{\alpha x_i + x_j} = 0 \\
&= -\frac{\alpha x_i}{(\alpha x_i + x_j)^2} (C + (1 - \alpha)x_i + 2K) = 0
\end{aligned}$$

Define  $x_i^*, x_j^*$  as the respective litigation spending of party i and j in equilibrium. Thus we have

$$x_i^* = 0 \text{ or } \frac{C + 2K}{\alpha - 1} \quad (15)$$

$$x_j^* = 0 \text{ or } \frac{\alpha(C + 2K)}{1 - \alpha} \quad (16)$$

and the second order conditions are:

$$\begin{aligned}
\frac{d^2V_i}{dx_i^2} &= \frac{-2\alpha x_j (\alpha x_i + x_j) (c + (\alpha - 1)x_j + 2\alpha K)}{(\alpha x_i + x_j)^4} \\
\frac{d^2V_j}{dx_j^2} &= \frac{-2\alpha x_i (\alpha x_i + x_j) (c + (1 - \alpha)x_i + 2K)}{(\alpha x_i + x_j)^4}
\end{aligned}$$

Since by assumption  $\alpha > 1$ , the solution  $x_j = \frac{\alpha(C+2K)}{1-\alpha} < 0$  is rejected. As a result, we have only two Nash equilibria,  $(x_i^*, x_j^*) = \left\{ (0, 0), \left( \frac{C+2K}{\alpha-1}, 0 \right) \right\}$ . I will take the second one as the solution as the first one is associated with an undefined winning probability.

**Lemma 13.** *Under the UK system, the optimal litigation spending is*

$$\begin{aligned}
x_i^* &= \frac{C + 2K}{\alpha - 1} \\
x_j^* &= 0
\end{aligned}$$

According to this solution, the equilibrium expected value from litigation is

$$\begin{aligned}
V_i^* &= \frac{\alpha x_i}{\alpha x_i + x_j} C - \frac{x_j}{\alpha x_i + x_j} (x_i + x_j + 2K) \\
&= C
\end{aligned} \quad (17)$$

$$\begin{aligned}
V_j^* &= -(C + x_i + x_j + 2K) \frac{\alpha x_i}{\alpha x_i + x_j} \\
&= -\left( C + \frac{C + 2K}{\alpha - 1} + 2K \right) \\
&= -\frac{\alpha}{\alpha - 1} (C + 2K)
\end{aligned} \quad (18)$$

This means that, under the UK system, the court is a costless solution to the innocent party. And the innocent party get compensated with certainty. This will completely deter cheating or shirking behavior, and essentially we can obtain the simple first best solution.

## 12.2 Ex-ante Payment Contracts

### 12.2.1 Case I (Incentive compatibility constraint binds)

Let us consider firstly where only the IC constraint binds. The maximization problem is

$$\max_{\tilde{e}, P_{\text{ante}}} \tilde{e} - P_{\text{ante}} \quad s.t. \quad \frac{\alpha(2+\alpha)}{(1+\alpha)^2} P_{\text{ante}} + K = \frac{\tilde{e}^2}{2}.$$

Hence,

$$\begin{aligned} \tilde{e}^* &= \frac{\alpha(2+\alpha)}{(1+\alpha)^2}, \\ P_{\text{ante}}^* &= \frac{\alpha(2+\alpha)}{2(1+\alpha)^2} - K \frac{(1+\alpha)^2}{\alpha(2+\alpha)}. \end{aligned}$$

In equilibrium,

$$\begin{aligned} EU_B^* &= \frac{\alpha(2+\alpha)}{2(1+\alpha)^2} + K \frac{(1+\alpha)^2}{\alpha(2+\alpha)} > 0, \\ EU_S^* &= \frac{\alpha(2+\alpha)}{2(1+\alpha)^2} - K \frac{(1+\alpha)^2}{\alpha(2+\alpha)} - \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^4} \\ &= \frac{\alpha(2+\alpha)}{2(1+\alpha)^4} - K \frac{(1+\alpha)^2}{\alpha(2+\alpha)}. \end{aligned}$$

The PC is satisfied if and only if:

$$EU_S \geq 0 \iff \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6} \geq K. \quad (19)$$

For the buyer sues the seller when the latter shirks, we need the *LC* being satisfied,

$$\begin{aligned} K &\leq \left( \frac{\alpha}{1+\alpha} \right)^2 P_{\text{ante}} = \frac{\alpha^3(2+\alpha)}{2(1+\alpha)^4} - K \frac{\alpha}{(2+\alpha)} \\ \iff K &\leq \frac{\alpha^3(2+\alpha)^2}{4(1+\alpha)^5}. \end{aligned}$$

It can be shown that equation (19) is more stringent, as we have

$$\frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6} < \frac{\alpha^3(2+\alpha)^2}{4(1+\alpha)^5},$$

where the inequality is a consequence of  $\frac{1}{1+\alpha} < \frac{\alpha}{2}$ . Since  $\alpha > 1$ , the inequality is true. Thus the only condition is the one satisfies *PC*. To summarize,

**Lemma 14.** *When  $K \leq \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6}$ , the optimal pure ex-ante payment contract will be,*

$$\tilde{e}^* = \frac{\alpha(2+\alpha)}{(1+\alpha)^2}, \quad P_{\text{ante}}^* = \frac{\alpha(2+\alpha)}{2(1+\alpha)^2} - K \frac{(1+\alpha)^2}{\alpha(2+\alpha)}.$$

The following three lemmas are the comparative statics with respect to  $K$  and  $\alpha$ .

**Lemma 15.** *The optimal  $\tilde{e}^*$  and  $P_{ante}^*$  are increasing in  $\alpha$ .  $P_{ante}^*$  is decreasing in  $K$ , while  $K$  has no effect on  $\tilde{e}^*$ .*

*Proof.*

$$\begin{aligned}\frac{d\tilde{e}^*}{d\alpha} &= \frac{1}{(1+\alpha)^3} > 0 \\ \frac{d\tilde{e}^*}{dK} &= 0 \\ \frac{dP_{ante}^*}{d\alpha} &= \frac{1}{(1+\alpha)^3} + \frac{2(1+\alpha)K}{\alpha^2(2+\alpha)^2} > 0 \\ \frac{dP_{ante}^*}{dK} &= -\frac{(1+\alpha)^2}{\alpha(2+\alpha)} < 0\end{aligned}$$

□

**Lemma 16.** *The expected payoff of the buyer is increasing with  $K$ , while the expected payoff of the seller is decreasing with  $K$ .*

*Proof.*

$$\frac{dEU_B}{dK} = \frac{(1+\alpha)^2}{\alpha(2+\alpha)} > 0, \quad \frac{dEU_S}{dK} = -\frac{(1+\alpha)^2}{\alpha(2+\alpha)} < 0.$$

□

**Lemma 17.** *The expected payoff of the buyer is increasing with  $\alpha$ , if  $K \leq \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^4}$ . While the expected payoff of the seller is increasing with  $\alpha$ , if  $K \leq \frac{(\alpha^2+2\alpha-1)\alpha^2(2+\alpha)^2}{2(1+\alpha)^6}$ .*

*Proof.*

$$\begin{aligned}\frac{dEU_B}{d\alpha} &= \frac{1}{(1+\alpha)^3} - \frac{2(1+\alpha)}{\alpha^2(2+\alpha)^2}K \\ \frac{dEU_B}{d\alpha} \geq 0 &\iff \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^4} \geq K \\ \frac{dEU_S}{d\alpha} &= \frac{1-2\alpha-\alpha^2}{(1+\alpha)^5} + \frac{2(1+\alpha)}{\alpha^2(2+\alpha)^2}K \\ \frac{dEU_S}{d\alpha} \geq 0 &\iff \frac{(\alpha^2+2\alpha-1)\alpha^2(2+\alpha)^2}{2(1+\alpha)^6} \geq K\end{aligned}$$

□

### 12.2.2 Case II (Participation constraint and incentive compatibility constraint bind)

When  $K$  becomes larger, the condition in lemma 14 may not be satisfied, which ensures both  $PC$  and  $LC$  to be satisfied. Under that situation, we will have solution at the kink point which both  $PC$  and  $IC$  are binding:

$$P_{ante} - \frac{\tilde{e}^2}{2} = \frac{P_{ante}}{(1+\alpha)^2} - K = 0.$$

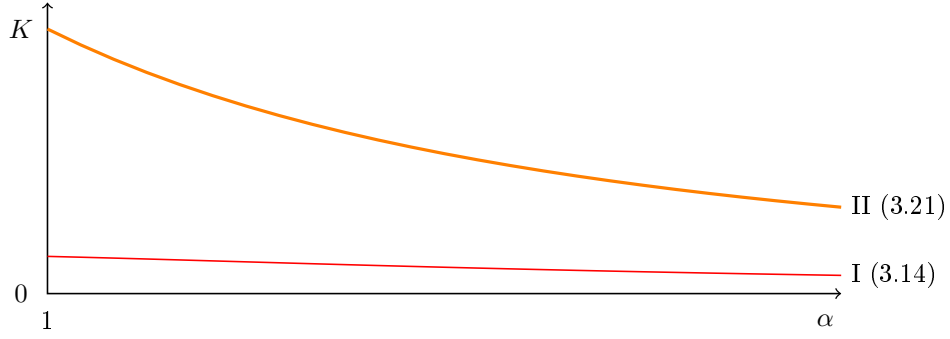


Figure 12: Legal environment for the two ex-ante payment contracts  
The area below the red curve (19) are the legal environments  $(\alpha, K)$  such that the case I ex-ante payment contract is feasible. And the region which below the orange thick curve (20) is where the case II ex-ante payment contract is feasible. Note that the two curves never intersect.

Then we get,

$$\begin{aligned} P_{\text{ante}}^* &= K(1 + \alpha)^2, \\ \tilde{e}^* &= \sqrt{2K}(1 + \alpha). \end{aligned}$$

Under this contract, buyer will always sue seller if he shirks, since  $K(1 + \alpha)^2 > K\left(\frac{1+\alpha}{\alpha}\right)^2$ . The buyer will offer this contract if

$$EU_B^* \geq 0 \implies \frac{2}{(1 + \alpha)^2} \geq K. \quad (20)$$

**Lemma 18.** When  $K \leq \frac{2}{(1+\alpha)^2}$ , the optimal pure ex-ante payment contract is:

$$\tilde{e}^* = \sqrt{2K}(1 + \alpha), \quad P_{\text{ante}}^* = K(1 + \alpha)^2.$$

We can show that the case II contract is applicable whenever case I contract is applicable by comparing the conditions.

**Lemma 19.**  $\frac{2}{(1+\alpha)^2} \geq \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6}$ .

*Proof.*  $\frac{2}{(1+\alpha)^2} \geq \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6}$  is equivalent to

$$4(1 + \alpha)^4 \geq \alpha^2(2 + \alpha)^2,$$

which can be written as  $4 + 16\alpha + 20\alpha^2 + 12\alpha^3 + 3\alpha^4 \geq 0$ . Given  $\alpha \geq 1$ , the inequality holds.  $\square$

To see it in diagram, see figure 12. The following lemma shows that the case I ex-ante payment contract is superior.

**Lemma 20.** The case I ex-ante payment contract will be offered whenever it is applicable.

The intuition of this result is clear, as the kink-point solution is only taken when the tangency is no longer applicable. Another way to see this, by Le Chatelier's principle, case I solution comes with only *IC* constraint, but case II comes with an extra constraint *PC*.



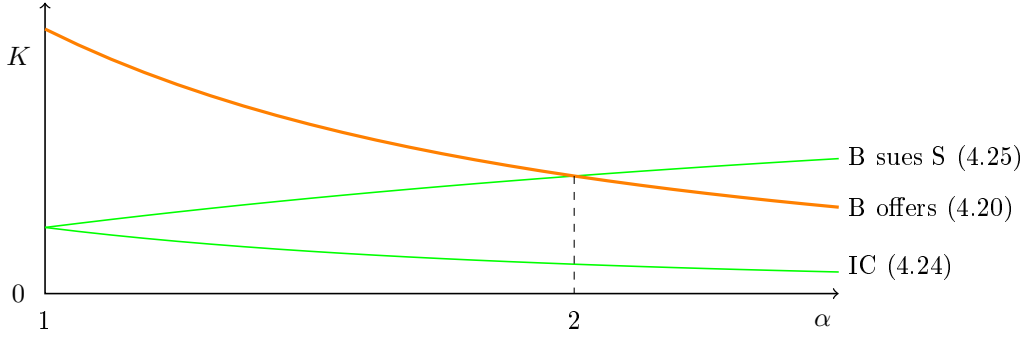


Figure 13: Legal environment for the second and third ex-ante payment contract  
The two green curves (21) and (22) are the corresponding constrains for the case III contract to be feasible. This diagram shows when  $\alpha$  is low, the feasible region of the case III contract is within the boundary of the case II contract, where under the orange thick curve (20), but when  $\alpha > 2$ , this is not true.

### 12.2.3 Case III (Participation constraint binds)

When  $K$  keep increasing, the solution will go back to the tangency, on the left of the intersection point of  $PC$  and  $IC$ , where only  $PC$  binds.

$$\max_{\tilde{e}, P_{\text{ante}}} \tilde{e} - P_{\text{ante}} \quad s.t. \quad P_{\text{ante}} = \frac{\tilde{e}^2}{2},$$

$$\tilde{e}^* = 1, \quad P_{\text{ante}} = \frac{1}{2}, \quad EU_P = \frac{1}{2}.$$

There are two constraints need to be satisfied, firstly the  $IC$  that the seller exert the required effort.

$$K \geq \frac{1}{2(1+\alpha)^2}. \quad (21)$$

The other one is that the buyer would sue the seller if the seller shirks,

$$K \leq \frac{\alpha^2}{2(1+\alpha)^2}. \quad (22)$$

**Lemma 21.** When the fixed cost of litigation is medium,  $\frac{1}{2(1+\alpha)^2} \leq K \leq \frac{\alpha^2}{2(1+\alpha)^2}$ , the optimal pure ex-ante payment contract will be,

$$\tilde{e}^* = 1, \quad P_{\text{ante}} = \frac{1}{2}.$$

### 12.2.4 Case IV (Participation constraint & legal constraint bind)

When  $K$  becomes really high such that  $LC$  lies on the price level associated with the efficient effort level, i.e. when  $K = \frac{\alpha^2}{2(1+\alpha)^2}$ ,  $LC$  will have effect on the solution. The solution will be at the

intersection point of  $PC$  and  $LC$ .

$$P_{\text{ante}} = \frac{\tilde{e}^2}{2} = K \left( \frac{1+\alpha}{\alpha} \right)^2$$

$$\tilde{e} = \sqrt{2K} \left( \frac{1+\alpha}{\alpha} \right)$$

To check that the  $IC$  is satisfied,

$$\frac{\alpha(2+\alpha)}{(1+\alpha)^2} P_{\text{ante}} + K \geq \frac{\tilde{e}^2}{2} \implies K \geq \frac{P_{\text{ante}}}{(1+\alpha)^2}.$$

By substitute  $P_{\text{ante}} = K \left( \frac{1+\alpha}{\alpha} \right)^2$  into the inequality, we have  $\alpha^2 \geq 1$ . Since by assumption  $\alpha > 1$ , the inequality holds. But the buyer will only offer this contract if this gives him positive payoff.

$$EU_B^* = \tilde{e} - P_{\text{ante}} \geq 0 \implies \sqrt{2K} \left( \frac{1+\alpha}{\alpha} \right) \geq K \left( \frac{1+\alpha}{\alpha} \right)^2,$$

which can be simplified as

$$K \leq \frac{2\alpha^2}{(1+\alpha)^2}. \quad (23)$$

**Lemma 22.** *The optimal pure ex-ante payment contract with binding  $PC$  and  $LC$  is, if  $\frac{\alpha^2}{2(1+\alpha)^2} \leq K \leq \frac{2\alpha^2}{(1+\alpha)^2}$ ,*

$$\tilde{e}^* = \sqrt{2K} \left( \frac{1+\alpha}{\alpha} \right), \quad P_{\text{ante}}^* = K \left( \frac{1+\alpha}{\alpha} \right)^2.$$

## 12.3 Ex-post Payment Contracts

### 12.3.1 Case I (Participation constraint and buyer's incentive compatibility constraint bind)

When  $K$  is low, we will have solution where  $PC$  and  $IC$

$$P_{\text{post}} = (1+\alpha)^2 K.$$

So seller will always sue buyer if he is not paying, as  $P_{\text{post}} = (1+\alpha)^2 K \geq \left( \frac{1+\alpha}{\alpha} \right)^2 K$ .

With the binding participation constraint,  $P_{\text{post}} - \frac{\tilde{e}^2}{2} = 0$ , we have

$$\tilde{e} = \sqrt{2K} (1+\alpha).$$

Which is the same as case II of ex-ante payment. Buyer will offer this contract if,

$$EU_B^* \geq 0 \implies \tilde{e}^* - P_{\text{post}}^* \geq 0,$$

which is equivalent to

$$K \leq \frac{2}{(1+\alpha)^2}. \quad (24)$$

**Lemma 23.** *When  $K \leq \frac{2}{(1+\alpha)^2}$ , the optimal ex-post payment contract is,*

$$\tilde{e}^* = \sqrt{2K}(1 + \alpha), \quad P_{post}^* = (1 + \alpha)^2 K.$$

### 12.3.2 Case II (Participation constraint binds)

Consider first the optimal contract that buyer will pay,

$$\begin{aligned} \max \tilde{e} - P_{post} \quad s.t. \quad & P_{post} - \frac{\tilde{e}^2}{2} \geq 0, \\ \tilde{e}^* = 1, \quad & P_{post}^* = \frac{1}{2}, \quad EU_B^* = \frac{1}{2}. \end{aligned}$$

So seller would sue buyer if,

$$K \leq \frac{\alpha^2}{2(1 + \alpha)^2}. \quad (25)$$

Buyer would pay if,

$$K \geq \frac{1}{2(1 + \alpha)^2}. \quad (26)$$

Notice that, this is exactly the same as the type III contract in the ex-ante payment.

**Lemma 24.** *For medium range of fixed cost,  $\frac{1}{2(1+\alpha)^2} \leq K \leq \frac{\alpha^2}{2(1+\alpha)^2}$ , the optimal ex-post payment contract is efficient, where*

$$\tilde{e}^* = 1, \quad P_{post}^* = \frac{1}{2}.$$

**Comparison: Case II Vs Case I** To compare this contract with the previous one,

$$EU_B^I \leq EU_B^{II} \implies (1 + \alpha) \left( \sqrt{2K} - K(1 + \alpha) \right) \leq \frac{1}{2},$$

which is equivalent as

$$K \geq \frac{1}{2(1 + \alpha)^2}. \quad (27)$$

But this is exactly the lower bound for the second ex-post payment contract to implement, for any other  $K$ ,  $EU_B^I \leq EU_B^{II}$ . So we have the following result.

**Lemma 25.** *The case II ex-post payment contract dominates the case I contract.*

### 12.3.3 Case III (Participation constraint & seller's legal constraint bind)

When  $K$  becomes higher,  $LC_S$  will be binding, and the solution is determined by the intersection point of  $PC$  and  $LC_S$ .

$$\begin{aligned} p_{post} &= \frac{\tilde{e}^2}{2} = K \left( \frac{1 + \alpha}{\alpha} \right)^2, \\ \tilde{e} &= \sqrt{2K} \left( \frac{1 + \alpha}{\alpha} \right). \end{aligned}$$

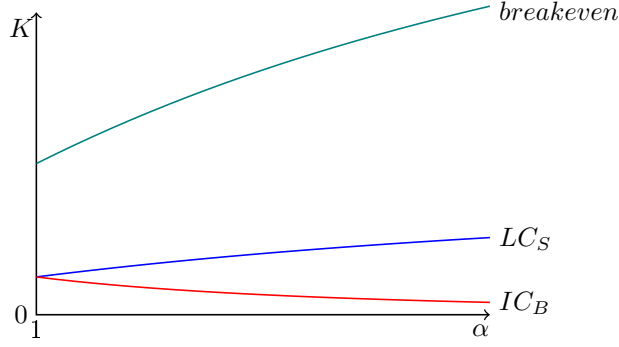


Figure 14: Legal environment for ex-post payment contract

For the region below the red curve 26, the case I ex-post payment contract is optimal. For the region between the blue (25) and red curve (26), the case II ex-post payment contract is optimal. For the region between the blue (25) and teal curve 28, the case III ex-post payment contract is optimal.

$IC_B$  must be satisfied as  $LC_S$  is binding, the only condition need to be checked is that the buyer is willing to offer this contract,

$$EU_B = \sqrt{2K} \left( \frac{1+\alpha}{\alpha} \right) - K \left( \frac{1+\alpha}{\alpha} \right)^2 \geq 0,$$

which is equivalent to

$$K \leq \frac{2\alpha^2}{(1+\alpha)^2}. \quad (28)$$

**Lemma 26.** *The optimal contract when PC and LC are binding, if  $K \leq \frac{2\alpha^2}{(1+\alpha)^2}$ , is*

$$\tilde{e}^* = \sqrt{2K} \left( \frac{1+\alpha}{\alpha} \right), \quad p_{post}^* = K \left( \frac{1+\alpha}{\alpha} \right)^2.$$

**Comparison: Case III Vs Case I** The comparison depends on

$$\sqrt{2K} \left( \frac{1+\alpha}{\alpha} \right) - K \left( \frac{1+\alpha}{\alpha} \right)^2 \geq EU_B^I \implies \geq \sqrt{2K} (1+\alpha) - K (1+\alpha)^2,$$

which is equivalent to

$$K \geq \frac{2\alpha^2}{(1+\alpha)^4}.$$

Thus we have a lower bound for which Case III ex-post contract is preferred to the Case I contract. A question follows naturally is whether this lower bound goes below the lower bound for the the Case II ex-post payment contract. The following shows that the answer is no.

$$3\alpha^2 - 2\alpha - 1 \frac{2\alpha^2}{(1+\alpha)^4} \geq \frac{1}{2(1+\alpha)^2} \implies \geq 0$$

The last inequality holds if  $\alpha \geq 1$ . So when  $K$  increases from zero, the type of ex-post contract will be offered is changed from Case I to Case II and then to Case III. See figure 14.

By comparing what we have so far, we get an important result.

**Proposition 11.**

*For every ex-post payment contract that buyer pays in equilibrium, there is an equivalent ex-ante payment contract, which having the same price and required effort.*

*Proof.*

By lemma 21 and 24, the case III ex-ante payment contract and the case II ex-post payment contract are essentially the same, except the timing of payment. By lemma 18 and 23, the case II ex-ante payment contract and the case I ex-post payment contract are essentially the same, except the timing of payment. By lemma 22 and 26, the case IV ex-ante payment contract and the case III ex-post payment contract are essentially the same, except the timing of payment. So for every ex-post payment contract, there is a corresponding ex-ante payment contract, the only difference is the timing of payment. □

**12.3.4 Case IV (Buyer default)**

Proposition 11 is true if we only consider the ex-post payment contract that buyer will pay. Now let us consider the optimal ex-post payment contract that buyer will not pay, where  $P_{\text{post}} \geq (1 + \alpha)^2 K$ . Under this case, the seller will sue the buyer for not paying, because  $P_{\text{post}} \geq K \left(\frac{1+\alpha}{\alpha}\right)^2$ . But still the seller will provide the required effort first. This may seem strange that the seller will do so, if the seller anticipates that the buyer will not pay. Actually the incentive constraint when the seller anticipates buyer default ( $IC_S^D$ ) is more stringent than the one he expects no default ( $IC_S$ ).

$$IC_S \geq IC_S^D \implies P_{\text{post}} - \frac{\tilde{e}^2}{2} \geq \left(\frac{\alpha}{1+\alpha}\right)^2 P_{\text{post}} - \frac{\tilde{e}^2}{2} - K,$$

which is equivalent to

$$K \geq -\frac{1+2\alpha}{(1+\alpha)^2} P_{\text{post}}.$$

The last inequality holds by assumption, as all variables are positive. Thus wrong expectation creates no problem. No matter what expectation the seller has, the seller will provide the required effort. The buyer will default, and then the seller sue. So in the following, we are going to maximize the expected payoff when the buyer defaults, subject to  $IC_S^D$ :

$$\max \tilde{e} - P_{\text{post}} \left(\frac{\alpha(2+\alpha)}{(1+\alpha)^2}\right) - K \quad s.t. \quad \left(\frac{\alpha}{1+\alpha}\right)^2 P_{\text{post}} - \frac{\tilde{e}^2}{2} - K \geq 0.$$

The solutions are

$$\begin{aligned} \tilde{e}^* &= \frac{\alpha}{2+\alpha}, \\ P_{\text{post}}^* &= \frac{(1+\alpha)^2}{2(2+\alpha)^2} + \left(\frac{1+\alpha}{\alpha}\right)^2 K. \end{aligned}$$

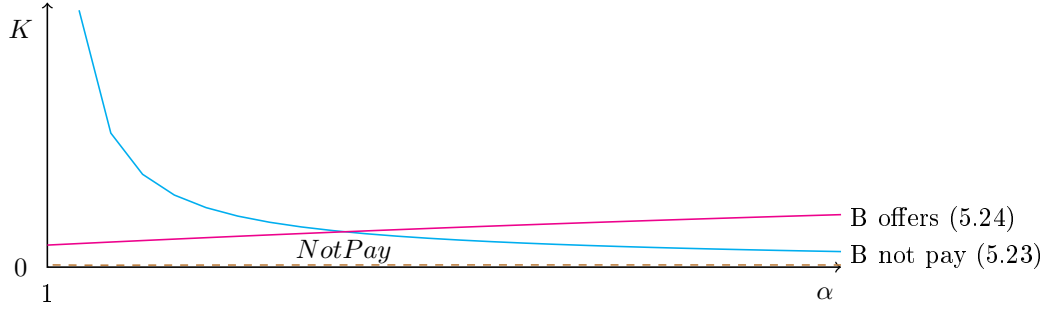


Figure 15: Legal environment for the buyer default ex-post payment contract  
The brown curve is equation 29. For the region in between the three curves, the buyer default ex-post payment contract is optimal.

The condition of buyer not paying is  $P_{\text{post}}^* \geq (1 + \alpha)^2 K$ , which is equivalent to

$$K \leq \frac{\alpha^2}{2(2 + \alpha)^2(\alpha^2 - 1)}.$$

The buyer will offer this contract if  $EU_B^* = \frac{\alpha}{2(2 + \alpha)} - \frac{2(1 + \alpha)}{\alpha} K \geq 0$ , which is equivalent to

$$K \leq \frac{\alpha^2}{4(1 + \alpha)(2 + \alpha)}.$$

**Lemma 27.** For low enough litigation fixed cost,  $K \leq \frac{\alpha^2}{2(2 + \alpha)^2(\alpha^2 - 1)}$  and  $K \leq \frac{\alpha^2}{4(1 + \alpha)(2 + \alpha)}$ , the optimal buyer default ex-post payment contract is

$$\tilde{e}^* = \frac{\alpha}{2 + \alpha}, \quad P_{\text{post}}^* = \frac{(1 + \alpha)^2}{2(2 + \alpha)^2} + \left(\frac{1 + \alpha}{\alpha}\right)^2 K.$$

**Comparison: Case IV Vs Case I** To compare the buyer default contract with the case I,

$$EU_B^{IV} \geq EU_B^I \implies \frac{\alpha}{2(2 + \alpha)} - \frac{2(1 + \alpha)}{\alpha} K \geq \sqrt{2K}(1 + \alpha) - (1 + \alpha)^2 K,$$

which is equivalent to

$$K \geq -\frac{\alpha^2 \left(2\sqrt{2(1 + \alpha)} - \alpha - 3\right)}{2(\alpha - 1)(\alpha^4 + 4\alpha^3 + 3\alpha^2 - 4\alpha - 4)}. \quad (29)$$

Note that, for  $\alpha > 1$ , the numerator is negative while the denominator is positive, thus the right hand side is positive. So the condition says that  $K$  cannot be extremely low for the buyer default contract to be chosen.

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