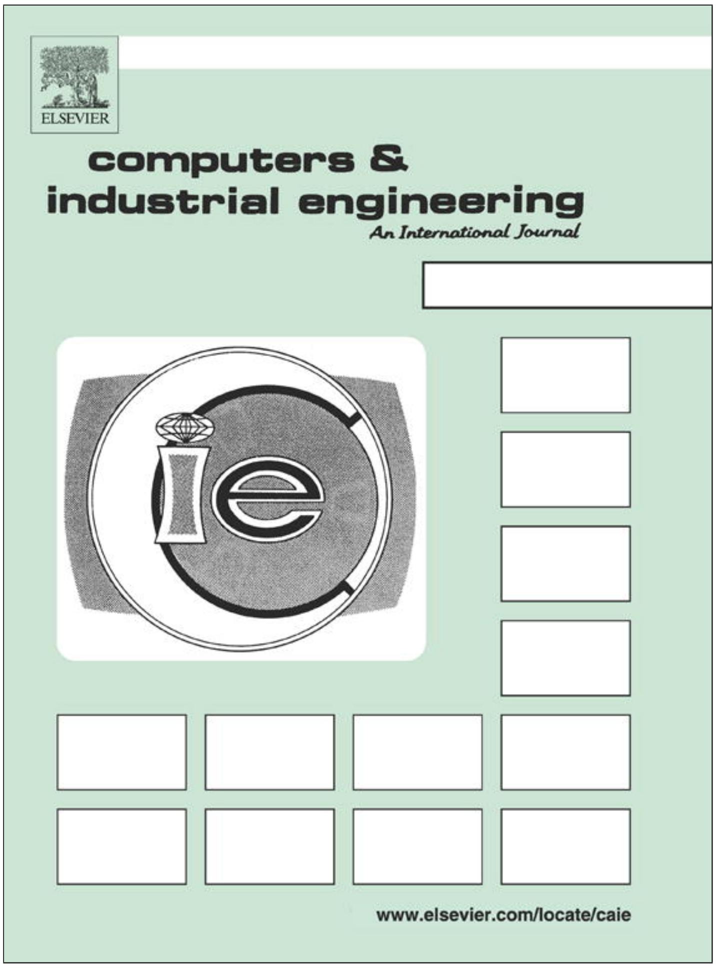

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Allocating fixed resources and setting targets using a common-weights DEA approach ☆

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ABSTRACT

Data envelopment analysis (DEA) is a data-driven non-parametric approach for measuring the efficiency of a set of decision making units (DMUs) using multiple inputs to generate multiple outputs. Conventionally, DEA is used in ex post evaluation of actual performance, estimating an empirical best-practice frontier using minimal assumptions about the shape of the production space. However, DEA may also be used prospectively or normatively to allocate resources, costs and revenues in a given organization. Such approaches have theoretical foundations in economic theory and provide a consistent integration of the endowment-evaluation-incentive cycle in organizational management. The normative use, e.g. allocation of resources or target setting, in DEA can be based on different principles, ranging from maximization of the joint profit (score), combinations of individual scores or game-theoretical settings. In this paper, we propose an allocation mechanism that is based on a common dual weights approach. Compared to alternative approaches, our model can be interpreted as providing equal endogenous valuations of the inputs and outputs in the reference set. Given that a normative use implicitly assumes that there exists a centralized decision-maker in the organization evaluated, we claim that this approach assures a consistent and equitable internal allocation. Two numerical examples are presented to illustrate the applicability of the proposed method and to contrast it with earlier work.

1. Introduction

Data envelopment analysis (DEA) pioneered by Farrell (1957) and later developed by Charnes, Cooper, and Rhodes (1978) is a non-parametric method, so-called CRS, to frontier analysis for measuring efficiency of a set of decision making units (DMUs). Mathematically, DEA utilizes a linear programming model which characterizes the relationship among multiple inputs and multiple outputs by envelopment of the observed data to determine a piecewise linear empirical best practice frontier. The DMUs placed on the frontier take unity score and they are called the frontier (efficient) DMUs. The radial or additive distance of the DMU from the frontier can be decomposed in different effects, distinguishing technical, scale, cost and congestion efficiency components. Moreover, the non-parametric approach allows the identification of real "peers" constituting the basis of comparison for the DMU, thereby providing managerially valuable information for performance analysis and improvement. In fact, DEA is the most well-published efficiency measurement method with over 4000 published papers (cf. Emrouznejad, Barnett, & Gabriel, 2008). The immediate and conventional justification and application of DEA is in ex post performance analysis, evaluating the situation for the individual DMU as well as the technology (frontier) after the fact. However, DEA can also be used for predictive, prospective and normative purposes by managers and organizations. Depending on application, the evaluator-principal may anticipate, direct or incentivize the DMUs to reposition in the production space. In a normative setting, the evaluator controls the inputs or outputs of the DMU by setting targets (for inputs or outputs) and/or by allocating fixed resources, products, revenues or budgets. Let us first clarify the difference between the terms "resource allocation" and "target setting" in the DEA terminology, following Beasley (2003, p. 208). The resource allocation may happen when the organization has restricted input resources or restricted output possibilities. In such circumstances, the organization must allocate the fixed input/output levels optimally among the DMUs. For example, adding additional raw material for processing among plants must be based on the overall profitability contribution of its use by any given plant. The target setting for input and output can be defined as a certain input/output value for each DMU without reference to organizational limitations. Thus, whereas resource allocation has a normative character, given a production possibility, target setting has a prospective

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flavor, potentially shifting the frontier to achieve unprecedented levels of performance. The reader immediately distinguishes the difference when considering e.g. a sales team, where resources could be staff and targets could be set in terms of sales volume.

Alluded to already in early work by Banker, Charnes, Cooper, and Clarke (1989), the use of DEA for activity planning received its theoretical foundations in the seminal work by Bogetoft (1993, 1994, 2000). Indeed, activity planning and resource allocation based on DEA-type frontiers is optimal under several different information economic settings. An influential model is the simple resource planning model by Golany and Tamir (1995) in which a resource vector is allocated among DMUs as to maximize the sum of joint output. Cook and Kress (1999) proposed a two-phase resource allocation mechanism to allocate the fixed cost between the DMUs using the principle of *Efficiency Invariance* and *Pareto [input] Optimality*. The first phase applied the output-oriented (input-oriented) CRS model to obtain the technical efficiency of the DMUs and the second phase allocated the fixed resource to all units where the efficiency of DMUs remains unchanged after resource allocation. Therefore, Cook and Zhu (2005) developed Cook and Kress's method (1999) into implementable a two-phase cost allocation method. The first phase of Cook and Zhu's method (2005) consists of three steps; (1) applying the conventional CRS model, (2) determining the efficient and inefficient DMUs, (3) obtaining the inefficiency values of all the inefficient units and the second phase of Cook and Zhu's method (2005) allocated the constant resource to DMUs based on the previous steps of the first phase. In fact, the cost allocation method proposed by Cook and Zhu (2005) only provided one feasible solution, which may not be optimal. With the similar assumption proposed by Cook and Kress (1999) on treating the costs allocation as an extra input, Beasley (2003) developed a five-phase DEA procedure for allocating fixed costs and setting output target to deal with the problem of non-uniqueness in Cook and Kress (1999) and determine a unique cost allocation by maximizing the average efficiency of all DMUs. The Beasley (2003) procedure was composed of (1) finding cross-efficiency (the maximum average DMU efficiency), (2) identifying the flexibility associated with the fixed resource allocation for each DMU, (3) minimizing the distance between the maximum and minimum proportions, over and above the minimum fixed resource for each DMU, (4) investigating whether there is any flexibility remaining with regard to the fixed resource allocation for each DMU, and (5) determining whether there is a unique fixed resource allocation. Amirteimoori and Kordrostami (2005) proposed an alternative DEA-based allocation approach to obtain a unique allocation in terms of combining the efficiency invariance proposed by Cook and Kress (1999) as well as taking account of the additional constraints proposed by Beasley (2003). Jahanshahloo, Hosseinzadeh Lotfi, Shoja, and Sanei (2004) indicated the shortcoming of Cook and Kress (1999)'s approach and they introduced a simple method to calculate a costs allocation without solving any linear programming model. Lin (2011) extended Cook and Zhu (2005)'s method for allocating fixed costs with some additional constraints based on the DEA technique. Lozano and Villa (2004) addressed an intraorganizational scenario where a centralized supervisor controls all DMUs and the supervisor not only wants units to be efficient but is also concerned about total input consumption and total output production. Yan, Wei, and Hao (2002) proposed a method in the inverse DEA for estimating inputs/outputs of a DMU when some or all of its input/output DMUs are changed such that the efficiencies are preserved. They considered preference cone constraints as well as using multi-objective programming (MOP) in their formulation. Also, their method argued how to execute the additional resource allocation problem in the inverse DEA problem. Korhonen and Syrjänen (2004) developed a method for treating resource allocation with a centralized decision of management. Their aim is to maximize the total output values of DMUs by allocating the fixed resources and they assumed that DMUs are able to change production in production possibility set (PPS). Jahanshahloo, Hosseinzadeh Lotfi, and Moradi (2005) presented a method for allocating a fixed output in a fair way among DMUs without solving any linear program. Amirteimoori and Shafiei (2006) proposed a DEA-based method for removing a fix resource from all DMUs in a fair way such that the efficiency of units before and after reduction remains unchanged. Li and Cui (2008) presented a resource allocation framework consisting of various returns to scale model, inverse DEA model, common weight analysis model, and extra resource allocation algorithm. Guedes de Avellar, Milioni, and Rabello (2007) developed a DEA model where a fixed input was fairly allocated among the DMUs, by assuming the existence of a geometric place with a spherical shape for the DEA frontier. Xiaoya and Jinchuan (2008) extended the resource allocation approach for VRS and inverse DEA formulations. Trappey and Chiang (2008) present an application of DEA in new product development activities and resource planning within a profit center for achieving the goal of maximal profit. Li, Yang, Liang, and Hua (2009) developed a DEA-based approach to allocating fixed cost among various DMUs based on the combination of the allocated cost with other cost measures to determine a unique allocation. In their approach the relationship between the allocated cost and the efficiency score were introduced and the fixed cost was considered as a dependent input. Pachkova (2009) proposed a DEA-based model to reallocate inputs in which her model was trade-off between the maximum allowed reallocation cost and the highest possible summation of the efficiency of all DMUs. Vaz, Camanho, and Guimarães (2010) applied the network-DEA model to evaluate the efficiency of the retail stores with several selling sections. In their approach, the VRS model was first used for evaluating the efficiency of similar sections in the stores, then, for efficiency improvement they applied the resource reallocation method developed in (Färe, Grabowski, Grosskopf, & Kraft, 1997) to set targets for the section. Amirteimoori and Mohaghegh Tabar (2010) proposed a three-phase DEA procedure in the presence of a fixed resource allocation and a fixed output target by defining an additional input and output for all DMUs, respectively. The first phase used the multiplier CRS model to determine the technical input-efficiency of DMUs, the second phase applied a mathematical model to allocate a fixed resource and set a fixed output target such that the DMUs become efficient, and the third phase re-evaluated the efficiencies of DMUs, similar to phase 1, in the presence of the new assigned input and output from phase 2. Lozano, Villa, and Canca (2011) employed several radial and non-radial DEA models for resource allocation and target setting in the presence of some integer inputs. Milioni, Avellar, Gomes, and Soares de Mello (2011) presented a parametric DEA method, namely the ellipsoidal frontier model, in the resource allocation context. Bi, Ding, Luo, and Liang (2011) proposed a common-weight DEA method for resource allocation and target setting in the parallel production system.

In target setting for DEA, the intuition may suggest the question to be trivial, as DEA per se indicates the "closest" projection on the efficiency frontier for inefficient DMUs. However, the seminal work by Golany (1988) established that preference information was vital in determining targets in DEA. Thanassoulis and Dyson (1992) extended this idea to allow for non-radial improvement targets on inputs or outputs to maximize decision maker utility. Athanassopoulos (1995) implemented the latter model in goal programming and DEA to two types of targets; one set related to overall effectiveness or profit, another set related directly to the efficiency of the DMUs. Similar approaches using preference weights and dual weights were suggested in Athanassopoulos (1996), Athanassopoulos and Triantis (1998). Athanassopoulos, Lambroukos, and Seiford (1999) utilized the DEA models to provide target setting scenarios based on two aspects. In the first aspect, the evaluation of the targets was implemented using the inputs and outputs simultaneously, and DMUs are permitted to get more resources for producing extra outputs. In the second aspect, the target setting process can be implemented in the presence of the decision makers. These approaches naturally lead us over to the question of dual weight setting in DEA.

Conventionally, DEA formulations draw on endogenous individual weights, which come with both pros and cons. The advantage is that each DMU can augment its efficiency using the suitable weights compared to the other DMUs while the disadvantage of freedom in the selection of weights is that solving different models provide different weights for inputs and outputs which may not be rational and acceptable from the decision maker. To deal with the disadvantage, several methods have been reported in the DEA literature. Cook, Roll, and Kazakov (1990) and Roll, Cook, and Golany (1991) are the first to introduce the common weights in DEA models for evaluating highway maintenance units. Thompson, Langemeier, Lee, Lee, and Trall (1990) shown the role of multiplier bounds in calculating the efficiency of the DMUs and they used a special case of Assurance Region in order to construct the linear homogeneous conditions on the multipliers in efficiency analysis. Cook and Kress (1990, 1991) proposed a subjective ordinal preference ranking based on DEA in the presence of the upper and lower bound on weights. Roll and Golany (1993) suggested an alternative method of treating factor weights in the DEA approach where their method involved two following steps: (1) "Normalizing" the inputs and outputs so that the magnitude of the parameters would not influence the model; (2) Setting restrictions on the weights of the model. Hosseinzadeh Lotfi, Jahanshahloo, and Memariani (2000) and Jahanshahloo, Memariani, Hosseinzadeh Lotfi, and Rezaei (2005) presented two different common-weight DEA models in which the efficiency of the DMUs can be obtained by a non-linear program in place of solving n linear programming models. Hosseinzadeh Lotfi et al. (2000) utilized the concept of MOP and the common set of weights to compute efficiency score of all DMUs. Jahanshahloo, Memarianiet, et al. (2005) presented a method based on the common weights to measure the efficiency and to rank the efficient DMUs in the two-step process. Amin and Toloo (2007) presented a model in order to find the most efficient DMUs by using a common set of weights. Liu and Hsuan Peng (2008) introduced a DEA method to specify a common set of weights in order to rank efficient DMUs. Jahanshahloo, Hosseinzadeh Lotfi, Khanmohammadi, Kazemimanesh, and Rezai (2010) used an ideal line for determining common multipliers for inputs and outputs to rank of the DMUs. Wang and Chin (2010) proposed a framework for measuring cross-efficiency via the common set of weights. Davoodi and Zhiani Rezai (2012) recently extended a common-weights DEA approach involving a linear programming problem to gauge the efficiency of the DMUs with respect to the multi-objective model.

In this paper, we have four main methodological contributions. First, we propose a common-weights DEA method to deal with zero-value weights and total weights flexibility. The method introduced by Liu and Hsuan Peng (2008) inspired us to propose this common-weights DEA model in this study. The aim of Liu and Hsuan Peng (2008)'s method is to rank the DEA efficient DMUs while our model (7) is able to obtain the efficiency score of all DMUs. Second, we create a new model to show how adequately the fixed costs or resources can be allocated to the DMUs and accordingly how adequately the expected common increase in all outputs set by the decision maker can be allocated to the DMUs. Third, by incorporating the created model all DMUs will be efficient. To the best of our knowledge, this study is the first one applying a common-weights DEA method for resource allocation when the efficiencies are taken into account. Finally, we apply the proposed model to two instances to demonstrate the features.

The outline of the paper is organized as follows. In Section 2, we introduce a procedure of finding a common set of weights by means of multi-objective program and goal programming concepts. In Section 3, we present the details of the proposed method based on the resources allocation and target setting. To illustrate the applicability of the proposed model a comparative study using two different data from Cook and Kress (1999) and Amirteimoori and Mohaghegh Tabar (2010) is presented in Section 4. The paper ends with some conclusions and future research directions in Section 5.

2. The common-weights DEA model

As starting point, we use a conventional radial input-oriented DEA formulation, cf. Charnes et al. (1978). Consider a set of n DMUs, $j = 1, \ldots, n$, and producing s outputs $y_{rj} (r = 1, \ldots, s)$ using m inputs $x_{ij} (i = 1, \ldots, m)$. The radial input-efficiency of DMU_o, $o \in \{1, \ldots, n\}$, under the assumption of constant returns to scale (CRS), can be obtained by solving the following linear programming problem:

$$\begin{aligned} \max \quad & \sum_{r=1}^{s} u_r y_{ro} \\ s.t. \quad & \sum_{i=1}^{m} v_i x_{io} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0, \quad \forall j \\ & u_r, v_i \geqslant \varepsilon, \quad \forall r, i. \end{aligned} \tag{1}$$

where u_r and v_i in model (1) are the dual weights assigned to the rth output and the ith input, respectively and ε presents a non-Archimedean infinitesimal constant. DMU_o is defined as [technically] input-efficient if and only if $\sum_{r=1}^{s} u_r^* y_{ro} = 1$ and there exists at least one optimal (u^*, v^*) of model (1) with $u^* \geqslant \varepsilon$ and $v^* \geqslant \varepsilon$.

The original CRS (Charnes et al., 1978) and VRS (Banker, Charnes, & Cooper, 1984) models involved a set of unbounded input and output weights because no restrictions were imposed on multipliers. Therefore a score of 100% efficiency can be achieved for a DMU through several various ways. For instance, a DMU can spread its weights equally among the different inputs and outputs, but in DEA a DMU mostly is a maverick by taking a huge weight on one or few factors and assigning a zero or very small weights ε to other factors. That is to say, the DEA model often gives excessively high or low values to multipliers in an attempt to drive the efficiency score as high as possible. Moreover, the weight flexibility in DEA provides often widely varying individual implicit resource prices for inputs and outputs that may be difficult to rationalize. In the four following situations, we need additional control on weights (Charnes, Cooper, Lewin, & Seiford, 1994): (i) the analysis directly declines some factors by assigning a zero (or epsilon) weight to a factor, (ii) the results deny the opinions of the decision maker, (iii) the decision maker has strong preferences about the relative importance of some given factors, and (iv) when the number of factors is proportionately large in comparison with the number of the DMUs under evaluation, the model fails to discriminate and most DMUs are classed as efficient.

To deal with the above-mentioned problems, many attempts have been explored further restricting weights in DEA (see e.g., Allen, Athanassopoulos, Dyson, & Thanassoulis, 1997 for a good overview). The common-weights DEA introduced by Cook et al. (1990) and Roll et al. (1991) is known as one of the popular methods in which all DMUs can be evaluated by the unique weights. The major aim of this method is to attain a common set of weights so that all DMUs simultaneously receive the highest (average) efficiency score. Therefore, contrary to the original DEA, the common-weights DEA method does not give the weight flexibility to the DMUs to achieve the score of 100% efficiency.

Multi-objective optimization (MOP) is the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints. Multi-objective fractional programming (MOFP) is a specific MOP method for fractional functions which are to be maximized subject to a set of constraints in order to tackle such complex and ill-structured decision problems.

Let us proceed with our earlier assumption that there are n DMUs under consideration with m inputs and r outputs. The following MOFP problem can be used to maximize the efficiency score of all DMUs simultaneously:

$$\begin{aligned} \max \quad & W = \left\{ \frac{\sum_{r=1}^{s} u_r y_{r1}}{\sum_{i=1}^{m} v_i x_{i1}}, \frac{\sum_{r=1}^{s} u_r y_{r2}}{\sum_{i=1}^{m} v_i x_{i2}}, \ldots, \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \right\} \\ s.t. \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1, \quad \forall j, \\ & u_r, v_i \geqslant \varepsilon, \quad \forall r, i. \end{aligned} \tag{2}$$

To solve the above MOFP problem, many methods have been developed in the optimization literature. Goal programming (GP) is one of the seminal methods for multi-objective optimization (see Tamiz, Jones, & Romero, 1998, for a recent survey). In the GP method, the decision maker is requested to set aspiration levels for the objective functions. Then, deviations from these aspiration levels are minimized as a preferred solution. An objective function jointly with an aspiration level is referred to as a goal. Based on the GP method, model (2) can be converted into the following non-linear model for identifying a set of common weights (e.g., Davoodi & Zhiani Rezai, 2012):

$$\begin{aligned} \min \quad & \sum_{j=1}^{n} (\varphi_j^- + \varphi_j^+) \\ s.t. \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} + \varphi_j^- - \varphi_j^+ = A_j, \quad \forall j, \quad (3a) \\ & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1, \quad \forall j, \quad (3b) \\ & \varphi_j^-, \varphi_j^+ \geqslant 0, u_r, v_i \geqslant \varepsilon, \quad \forall j, r, i. \end{aligned} \tag{3}$$

where A_j, $j = 1,\ldots,n$, presents the goal of the jth objective function and φ_j^- and φ_j^+ are the under-achievement (so-called negative deviation) and over-achievement (so-called positive deviation) of the jth goal, respectively. A_j is set to unity in model (3) since in the conventional DEA models, each DMU desires to maximize the efficiency score.

In fact, the deviational variables φ_j^- and φ_j^+ helps the jth objective function, $\sum_{r=1}^{s} u_r y_{rj} / \sum_{i=1}^{m} v_i x_{ij}$, to approach its goal ($A_j = 1$). Given constraint (3b), the positive deviational variables φ_j^+ cannot take the positive value in the constraint (3a) (i.e., $\varphi_j^+ = 0$). In such case, the constraints (3b) becomes redundant and the constraint (3a) can be rewritten as

$$\sum_{r=1}^{s} u_r y_{rj} + \varphi_j^- \left(\sum_{i=1}^{m} v_i x_{ij} \right) = \sum_{i=1}^{m} v_i x_{ij}, \quad \forall j,$$

We cannot attain the linear programming problem from model (3) by using the above non-linear constraints. To linearize the solving model, we take into account the concept of the GP method to propose a new model. Consider the constraints $\sum_{r=1}^{s} u_r y_{rj} / \sum_{i=1}^{m} v_i x_{ij} \leqslant 1$ where $A_j = 1$ in model (2). In order to achieve the goal of $\mathrm{DMU_j}$ (unity value for the efficiency score), fraction $\sum_{r=1}^{s} u_r y_{rj} / \sum_{i=1}^{m} v_i x_{ij}$ must be increased by the numerator increasing and/or the denominator decreasing. To implement this, the DMU must minimize the sum of the total virtual gaps to the benchmarking frontier by adding φ_j^+ to $\sum_{r=1}^{s} u_r y_{rj}$ and taking φ_j^- away from $\sum_{i=1}^{m} v_i x_{ij}$. Consequently, the MOFP model (2) can be converted to the following model:

$$\begin{aligned} \min \quad & \sum_{j=1}^{n} (\varphi_j^- + \varphi_j^+) \\ s.t. \quad & \frac{\sum_{r=1}^{s} u_r y_{rj} + \varphi_j^+}{\sum_{i=1}^{m} v_i x_{ij} - \varphi_j^-} = 1, \quad \forall j, \quad (4a) \\ & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1, \quad \forall j, \quad (4b) \\ & \varphi_j^-, \varphi_j^+ \geqslant 0, u_r, v_i \geqslant \varepsilon, \quad \forall j, r, i. \end{aligned} \tag{4}$$

Obviously the constraints (4b) are redundant in the presence of (4a) and can be omitted from model (4). We obtain the linear programming model (5) by converting the proportional form of constraints in (4) to the linear forms using the cross-multiplication method.

$$\begin{aligned} \min \quad & \sum_{j=1}^{n} (\varphi_j^- + \varphi_j^+) \\ s.t. \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \varphi_j^- + \varphi_j^+ = 0, \quad \forall j, \\ & \varphi_j^-, \varphi_j^+ \geqslant 0, u_r, v_i \geqslant \varepsilon, \quad \forall j, r, i. \end{aligned} \tag{5}$$

The above model can be simplified to the following linear programming by substitution $\varphi_j^- + \varphi_j^+$ with φ_j:

$$\begin{aligned} \min \quad & \sum_{j=1}^{n} \varphi_j \\ s.t. \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \varphi_j = 0, \quad \forall j, \\ & \varphi_j \geqslant 0, u_r, v_i \geqslant \varepsilon, \quad \forall j, r, i. \end{aligned} \tag{6}$$

Definition 1. $\mathrm{DMU_j}$, $j = 1,\ldots,n$, is non-dominated if and only if $\varphi_j^* = 0, j = 1, \ldots, n$, in model (6).

Then if we let $(u_r^*, v_i^*, \varphi_j^*)$, $\forall r, i, j$ are the optimal solutions of model (6), the efficiency scores of $\mathrm{DMU_j}$, $j = 1,\ldots,n$, can be obtained as:

$$\theta_j^* = \frac{\sum_{r=1}^{s} u_r^* y_{rj}}{\sum_{i=1}^{m} v_i^* x_{ij}} = 1 - \frac{\varphi_j^*}{\sum_{i=1}^{m} v_i^* x_{ij}}, \quad \forall j \tag{7}$$

Definition 2. $\mathrm{DMU_j}$, $j = 1,\ldots,n$, is non-dominated if and only if $\theta_j^* = 1, j = 1, \ldots, n$, in Eq. (7).

3. The proposed model for resource allocation and target setting

In the real life problems, there are some centralized situations that the decision maker desires (or is obliged) to allocate additional resources to the inputs of the DMUs and to define a target for the output-level of the DMUs. For instance, assume that management of a chain store makes a decision to allocate a new product with a scarce capacity between the present stores, and management expects to attain certain revenue with regard to selling the new product. Therefore, the assigned additional products to each DMU can be treated as an additional input resource and the imposed revenue level through the DMUs can be considered as an additional output. It is obvious that management of a chain store is increasingly eager to obtain 100% efficiency score. Therefore, the major aim is to identify the resource allocation and setting output target such that all DMUs become efficient. Note also the important centralized perspective in our example, the manager for the DMUs has no short-term interest in starving DMUs that may have been temporarily inefficient in the last period, nor congesting those DMUs that may have been on the frontier last period. Under mild assumptions regarding dual output and input prices, the manager is more interested to assure that all units are represented at the frontier. However, since the evaluation is centralized and the units are assumed homogenous, there is no rationale for using endogenous individual weights in the DEA evaluation. In this section, we propose a new model for allocating resources and setting output targets adequately.

Let us consider a system (organization) consisting of n independent DMUs under the evaluation process that each DMU$_j$, $j = 1, \ldots, n$, use m inputs, $x_{ij} \in R^+, (i = 1, \ldots, m; j = 1, \ldots, n)$ to produce s outputs, $y_{rj} \in R+, (r = 1, \ldots, s; j = 1, \ldots, n)$. In this centralized system, we suppose that the organization has q additional resources $F_k \in R+$, $k = 1, \ldots, q$ and it wants to allocate these resources to each DMU. Accordingly, the organization expects to achieve p fixed outputs, $G_w \in R+$, $w = 1, \ldots, p$, as targets set for each DMU. The non-negative variables $\bar{f}_{kj}$ and $\bar{g}_{wj}$ present the allocated inputs and allocated outputs to DMU$_j$, respectively. Thus, the relations $\sum_{j=1}^{n} \bar{f}_{kj} = F_k, \forall k$ and $\sum_{j=1}^{n} \bar{g}_{wj} = G_w, \forall w$, must be held. Hence, we create the following system:

$$\begin{aligned}
&\frac{\sum_{r=1}^{s} u_r y_{rj} + \sum_{w=1}^{p} u_{s+w} \bar{g}_{wj}}{\sum_{i=1}^{m} v_i x_{ij} + \sum_{k=1}^{q} v_{m+k} \bar{f}_{kj}} = 1, \quad \forall j, \quad (8a)\\
&\sum_{j=1}^{n} \bar{f}_{kj} = F_k, \quad \forall k, \quad (8b)\\
&\sum_{j=1}^{n} \bar{g}_{wj} = G_w, \quad \forall w, \quad (8c)\\
&u_r, u_{s+w}, v_i, v_{m+k} \geqslant \varepsilon, \; \bar{f}_{kj}, \bar{g}_{wj} \geqslant 0, \quad \forall r, i, k, w, j. \quad (8d)
\end{aligned} \tag{8}$$

In the above system, constraints (8a) guarantees that the efficiency score of each DMU reaches to unity based on defining extra inputs for the resources allocation and an extra outputs for target setting. The constraints (8b) and (8c) state that the sum of the resources allocation and target setting are equal to F_k and G_w, respectively. Due to the non-linearity of (8) we apply the following alteration variables

$$u_{s+w} \bar{g}_{wj} = g_{wj}, \qquad v_{m+k} \bar{f}_{kj} = f_{kj}.$$

so that the system (8) results in the following system:

$$\begin{aligned}
&\frac{\sum_{r=1}^{s} u_r y_{rj} + \sum_{w=1}^{p} g_{wj}}{\sum_{i=1}^{m} v_i x_{ij} + \sum_{k=1}^{q} f_{kj}} = 1, \quad \forall j, \quad (9a)\\
&\sum_{j=1}^{n} f_{kj} = v_{m+k} F_k, \quad \forall k, \quad (9b)\\
&\sum_{j=1}^{n} g_{wj} = u_{s+w} G_w, \quad \forall w, \quad (9c)\\
&u_r, u_{s+w}, v_i, v_{m+k} \geqslant \varepsilon, f_{kj}, g_{wj} \geqslant 0, \quad \forall r, i, k, w, j. \quad (9d)
\end{aligned} \tag{9}$$

To take into account the effects of the present input and output values in allocating resources and setting output targets, we define λ_j and μ_j multipliers assigned to all additional inputs and all additional outputs for a certain DMU, respectively, as $(1/m)\sum_{i=1}^{m} [x_{ij}/\sum_{t=1}^{n} x_{it}]$, and, $(1/s)\sum_{r=1}^{s} [y_{rj}/\sum_{t=1}^{n} y_{rt}]$, where $\sum_{j=1}^{n} \lambda_j = \sum_{j=1}^{n} \mu_j = 1$ $(j = 1, \ldots, n)$. Therefore, $\lambda_j F_k$ and $\mu_j G_w$ can be utilized for the input allocation and the output setting of jth the DMU, respectively. In the literature, there are several methods to solve system (9) such as Gauss-Jordan, Gaussian elimination (e.g., see Ralston & Rabinowitz, 1978).

We can show that constraints (9a) always are feasible by considering the effect of the multipliers λ_j and μ_j in the inputs and outputs, respectively. Notice that, similar to (9a), all DMUs will be efficient simultaneously. When we use the multipliers λ_j and μ_j, it is possible that infeasibility occurs for system (9). Hence, (9) cannot be solved via the conventional methods. Consequently, we define extra variables for a linear programming model based on the GP concept. In detail, we use the negative and positive deviational variables for f_{kj} and g_{wj} denoted by $(\alpha_{kj}^-, \alpha_{kj}^+)$ and $(\beta_{wj}^-, \beta_{wj}^+)$, to reach the goals $v_{m+k}\lambda_j F_k$ and $u_{s+w}\mu_j G_w$ respectively, in order to assure feasibility of constraints (9b) and (9c). In other word, our main concern is to have a feasible system for (9) in the presence of the multipliers λ_j and μ_j. Note here that the multipliers λ_j and μ_j never influence (9a) while constraints (9b) and (9c) might be infeasible. Rationally speaking, we minimize the sum of the defined deviations to achieve our goal, viz.

$$\begin{aligned}
\min \; & \sum_{j=1}^{n} \left(\sum_{k=1}^{q} (\alpha_{kj}^- + \alpha_{kj}^+) + \sum_{w=1}^{p} (\beta_{wj}^- + \beta_{wj}^+) \right)\\
s.t. \; & \frac{\sum_{r=1}^{s} u_r y_{rj} + \sum_{w=1}^{p} g_{wj}}{\sum_{i=1}^{m} v_i x_{ij} + \sum_{k=1}^{q} f_{kj}} = 1, \;\; \forall j,\\
& f_{kj} + \alpha_{kj}^- - \alpha_{kj}^+ = v_{m+k} \lambda_j F_k, \;\; \forall k, j,\\
& g_{wj} + \beta_{wj}^- - \beta_{wj}^+ = u_{s+w} \mu_j G_w, \;\; \forall w, j,\\
& \sum_{j=1}^{n} f_{kj} = v_{m+k} F_k, \;\; \forall k,\\
& \sum_{j=1}^{n} g_{wj} = u_{s+w} G_w, \;\; \forall w,\\
& u_r, u_{s+w}, v_i, v_{m+k} \geqslant \varepsilon, f_{kj}, g_{wj}, \alpha_{kj}^-, \alpha_{kj}^+, \beta_{wj}^-, \beta_{wj}^+ \geqslant 0, \;\; \forall r, i, k, w, j.
\end{aligned} \tag{10}$$

The above model is fractional programming problem and it can be converted to the linear programming problem using the cross-multiplication method as follows:

$$\begin{aligned}
\min \; & \sum_{j=1}^{n} \left(\sum_{k=1}^{q} (\alpha_{kj}^- + \alpha_{kj}^+) + \sum_{w=1}^{p} (\beta_{wj}^- + \beta_{wj}^+) \right)\\
s.t. \; & \sum_{r=1}^{s} u_r y_{rj} + \sum_{w=1}^{p} g_{wj} - \left(\sum_{i=1}^{m} v_i x_{ij} + \sum_{k=1}^{q} f_{kj} \right) = 0, \;\; \forall j,\\
& f_{kj} + \alpha_{kj}^- - \alpha_{kj}^+ = v_{m+k} \lambda_j F_k, \;\; \forall k, j,\\
& g_{wj} + \beta_{wj}^- - \beta_{wj}^+ = u_{s+w} \mu_j G_w, \;\; \forall w, j,\\
& \sum_{j=1}^{n} f_{kj} = v_{m+k} F_k, \;\; \forall k,\\
& \sum_{j=1}^{n} g_{wj} = u_{s+w} G_w, \;\; \forall w,\\
& u_r, u_{s+w}, v_i, v_{m+k} \geqslant \varepsilon, f_{kj}, g_{wj}, \alpha_{kj}^-, \alpha_{kj}^+, \beta_{wj}^-, \beta_{wj}^+ \geqslant 0, \;\; \forall r, i, k, w, j.
\end{aligned} \tag{11}$$

Theorem 1. There always exists a feasible solution to model (11).

Proof. Obviously, we have the following feasible solution to (11):

$$\alpha_{kj} = \sum_{r=1}^{s} u_r y_{rj}, \quad \forall k, j, \;\; \beta_{wj} = \sum_{i=1}^{m} v_i x_{ij}, \quad \forall w, j,$$

$$g_{wj} = \beta_{wj}, \quad \forall w, j, \quad f_{kj} = \alpha_{kj}, \quad \forall k, j,$$

$$u_r = \varepsilon, \quad \forall r, \quad u_{s+w} = \frac{\sum_{j=1}^{n} g_{wj}}{G_w}, \quad \forall w,$$

$$v_i = \varepsilon, \quad \forall i, \quad v_{m+k} = \frac{\sum_{j=1}^{n} f_{kj}}{F_k}, \quad \forall k,$$

$$\alpha_{kj}^{-} = \begin{Bmatrix} 0, & f_{kj} \geqslant v_{m+k}\lambda_j F_k, & \forall k, \forall j \\ v_{m+k}\lambda_j F_k - f_{kj}, & f_{kj} < v_{m+k}\lambda_j F_k, & \forall k, \forall j \end{Bmatrix},$$

$$\alpha_{kj}^{+} = \begin{Bmatrix} f_{kj} - v_{m+k}\lambda_j F_k, & f_{kj} \geqslant v_{m+k}\lambda_j F_k, & \forall k, \forall j \\ 0, & f_{kj} < v_{m+k}\lambda_j F_k, & \forall k, \forall j \end{Bmatrix},$$

$$\beta_{wj}^{-} = \begin{Bmatrix} 0, & g_{wj} \geqslant u_{s+w}\mu_j G_w, & \forall w, \forall j \\ u_{s+w}\mu_j G_w - g_{wj}, & g_{wj} < u_{s+w}\mu_j G_w, & \forall w, \forall j \end{Bmatrix},$$

$$\beta_{wj}^{+} = \begin{Bmatrix} g_{wj} - u_{s+w}\mu_j G_w, & g_{wj} \geqslant u_{s+w}\mu_j G_w, & \forall w, \forall j \\ 0, & g_{wj} < u_{s+w}\mu_j G_w, & \forall w, \forall j \end{Bmatrix}.$$

Therefore, the proof is accomplished. □

Notice that Theorem 1 guarantees that all DMUs will be efficient with regard to the cost allocation and target setting.

Corollary. The optimal solutions from model (11) are non-negative and bounded.

After obtaining the optimal solution $(u^*, v^*, f_j^*, g_j^*, \alpha_j^{-*}, \alpha_j^{+*}, \beta_j^{-*}, \beta_j^{+*})$ from model (11), we set $f_{kj}^*, g_{wj}^*, v_{m+k}^*$ and u_{s+w}^* in the equations $u_{s+w}\bar{g}_{wj} = g_{wj}$ and $v_{m+k}\bar{f}_{kj} = f_{kj}$ to identify the optimal resource allocation values, $\bar{f}_{kj}^*$, and target setting values, $\bar{g}_{wj}^*$. As mentioned earlier we take into account the resource allocation and target setting values as additional inputs and additional outputs, respectively. Therefore, model (2) enables us to examine the efficiency of DMUs after allocating resources and setting output targets. In the presence of these additional inputs and outputs, model (2) can be converted to the following model:

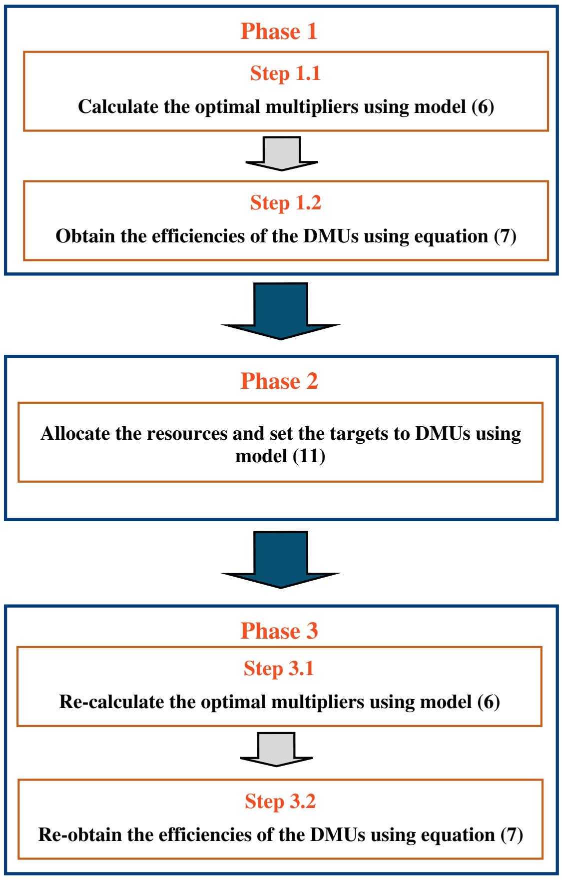

Fig. 1. The proposed framework.

$$\begin{aligned} \max \quad & F = \left\{ \frac{\sum_{r=1}^{s} u_r y_{r1} + \sum_{w=1}^{p} u_{s+w}\bar{g}_{w1}}{\sum_{i=1}^{m} v_i x_{i1} + \sum_{k=1}^{q} v_{m+k}\bar{f}_{k1}}, \frac{\sum_{r=1}^{s} u_r y_{r2} + \sum_{w=1}^{p} u_{s+w}\bar{g}_{w2}}{\sum_{i=1}^{m} v_i x_{i2} + \sum_{k=1}^{q} v_{m+k}\bar{f}_{k2}}, \ldots, \frac{\sum_{r=1}^{s} u_r y_{rn} + \sum_{w=1}^{p} u_{s+w}\bar{g}_{wn}}{\sum_{i=1}^{m} v_i x_{in} + \sum_{k=1}^{q} v_{m+k}\bar{f}_{kn}} \right\} \\ s.t. \quad & \frac{\sum_{r=1}^{s} u_r y_{rj} + \sum_{w=1}^{p} u_{s+w}\bar{g}_{wj}}{\sum_{i=1}^{m} v_i x_{ij} + \sum_{k=1}^{q} v_{m+k}\bar{f}_{kj}} \leqslant 1, \quad \forall j, \\ & u_r, u_{s+w}, v_i, v_{m+k} \geqslant \varepsilon, \; \bar{f}_{kj}, \bar{g}_{wj} \geqslant 0, \quad \forall r, i, k, w, j. \end{aligned} \qquad (12)$$

The following proposition shows the relation between models (2) and (12).

We summarize the proposed procedure in this study using the following three structured successive phases (see Fig. 1):

Phase 1: Calculate the optimal multipliers using the CSW model (6) consisting of n constraints and $n + m + s$ variables. Then Eq. (7) is used to obtain the efficiency score of each DMU.

Phase 2: Allocate q resources and set p targets for DMUs simultaneously using model (11) in order to receive the efficient DMUs. Model (11) consists of $n(p + q + 1) + p + q$ constraints and $n(3q + 3p) + m + s + p + q$ non-negative and positive variables.

Phase 3: Re-calculate the optimal multipliers using model (6) with considering q additional inputs and p additional outputs derived from the previous phase. We note that according to Theorem 1, all DMUs are technically efficient after allocation.

4. Numerical examples

In this section, we use two numerical examples to illustrate the applicability and efficacy of the proposed method. We first consider a hypothetical example proposed by Cook and Kress (1999)[1] for resource allocation followed by a second example from Amirteimoori and Mohaghegh Tabar (2010) for simultaneous resource allocation and target setting.[2] We make the comparison between the proposed method and the respective original methods.

[1] Also used in Beasley (2003) and Cook and Zhu (2005).

[2] The method of Amirteimoori and Mohaghegh Tabar (2010) is hereafter called "*AM method*".

Table 1
Input and output data and efficiency scores for 12 DMUs (Cook & Kress, 1999).

DMU	x_1	x_2	x_3	y_1	y_2	Efficiency before allocation	
						Cook and Kress (1999)	Proposed method
1	350	39	9	67	751	0.757	0.649
2	298	26	8	73	611	0.926	0.641
3	422	31	7	75	584	0.746	0.439
4	281	16	9	70	665	1.000	0.736
5	301	16	6	75	445	1.000	0.488
6	360	29	17	83	1070	0.961	0.892
7	540	18	10	72	457	0.862	0.279
8	276	33	5	78	590	1.000	0.672
9	323	25	5	75	1074	1.000	1.000
10	444	64	6	74	1072	0.833	0.713
11	323	25	5	25	350	0.333	0.326
12	444	64	6	104	1199	1.000	0.810

Table 2
Resulting resource allocation for the Cook and Kress (1999) problem.

DMU	Cook and Kress	Beasley	Cook and Zhu	Proposed method
1	14.520	6.780	11.220	8.199
2	6.740	7.210	0.000	7.462
3	9.320	6.830	16.950	4.284
4	5.600	8.470	0.000	9.301
5	5.790	7.080	0.000	4.807
6	8.150	10.060	15.430	15.370
7	8.860	5.090	0.000	0.000
8	6.260	7.740	0.000	7.339
9	7.310	15.110	17.620	16.330
10	10.080	10.080	21.150	11.598
11	7.310	1.580	17.620	0.000
12	10.080	13.970	0.000	15.310
Sum	100.020	100.000	99.990	100.000

4.1. Example 1

In this numerical example, there are 12 DMUs, three inputs $\{x_1,x_2,x_3\}$ and two outputs $\{y_1,y_2\}$ as presented in Table 1. Similar to Beasley (2003) and Cook and Zhu (2005) we assume that a fixed resource of 100 units is to be allocating among the DMUs.

Cook and Kress (1999) allocated the fixed cost between the DMUs in the two phases according to two principles; invariance and pareto-minimality. In the first phase, Cook and Kress (1999) applied the output-oriented (input-oriented) CRS model to obtain the technical efficiency of the DMUs. The 7th column of Table 1 shows the (input-oriented) efficiency score, yielding five technically input-efficient DMUs; 4, 5, 8, 9, 12. In the second phase, they allocated the fixed resource to all units where the efficiency of DMUs remains unchanged after resource allocation as presented in the 2nd column of Table 2. To address the non-uniqueness problem for allocated resources in the presence of multiple efficient units, they used cone-ratio constraints. In addition, their resource allocation method was only dependent on the input, and the output values do not influence in determining the amount of allocation (Beasley, 2003, p.208). For example, as shown in Table 2, the cost allocation of Cook and Kress (1999) for DMU_9 and DMU_{11} are 7.31 with identical inputs and different outputs. Note also that the fixed resource of 100 units is precisely allocated between the DMUs by Beasley (2003) and the proposed method in this study (see the last row of Table 2).

To deal with the problem of non-uniqueness in Cook and Kress (1999), Beasley (2003) created the following four-step DEA procedure for allocating fixed resources:

1. Finding cross-efficiency (the maximum average DMU efficiency).
2. Identifying the flexibility associated with fixed resource allocation for each DMU.
3. Minimizing $p_{max} - p_{min}$ where p_{max} and p_{min} are defined as the maximum and minimum proportions, over and above the minimum fixed resource for each DMU.
4. Investigating whether there is any flexibility remaining with regard to the fixed resource allocation for each DMU.
5. Determining whether there is a unique fixed resource allocation.

Cook and Zhu (2005) extended the method of Cook and Kress (1999) by proposing a two-phase approach. The first phase involves three following steps:

1. This step is the same as the first phase of Cook and Kress (1999) (see the 7th column of Table 1).
2. Classifying the DMUs in the two classes; [technically] efficient and inefficient DMUs.
3. Determining the inefficiency values for all inefficient units.

In the second phase, Cook and Zhu (2005) allocated the fixed resource to DMUs based on step 3 while keeping the classification of step 2 as well as preserving the efficiency scores of step 1. The assigned fixed resource of Cook and Zhu (2005) is presented in the 4th column of Table 2. In fact, Cook and Zhu (2005)'s cost allocation merely provides one feasible solution and not necessary the optimal solution.

We now apply the proposed three-phase procedure in this study (see Fig. 1) to allocate a fixed cost of 100 units across 12 DMUs. The first phase calculates the efficiency score of DMUs using model (6) and Eq. (7) as shown in the last column of Table 1. The second phase allocates resources and sets targets, simultaneously, for DMUs by means of model (11). This numerical example is a special case of the proposed model because of absence of target setting (i.e., $G_w = 0$). The result of the resource allocation is reported in the last column of Table 2. The last phase re-evaluates the efficiency of DMUs after allocation using model (6) only to obtain the optimal multipliers, since the efficiency is assured by Theorem 1. In Table 2, DMU_9 receives the highest resource allocation, 16.330, with our method compared to others because DMU_9 is efficient before allocating cost and this is the reason why it is allocated a much higher fixed cost allocation. Similarly, in our method DMU_7 and DMU_{11} receives the lowest resource allocation, 0, since these units exhibit the worst performance in the production set.

The results of Beasley (2003)'s method presented in the 3rd column of Table 2, reveal the similarity of resource allocation of DMU_7, DMU_9 and DMU_{11} with our method. In other words, DMU_9 takes the highest amount of resource allocation (i.e., 15.11) while DMU_7 and DMU_{11} take the lowest amounts of resource allocation (i.e., 5.090 and 1.580, respectively). Indeed, the Spearman rank-order correlation between the results of Beasley (2003) and our method is 0.

However, in Cook and Zhu's (2005) method DMU_4, DMU_5, DMU_8 and DMU_{12} are efficient but receive no resource allocation at all. Moreover, in Cook and Zhu, DMU_{11} receives the highest resource allocation in the set, although it is by far the weakest performer with an efficiency score of 0.333 in Farrell radial input-oriented efficiency. The allocation result of Cook and Zhu (2005) differs substantially from those of the other three approaches (Spearman rank-order coefficient 0.206, $t = 0.67$). Note that Cook and Zhu (2005) stated that Cook and Kress (1999) and Beasley (2003) are two different approaches because of using different underlying assumptions.

4.2. Example 2

This example includes 20 DMUs with three inputs and three outputs as reported in (Amirteimoori & Mohaghegh Tabar, 2010, Table 2, p. 3038).

Table 3
Resulting efficiency, resource allocation and target setting of the AM method and the proposed method.

DMU_j	Efficiency before allocation		Efficiency after allocation		Resource allocation ($\bar{f}_{1j}$)		Target setting ($\bar{g}_{1j}$)	
	AM method	Proposed method	AM method	Proposed method	AM method	Proposed method	AM method	Proposed method
1	1.000	0.796	1.000	1.000	13	6.118	28	28.970
2	0.711	0.675	0.711	1.000	4	0.000	20	23.076
3	0.896	0.485	0.896	1.000	11	0.000	9	41.799
4	0.596	0.501	0.598	1.000	7	0.000	9	29.145
5	1.000	0.478	1.000	1.000	11	0.000	6	28.825
6	1.000	1.000	1.000	1.000	0	4.364	21	0.000
7	0.704	0.597	0.704	1.000	11	0.000	6	22.133
8	1.000	0.940	1.000	1.000	0	3.202	14	13.878
9	1.000	1.000	1.000	1.000	0	4.063	22	14.984
10	0.523	0.282	0.530	1.000	10	0.000	5	21.509
11	0.668	0.480	0.776	1.000	9	0.000	27	52.552
12	1.000	0.748	1.000	1.000	0	0.000	25	20.855
13	0.958	0.910	1.000	1.000	0	6.099	22	11.018
14	0.994	0.562	1.000	1.000	0	0.000	27	32.011
15	1.000	1.000	1.000	1.000	6	12.591	61	51.634
16	1.000	1.000	1.000	1.000	18	119.404	82	0.000
17	0.942	0.793	0.951	1.000	34	0.000	72	67.222
18	1.000	0.757	1.000	1.000	6	0.000	56	65.325
19	1.000	0.926	1.000	1.000	13	19.159	68	62.655
20	0.891	0.809	0.891	1.000	21	0.000	39	32.409
				Sum	174	175	619	620

We assume that a central decision maker allocates 175 resource [units] to the DMUs and subsequently imposes 620 units as an output target (i.e., $F_1 = 175$ and $G_1 = 620$). The 2nd and 4th columns of Table 3 show the efficiency scores of the AM method prior to and after resource allocation, respectively, while the 6th and 8th columns present the amounts of the resource and output targets allocated among the DMUs. After resource and target allocation in the AM method, 20% of DMUs record efficiency improvements while the efficiency scores of 75% of DMUs stay unchanged.

According to the proposed framework, we first calculate the efficiencies of the DMUs using model (6) and Eq. (7) presented in the 3rd column of Table 3. We then use the proposed model (11) to obtain the optimal resource allocation as well as setting targets for all units. The results are presented in the 7th and 9th columns of Table 3.

As shown in Table 3, the $\bar{f}_{1j}$ and $\bar{g}_{1j}$, $j = 1, \ldots, 20$ denote the new allocated input and output for each DMU, respectively. All DMUs become efficient if we re-evaluate the DMUs via model (6) and Eq. (7) in the presence of $\bar{f}_{1j}$ and $\bar{g}_{1j}$ as an extra input and output. As a result, this example shows that the proposed model enables us to achieve our goal which is to receive the efficient DMUs after incorporating the resource allocation and setting output target. Note here that in Table 3, the values of some $\bar{f}_{1j}$ are zero because these DMUs are at the efficient frontier without additional allocated resources.

We here compare the proposed framework in this study with the AM method. The procedure of the AM method can be summarized in the three following phases:

- *Phase 1*. The multiplier CRS model (1) with $n + 1$ constraints and $m + s$ variables is applied n times to determine the technical input-efficiency of n DMUs.
- *Phase 2*. According to the authors' claim, a fixed resource is allocated and a fixed output target is set to receive the efficient DMUs. However, the AM method was not able to allocate the fixed input and output adequately to get the efficient DMUs (see Amirteimoori & Mohaghegh Tabar, 2010, Table 2, p. 3038). The AM model includes $6n + 2$ constraints and $7n + m + s + 2$ variables including $2n + m + s + 2$ non-negative variables and $5n$ free variables.
- *Phase 3*. The AM method re-evaluates the efficiencies of DMUs by using n times of the multiplier CRS model with respect to the new assigned input and output attained from preceding stage. In this stage, each DMU involves $m + 1$ inputs and $s + 1$ outputs which have one more input and output compared with stage 1.

Similarly, the procedure proposed in this paper consists of the three phases (see Fig. 1) where there are q resources and p targets for allocation. The first phase of the proposed method solves $(n - 1)$ linear programming models fewer in comparison with the AM method.

For comparison with the second phase of the AM method, it needs $q = 1$ and $p = 1$ substitutes in order to have one fixed resource and one fixed output target. As a result, our proposed model contains $3n + 2$ constraints and $6n + m + s + 2$ variables and it includes fewer constraints and variables compared to the AM method. On the basis of Theorem 1, the last phase of the proposed method can be omitted while the AM method requires the solution of n linear programming models.

In brief, the proposed framework in this paper is able to allocate q resources and set p targets simultaneously with less complexity and higher efficacy[3] than the AM method.

5. Concluding remarks and future research directions

The intuition behind the two concepts brought together in this paper is easy to understand when taking a managerial perspective on efficiency evaluation.

First, whereas the cautious paradigm in non-parametric frontier analysis prescribes the use of individual endogenous dual weights for the evaluation, this makes little sense in an applied setting under a common management. Relative prices and values of inputs and outputs may be unknown to the manager, but clearly not deviating widely between the units under evaluation. To address this

[3] Furthermore, it is important to note here that Amirteimoori and Mohaghegh Tabar (2010, p. 3039) determined the results of the example using the integer-valued DEA of Lozano and Villa (2006) while in their paper a standard DEA formulation was presented for resource allocation and target setting.

issue, we apply a common weights approach, deriving endogenous common dual weights for all units. Using an approach equivalent to goal programming, the problems of sub-dimensionality and computational complexity are resolved.

Second, a manager may employ an incentive system to promote efficiency ex ante, but organizational budgeting (and day-to-day management) is all about concurrent resource allocation and target setting for the plants, groups or departments under his influence. Some earlier approaches have employed concepts based on efficiency invariance, i.e. the resource allocation should not change the previous efficiency. We argue that the manager will explicitly violate this principle for reasons of joint maximization of system performance and incentive provision to the individual units. In organizations with layers of bureaucracy, the allocation of resources and associated budgets are often important elements in the managerial utility function, career development and motivation. A counterintuitive resource allocation, depriving efficient units of resources may, in an applied setting, perhaps achieve static efficiency at the expense of dynamic efficiency. The numerical results from the two published examples clearly illustrate these two points, which yet have to be confirmed from a behavioral viewpoint.

In short, the main contributions of the paper are fourfold: (1) we develop the common-weights DEA method to deal with total weights flexibility in DEA; (2) we propose an alternative mathematical model to allocate the fixed resources to the units along with setting the expected common increase of the targets to the units in a fair way; (3) the optimal solution of the proposed model always assigns an efficiency score of unity to all DMUs; (4) we compare the proposed framework with the present methods in the literature.

The approach in this paper contributes to a field where rich opportunities for modeling are opened in terms of e.g. multi-stage evaluation in techno-economic systems such as supply chains. Here, the current assumption about a single omniscient decision-maker must be modified, giving raise to interesting interaction between internal resource allocation and external target setting. It is plausible that the efficient formulation in this paper may be useful to model these multi-stage systems without exploding in complexity. Other issues concern extensions to dynamic settings to pursue consistent allocation schemes over time, using dynamic decompositions. Finally, the production technology may also be refined to a general case including categorical, environmental, integer-valued and interval data for the input and output parameters.

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