Abstract

We consider a model where investors can invest directly or search for an asset manager, information about assets is costly, and managers charge an endogenous fee. The efficiency of asset prices is linked to the efficiency of the asset management market: if investors can find managers more easily, more money is allocated to active management, fees are lower, and asset prices are more efficient. Informed managers outperform after fees, uninformed managers underperform after fees, and the net performance of the average manager depends on the number of “noise allocators.” Finally, we show why large investors should be active and discuss broader implications and welfare considerations.
In the real world, asset managers play a central role in making markets efficient as their size allows them to spend significant resources on acquiring and processing information. The asset management market is subject to its own frictions, however, since investors must search for informed asset managers. Indeed, institutional investors — e.g., pension funds and insurance companies — literally fly around the world to examine asset managers in person. They seek to assess the manager’s investment process, the number and quality of the investment professionals, the trading infrastructure, information flow, and risk management, and perform due diligence on the back office, valuation practices, the custody of the assets, IT security, disaster recovery plan, and so on. Similarly, individual investors search for asset managers, some via local branches of financial institutions, others with the aid of investment advisors, and yet others via the internet or otherwise. How do the search frictions in the market for asset management affect the efficiency of the underlying security market? Which types of securities are likely to be priced efficiently? What determines asset management fees? How large of an outperformance can investors expect from asset managers before and after fees? Which type of investors should use active, rather than passive, investing?

We seek to address these questions in a model with two levels of frictions: (a) investors’ search frictions of finding and vetting asset managers and (b) asset managers’ cost of collecting information about assets. The levels of inefficiency in the security market and the market for asset management are closely linked, yielding several new predictions: (1) Informed managers outperform after fees, uninformed managers underperform after fees, and the net performance of the average manager depends on the number of “noise allocators.” (2) If investors can find managers more easily, more money is allocated to active management, fees are lower, and security prices are more efficient. (3) As search costs diminish, asset prices become efficient in the limit, even if information-collection costs remain large. (4) Managers of complex assets earn larger fees and are fewer, and such assets are less efficiently priced. (5) Allocating to active managers is attractive for large or sophisticated investors with small search cost, while small or unsophisticated investors should be passive. (6) Finally, we discuss the economic magnitude of our predictions and welfare considerations.
As a way of background, the key benchmark is that security markets are perfectly efficient (Fama (1970)), but this leads to two paradoxes: First, no one has an incentive to collect information in an efficient market, so how does the market become efficient (Grossman and Stiglitz (1980))? Second, if asset markets are efficient, then positive fees to active managers implies inefficient markets for asset management (Pedersen (2015)).

Grossman and Stiglitz (1980) show that the first paradox can be addressed by considering informed investing in a model with noisy supply, but, when an agent has collected information about securities, she can invest on this information on behalf of others, so professional asset managers arise naturally as a result of the returns to scale in collecting and trading on information (Admati and Pfeiderer (1988), Ross (2005), Garcia and Vanden (2009)). Therefore, we introduce professional asset managers into the Grossman-Stiglitz model, allowing us to study the efficiency of asset markets jointly with the efficiency of the markets for asset management.

One benchmark for the efficiency of asset management is provided by Berk and Green (2004), who consider the implications of perfectly efficient asset-management markets (in the context of exogenous and inefficient asset prices). In contrast, we consider an imperfect market for asset management due to search frictions, consistent with the empirical evidence of Sirri and Tufano (1998), Jain and Wu (2000), Hortacsu and Syverson (2004), and Choi, Laibson, and Madrian (2010). We focus on investors’ incentive to search for informed managers and managers’ incentives to acquire information about assets with endogenous prices, abstracting from how agency problems and imperfect contracting can distort asset prices (Shleifer and Vishny (1997), Stein (2005), Cuoco and Kaniel (2011), Buffa, Vayanos, and Woolley (2014)).

Our equilibrium works as follows. Among a large group of asset managers, an endogenous number decide to acquire information about a security. Investors must decide whether to expend search costs to find one of the informed asset managers. In an interior equilibrium, investors are indifferent between passive investing (i.e., investing that does not rely on information collection) and searching for an informed asset manager. Investors do not
collect information on their own, since the costs of doing so are higher than the benefits to an individual due to the relatively high equilibrium efficiency of the asset markets. This high equilibrium efficiency arises from investors’ ability to essentially “share” information collection costs by investing through an asset manager.\(^1\) When an investor meets an asset manager, they negotiate a fee, and asset prices are set in a competitive noisy rational expectations market. The economy also features a group of “noise traders” (or “liquidity traders”) who take random security positions as in Grossman-Stiglitz. Likewise, we introduce a group of “noise allocators” who allocate capital to a random group of asset managers, e.g., because they place trust in these managers as modeled by Gennaioli, Shleifer, and Vishny (2015).

We solve for the equilibrium number of investors who invest directly, respectively through managers, the equilibrium number of asset managers, the equilibrium management fee, and the equilibrium asset prices. The model features both search and information frictions, but the solution is surprisingly simple and yields a number of clear new results.

First, we show that informed managers outperform before and after fees, while uninformed managers naturally underperform after fees. Investors who search for asset managers must be compensated for their search and due diligence costs, and this compensation comes in the form of expected outperformance after fees. Investors are indifferent between passive investing and active investing in an interior equilibrium so a larger search cost must be associated with a larger outperformance by active investors. Noise allocators invest mostly with uninformed managers and therefore experience underperformance after fees. The value-weighted average manager (which equals the average investor) outperforms after fees if the number of noise allocators is small, and underperforms if many noise allocators exist.

The model helps explain a number of empirical regularities on the performance of asset managers that are puzzling in light of the existing literature. Indeed, our prediction that some managers should be able to outperform passive investing before fees is consistent with evidence of mutual fund returns (Grinblatt and Titman (1989), Wermers (2000), Kacperczyk,\(^1\)

\(^1\)In our benchmark model with symmetric investors, no one collects information on their own; one could consider an extension with investors with different abilities, in which case some investors may collect information on their own.
Sialm, and Zheng (2008), Kosowski, Timmermann, Wermers, and White (2006)) and even after fees (Kosowski, Timmermann, Wermers, and White (2006), Fama and French (2010), Kacperczyk, Nieuwerburgh, and Veldkamp (2014)). In our model this outperformance is expected as compensation for the investors' search costs, but it is puzzling in light of the prediction of Fama (1970) that all managers underperform after fees, and the prediction of Berk and Green (2004) that all managers deliver zero outperformance after fees. If we take a broader view, the evidence suggests that the average active U.S. equity mutual fund underperforms after fees (e.g., Carhart (1997), but see Berk and Binsbergen (2012) for a critique), which could be consistent with the presence of noise allocators who pay fees to uninformed mutual funds.

The evidence for hedge funds suggest that they may outperform after fees (Kosowski, Naik, and Teo (2007), Fung, Hsieh, Naik, and Ramadorai (2008), Jagannathan, Malakhov, and Novikov (2010)) and likewise for private equity funds, where the puzzle in the literature in the performance persistence (Kaplan and Schoar (2005)). The larger outperformance of informed hedge funds and private equity funds is consistent with our model under the assumption that investors face larger search costs in these segments. Indeed, the needed due diligence (and work to convince a pension fund’s own board of directors) is likely larger for such alternative investments, leading to a higher required net return in equilibrium, again a new prediction of the search framework. Our model’s new prediction that searching investors should be able to find, at a cost, an asset manager that delivers a positive expected net return also suggests that funds of funds may be able to add value, as documented by Ang, Rhodes-Kropf, and Zhao (2008).

The model also generates a number of implications of cross-sectional and time-series variation in search costs. The important observation is that, if search costs are lower such that investors more easily can identify informed managers, then more money is allocated to active management, fees are lower, and security markets are more efficient. If investors’ search costs go to zero and the pool of potential investors is large, then the asset market becomes efficient in the limit. Indeed, as search costs diminish, fewer and fewer asset managers with
more and more asset under management collect smaller and smaller fees (both per investor and in total), and this evolution makes asset prices more and more efficient even though information-collection costs remain constant (and potentially large). It may appear surprising (and counter to the result of Grossman and Stiglitz (1980)) that markets can become close to efficient despite large information collection costs, but this result is driven by the fact that the costs are shared by investors through an increasingly consolidated group of asset managers.

We discuss how these model-implied effects of changing search costs can help explain cross-sector, cross-country, and time-series evidence on the efficiency, fees, and asset management industry for mutual funds, hedge funds, and private equity and gives rise to new tests. For instance, if search costs have diminished over time as information technology has improved, our model predicts that markets should have become more efficient consistent with the empirical evidence of Wurgler (2000) and Bai, Philippon, and Savov (2013), and linked to the amount of assets managed by hedge funds and other professional traders (Rosch, Subrahmanyam, and van Dijk (2015)).

We also consider the effect of the magnitude of information-collection costs. Higher information-collection costs leads to fewer active investors, fewer asset managers, higher fees, and lower asset market efficiency. One can interpret a high information-collection cost as a “complex” asset and, hence, the result can be stated as saying that complex assets have fewer asset managers, higher asset management fees, and lower efficiency, predictions that we relate to the empirical literature.

The paper is related to a large body of research in addition to that cited above. We discuss the empirical literature in detail in relation to our empirical predictions in Section 5. The related theoretical literature includes, in addition to the papers cited above, noisy rational expectations models (Grossman (1976), Hellwig (1980), Diamond and Verrecchia (1981), Admati (1985)) and other models of informed trading (Glosten and Milgrom (1985), Kyle (1985)), information acquisition (Van Nieuwerburgh and Veldkamp (2010), Kacperczyk, Van Nieuwerburgh, and Veldkamp (2014)), and noise trading (Black (1986)); search
models in finance (Duffie, Gārleanu, and Pedersen (2005), Lagos (2010)); and models of asset management (Pastor and Stambaugh (2012), Vayanos and Woolley (2013), Stambaugh (2014)).

The next section lays out the basic model, Section 2 provides the solution, and Section 3 derives the key properties of the equilibrium. Section 4 considers further applications of the framework, including the result that large investors should be active while small investors should be passive because it is more economic for a large investor to incur search costs to find an informed manager. This section also illustrates the economic magnitude of the predicted effects and welfare considerations. Section 5 lays out the empirical predictions of the model and their relation to the existing evidence, while Section 6 concludes. Appendix A describes the real-world issues related to search and due diligence of asset managers and Appendix B contains proofs.

1 Model of Assets and Asset Managers

1.1 Investors and Asset Managers

The economy features two types of agents trading in a financial market: investors and asset managers. Both investors and managers can obtain a signal about the asset value by paying a fixed cost $k$, but while investors can only trade on their own behalf, managers have the ability to manage funds on behalf of a group of investors.

More specifically, there exist $N$ optimizing investors and each investor can either (i) invest directly in asset markets without the signal, (ii) invest directly in asset markets after having acquired the signal, or (iii) invest through an asset manager. Due to economies of scale, a natural equilibrium outcome is that investors do not acquire the signal, but, rather, invest as uninformed or through a manager. We highlight below some weak conditions under which all realistic equilibria with a positive number of informed managers take this form, and we therefore rule out that investors acquire the signal. Consequently in our equilibria we focus on the number of investors who make informed investments through a manager $I$, inferring
the number of uninformed investors as the residual, \( N - I \).

The cost of setting up an asset management firm and accepting inflows is zero, so an unlimited number of asset managers exist. However, of these asset managers, only \( M \) elect to pay a cost \( k \) to acquire the signal and thereby become informed asset managers. The number of informed asset managers is determined as part of the equilibrium, and we think of the sets of managers and investors as continua (i.e., \( M \) is the mass of informed managers).\(^2\)

All agents act competitively, taking as given the actions of others.

To invest with an informed asset manager, investors must search for, and vet, managers, which is a costly activity. Specifically, the cost of finding an informed manager and confirming that he has the signal (i.e., performing due diligence) is \( c(M, I) \), which depends on both the number of informed asset managers \( M \) and the number of investors \( I \) in these asset management firms. We make the natural assumption that finding an informed manager is easier when there are more of them (e.g., because more informed managers means that the fraction of all managers with information is larger, or because the geographical distance between investors and managers is smaller)\(^3\) and fewer investors. Mathematically, this assumption means that the search cost \( c \) decreases weakly with \( M \) and increases weakly with \( I \), that is, \( c_M \leq 0 \) and \( c_I \geq 0 \) using familiar notation for partial derivatives.\(^4\) Furthermore, we require \( c(0, I) = \infty \) for all \( I \) — i.e., it is impossible to be matched with a manager if there aren’t any.\(^5\) The search cost \( c \) captures the realistic feature that most investors spend significant resources finding an asset manager they trust with their money as described in detail in Appendix A.

We assume that all agents have constant absolute risk aversion (CARA) utility over end-of-period consumption with risk-aversion parameter \( \gamma \) (following Grossman and Stiglitz

\(^2\)Treating agents as a continuum keeps the exposition as simple as possible, but the model’s properties also obtain in a limit of a finite-investor model.

\(^3\)Sialm, Sun, and Zheng (2014) find that “funds of hedge funds overweight their investments in hedge funds located in the same geographical areas” and have an information advantage in doing so, consistent with the similar results for individual investors’ stock investments due to Coval and Moskowitz (1999).

\(^4\)We note that many of our results hold for a broader class of search-cost specifications that need not satisfy these monotonicity conditions. Our performance results in Proposition 3, in particular, hold for any \( c \) function.

\(^5\)We require continuity of \( c \) only on \([0, \infty)^2 \setminus \{(0, 0)\}\).
For convenience, we express the utility as certainty-equivalent wealth — hence, with end-date wealth $\tilde{W}$, an agent’s utility is $-\frac{1}{\gamma} \log(E(e^{-\gamma \tilde{W}}))$. Each investor is endowed with an initial wealth $W$ while managers have a zero initial wealth (without loss of generality).

When an investor has found an asset manager and confirmed that the manager has the technology to obtain the signal, they negotiate the asset management fee $f$. The fee is set through Nash bargaining and, at this bargaining stage, all costs are sunk — both the manager’s information acquisition cost and the investor’s search cost.

A manager who does not pay the cost $k$ receives no asset inflows from searching investors. The utility of an informed asset manager is given by

$$
-\frac{1}{\gamma} \log \left( E \left[ e^{-\gamma (fI/M - k)} \right] \right) = f \frac{I}{M} - k,
$$

where $I/M$ is the number of investors per manager, relying informally on the law of large numbers.\textsuperscript{6} Hence, $f I/M$ is the manager’s total fee revenue and $k$ is his cost of operation.

Lastly, the economy features a group of “noise traders” and one of “noise allocators.” As in Grossman and Stiglitz (1980), noise traders buy an exogenous number of shares of the security, $Q - q$, as described below. Noise traders create uncertainty about the supply of shares and are used in the literature to capture that it can be difficult to infer fundamentals from prices. Noise traders are also called “liquidity traders” in some papers and their demand can be justified by a liquidity need, hedging demand, or behavioral reasons. They are characterized by the fact that their trades are not solely motivated by informational issues.

Following the tradition of noise traders, we introduce the concept of “noise allocators,” of total mass $A$, who allocate their funds across randomly chosen asset managers, paying the general fee $f$. Noise allocators could play a similar role in the market for asset management as noise traders do in the market for assets — specifically, noise allocators can make it difficult for searching investors to determine whether a manager is informed by looking at whether he has other investors, although we don’t model this feature. Further, since noise allocators tend to invest with uninformed asset managers, they change the performance characteristics in

\textsuperscript{6}Alternatively, managers can be taken to be risk neutral, with exactly the same outcome.
the distributions of managers and investors. In fact, given the existence of an infinite number of managers, (virtually) all their investments go to uninformed managers.\footnote{Our results also apply qualitatively if we consider a finite number of uninformed managers or a small entry cost for being an uninformed manager. We view our model with an infinite number of uninformed managers as the limit as the number of uninformed managers tends to infinity (or their entry cost tends to zero), and noise allocators randomly pick an asset manager from a uniform distribution.} Noise allocators may allocate based on trust, as proposed by Gennaioli, Shleifer, and Vishny (2015).

### 1.2 Assets and Information

We adopt the asset-market structure of Grossman and Stiglitz (1980), aiming to focus on the consequences of introducing asset managers into this framework. Specifically, there exists a risk-free asset normalized to deliver a zero net return, and a risky asset with payoff $v$ distributed normally with mean $m$ and standard deviation $\sigma_v$.

Agents can obtain a signal $s$ of the payoff, where

$$s = v + \varepsilon.$$  \hspace{1cm} (2)

The noise $\varepsilon$ has mean zero and standard deviation $\sigma_\varepsilon$, is independent of $v$, and is normally distributed.

The risky asset is available in a stochastic supply given by $q$, which is jointly normally distributed with, and independent of, the other exogenous random variables. The mean supply is $Q$ and the standard deviation of the supply is $\sigma_q$. We think of the noisy supply as the number of shares outstanding $Q$ plus the supply $q - Q$ from the noise traders.

Given this asset market, uninformed investors buy a number of shares $x_u$ as a function of the observed price $p$, to maximize their utility $u_u$, taking into account that the price $p$ may reflect information about the value:

$$u_u(W) = -\frac{1}{\gamma} \log \left( \mathbb{E}_{x_u} \left\{ \max_{x_u} \mathbb{E} \left( e^{-\gamma(W+x_u(v-p))} | p \right) \right\} \right) = W + u_u(0) \equiv W + u_u.$$ \hspace{1cm} (3)

We see that, because of the CARA utility function, an investor’s wealth level simply
shifts his utility function and does not affect his optimal behavior. Therefore, we define the scalar \( u_u \) as the wealth-independent part of the utility function (a scalar that naturally depends on the asset-market equilibrium, in particular the price efficiency).

Asset managers observe the signal and invest in the best interest of their investors. This informed investing gives rise to the gross utility \( u_i \) of an active investor (i.e., not taking into account his search cost and the asset management fee — we study those, and specify their impact on the ex-ante utility, later):

\[
u_i(W) = -\frac{1}{\gamma} \log \left( E \left[ \max_{x_i} E \left( e^{-\gamma(W+x_i(u-v))} | p, s \right) \right] \right) = W + u_i(0) \equiv W + u_i.
\] (4)

As above, we define the scalar \( u_i \) as the wealth-independent part of the utility function. The gross utility of an active investor differs from that of an uninformed via conditioning on the signal \( s \).

1.3 Equilibrium Concept

We first consider the (partial) equilibrium in the asset market given the number of active investors \( I \): An asset-market equilibrium is an asset price \( p \) such that the asset market clears,

\[
q = (N - I + A)x_u + Ix_i,
\] (5)

for the uninformed investors’ demand \( x_u \) that maximizes their utility (3) given \( p \) and the demand from investors using asset managers \( x_i \) that maximizes their utility (4) given \( p \) and the signal \( s \). The market clearing condition equates the noisy supply \( q \) with the total demand from the \( N - I \) uninformed investors and the \( I \) informed investors.

Second, we define a general equilibrium for assets and asset management as a number of asset managers in operation \( M \), a number of active investors \( I \), an asset price \( p \), and an asset management fee \( f \) such that (i) no manager would like to change his decision of whether to acquire information, (ii) no investor would like to switch status from active (with an associated utility of \( W + u_i - c - f \)) to passive (conferring utility \( W + u_u \)) or vice-versa, (iii)
the price is an asset-market equilibrium, and (iv) the asset management fee is the outcome of Nash bargaining.

2 Solving the Model

2.1 Asset-Market Equilibrium

We first derive the asset-market equilibrium. The price \( p \) of the risky asset is determined as in a market in which \( I \) investors have the signal (because their portfolios are chosen by informed managers) and the remaining \( N - I + A \) investors are uninformed, i.e., the asset-market equilibrium is as in Grossman and Stiglitz (1980). We consider only their linear asset-market equilibrium and, for completeness, we record the main results in this section.\(^8\)

In the linear equilibrium, an informed agent’s demand for the asset is a linear function of prices and signals and the price is a linear function of the signal and the noisy supply:

\[
p = \theta_0 + \theta_s \left( (s - m) - \theta_q (q - Q) \right),
\]

where, as we show in the appendix, the coefficients are given by

\[
\theta_0 = m - \frac{\gamma Q \text{var}(v|s)}{I + (N - I + A) \frac{\text{var}(v|s)}{\text{var}(v|p)}} \tag{7}
\]

\[
\theta_s = \frac{I \frac{\sigma_v^2}{\sigma_v^2 + \sigma_q^2} + (N - I + A) \frac{\text{var}(v|s)}{\text{var}(v|p)} \frac{\sigma_v^2}{\sigma_v^2 + \theta_q^2 \sigma_q^2}}{I + (N - I + A) \frac{\text{var}(v|s)}{\text{var}(v|p)}} \tag{8}
\]

\[
\theta_q = \gamma \frac{\sigma_v^2}{I}. \tag{9}
\]

As we see, the equilibrium price depends on the ratio \( \frac{\text{var}(v|s)}{\text{var}(v|p)} \), which is given explicitly in Proposition 1 and has an important interpretation. Indeed, following Grossman and Stiglitz

\(^8\)Our results in this section differ slightly from those of Grossman and Stiglitz (1980) because of differences in notation (not just in the naming of variables, but also in the modeling of the information structure), but there exists a mapping from our results to theirs. Palvolgyi and Venter (2014) derive interesting non-linear equilibria in the Grossman and Stiglitz (1980) model.
(1980), we define the efficiency (or informativeness) of asset prices based on this ratio. For convenience, we concentrate on the quantity

$$
\eta \equiv \log\left(\frac{\sigma_{vp}}{\sigma_{vs}}\right) = \frac{1}{2} \log\left(\frac{\text{var}(v|p)}{\text{var}(v|s)}\right),
$$

which represents the price inefficiency. This quantity records the amount of uncertainty about the asset value for someone who only knows the price $p$, relative to the uncertainty remaining when one knows the signal $s$. The price inefficiency is a positive number, $\eta \geq 0$, and a higher $\eta$ corresponds to a more inefficient asset market. Naturally, a zero inefficiency corresponds to a price that fully reveals the signal.

The relative utility of investing based on the manager’s information versus investing as uninformed, $u_i - u_u \geq 0$, also plays a central role in the remainder of the paper. We can also think of it as a measure of the outperformance of informed investors relative to uninformed ones. As we shall see, the relative utility is central for our analysis for several reasons: It affects investors’ incentive to search for managers, the equilibrium asset management fee, and managers’ incentive to acquire information. Importantly, in equilibrium, investors’ relative utility is linked to the asset price inefficiency $\eta$, and both depend on the number of active investors as described in the following proposition.

**Proposition 1** There exists a unique linear asset-market equilibrium given by (6)–(9). In the linear asset-market equilibrium, the utility differential between informed and uninformed investors, $u_i - u_u$, is given by the inefficiency of the price, $\eta$:

$$
\gamma(u_i - u_u) = \eta.
$$

Further, $\eta$ is decreasing in the number of active investors $I$ and can be written as

$$
\eta = -\frac{1}{2} \log \left(1 + \frac{\gamma^2 \sigma_a^2 \sigma_e^2}{I^2 + \gamma^2 \sigma_a^2 \sigma_e^2 \sigma_v^2 + \sigma_v^2}\right) \in (0, \infty).
$$

Naturally, when there are more active investors (i.e., larger $I$), asset prices become more
efficient (lower $\eta$), implying that informed and uninformed investors receive more similar utility (lower $u_i - u_u$). We note that the asset price efficiency does not depend directly on the number of asset managers $M$. What determines the asset price efficiency is the risk-bearing capacity of agents investing based on the signal, and this risk-bearing capacity is ultimately determined by the number of active investors (not the number of managers they invest through). The number of asset managers does affect asset price efficiency indirectly, however, since the number of active investors and asset managers are determined jointly in equilibrium as we shall see.

### 2.2 Asset Management Fee

The asset-management fee is set through Nash bargaining between an investor and a manager. The bargaining outcome depends on each agent’s utility in the events of agreement vs. no agreement (the latter is called the “outside option”). For the investor, the utility in an agreement of a fee of $f$ is $W - c - f + u_i$. If no agreement is reached, the investor’s outside option is to invest as uninformed with his remaining wealth, yielding a utility of $W - c + u_u$ as the cost $c$ is already sunk. This outside option is equal to the utility of searching again for another manager in an interior equilibrium. Hence, we can think of the investor’s bargaining threat as walking away to invest on his own or to find another manager. In other words, in a search market, managers engage in imperfect competition which determines the fee and the equilibrium entry.

Similarly, if $o$ is the outside option of the manager, then $o + f$ is the utility achieved following an agreement (the cost $k$ is sunk and there is no marginal cost to taking on the investor). The bargaining outcome maximizes the product of the utility gains from agreement:

$$ (u_i - f - u_u) f. $$ (13)
The objective (13) is maximized by the asset management fee $f$ given by

$$f = \frac{1}{2} \eta \gamma,$$  
[equilibrium asset management fee] (14)

using that $u_i - u_u = \eta / \gamma$ based on Equation (11). This equilibrium fee is simple and intuitive: The fee would naturally have to be zero if asset markets were perfectly efficient, so that no benefit of information existed ($\eta = 0$), and it increases in the size of the market inefficiency.

We next derive the investors’ and managers’ decisions in an equally straightforward manner. Indeed, an attractive feature of this model is that it is very simple to solve, yet provides powerful results.

### 2.3 Investors’ Decision to Search for Asset Managers

An investor optimally decides to look for an informed manager as long as

$$u_i - c - f \geq u_u$$  
(15)

or, recalling the equality $\eta = \gamma(u_i - u_u)$,

$$\eta \geq \gamma(c + f).$$  
(16)

This relation must hold with equality in an “interior” equilibrium (i.e., an equilibrium in which strictly positive amounts of investors decide to invest as uninformed and through asset managers — as opposed to all investors making the same decision). Inserting the equilibrium asset management fee (14), we have already derived the investor’s indifference condition, $\gamma c = \frac{1}{2} \eta$.

Using similar straightforward arguments, we see that an investor would prefer using an asset manager to acquiring the signal singlehandedly provided $k \geq c + f$. Using the equilibrium asset management fee derived in Equation (14), the condition that asset management is preferred to buying the signal can be written as $k \geq 2c$. In other words, finding an as-
set manager should cost at most half as much as actually being one, which seems to be a
condition that is clearly satisfied in the real world. We can also make use of (17) to express
this condition equivalently as $I \geq 2M$, i.e., there must be at least two investors for every
manager, another realistic implication.

2.4 Entry of Asset Managers

A prospective asset manager must pay the cost $k$ to acquire information and then, in equi-
librium, manages the capital of $I/M$ investors. Therefore, she chooses to enter and become
an active manager provided that the total fee revenue covers the cost of operations:

$$f \frac{I}{M} \geq k.$$ (17)

This manager condition must hold with equality for an interior equilibrium, and we can
easily insert the equilibrium fee (14) to get $M = \frac{\eta I}{2\gamma k}$.

2.5 General Equilibrium for Assets and Asset Management

We have arrived at following two indifference conditions:

$$\frac{\eta(I)}{2\gamma} = c(M, I) \quad \text{[investors’ indifference condition]} \quad (18)$$

$$M = \frac{\eta(I)I}{2\gamma k}, \quad \text{[asset managers’ indifference condition]} \quad (19)$$

where $\eta$ is a function of $I$ given explicitly by (12). Hence, solving the general equilibrium
comes down to solving these two explicit equations in two unknowns ($I, M$). Recall that a
general equilibrium for assets and asset management is a four-tuple $(p, f, I, M)$, but we have
eliminated $p$ by deriving the market efficiency $\eta(I)$ in a (partial) asset market equilibrium
and we have eliminated $f$ by expressing it in terms of $\eta$. We can solve equations (18)–
(19) explicitly when the search-cost function $c$ is specified appropriately as we show in the
following example, but the remainder of the paper provides general results and intuition for
general search-cost functions.

**Example: Closed-Form Solution.** A cost specification motivated by the search literature is

\[
c(M, I) = \bar{c} \left( \frac{I}{M} \right)^\alpha \text{ for } M > 0 \quad \text{and} \quad c(M, I) = \infty \text{ for } M = 0,
\]

where the constants \( \alpha > 0 \) and \( \bar{c} > 0 \) control the nature and magnitude of search frictions. With this search cost function, equations (18)–(19) can be combined to yield

\[
\eta = 2\gamma (\bar{c}k^\alpha)^{\frac{1}{1+\alpha}},
\]

which shows how search costs and information costs determine market inefficiency \( \eta \).

We then derive the equilibrium number if active investors \( I \) from (12):

\[
I = \frac{\gamma \sigma_q \sigma_v}{\sigma^2_{\varepsilon} + \sigma^2_v} \sqrt{\frac{1}{1 - e^{-2\eta}} - 1} = \frac{\gamma \sigma_q \sigma_v}{\sigma^2_{\varepsilon} + \sigma^2_v} \sqrt{\frac{1}{1 - e^{-4\gamma (\bar{c}k^\alpha)^{\frac{1}{1+\alpha}}}} - 1}, \tag{22}
\]

as long as the resulting value of \( I \) is smaller than the total number of investors \( N \), otherwise the equilibrium is the corner solution \( I = N \).

When \( \eta \) is small — a reasonable value is \( \eta = 6\% \), as we show in Section 4.3 — we can approximate the number of active investors more simply as

\[
I \approx \frac{\gamma \sigma_q \sigma_v}{(2\eta)^{1/2} (\sigma^2_{\varepsilon} + \sigma^2_v)^{1/2}} = \frac{\gamma^{1/2}}{2(\bar{c}k^\alpha)^{\frac{1}{1+\alpha}}} \frac{\sigma_q \sigma_v}{(\sigma^2_{\varepsilon} + \sigma^2_v)^{1/2}}, \tag{23}
\]

illustrating more directly how search costs \( \bar{c} \) and information costs \( k \) lower the number of active investors \( I \), while risk aversion \( \gamma \) and noise trading \( \sigma_q \) raise \( I \). The number of informed managers \( M \) in equilibrium is:

\[
M = \left( \frac{\bar{c}}{k} \right)^{\frac{1}{1+\alpha}} I, \tag{24}
\]

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so the number of managers per investor $M/I$ depends on the magnitude of the search cost $\bar{c}$ relative to the information cost $k$.

Figure 1 provides a graphical illustration of the determination of equilibrium as the intersection of the managers’ and investors’ indifference curves. The figure is plotted based on the parametric example above, but it also illustrates the derivation of equilibrium for a general search function $c(M, I)$.

Specifically, Figure 1 shows various possible combinations of the numbers of active investors, $I$, and asset managers, $M$. The solid blue line indicates investors’ indifference condition (18). When $(I, M)$ is to the North-West of the solid blue line, investors prefer to search for asset managers because managers are easy to find and attractive to find due to the limited efficiency of the asset market. In contrast, when $(I, M)$ is South-East of the blue line, investors prefer to be passive as the costs of finding a manager is not outweighed by the benefits. The indifference condition is naturally increasing as investors are more willing to be active when there are more asset managers.

Similarly, the dashed red line shows the managers’ indifference condition (19). When $(I, M)$ is above the red line, managers prefer not to incur the information cost $k$ since too many managers are seeking to service the investors. Below the red line, managers want to become informed asset managers. Interestingly, the manager indifference condition is hump shaped for the following reason: When the number of active investors increases from zero, the number of informed managers also increases from zero, since the managers are encouraged to earn the fees paid by searching investors. However, the total fee revenue is the product of the number of active investors $I$ and the fee $f$. The equilibrium asset management fee decreases with number of active investors because active investment increases the asset-market efficiency, thus reducing the value of the asset management service. Hence, when so many investors have become active that this fee-reduction dominates, additional active investment decreases the number of informed managers.

---

We use the following parameters: $N = A = 10^8/2$, $\gamma = 3 \times 10^{-5}$ corresponding to a relative risk aversion $\gamma^R = 3$ and average invested wealth $W = 10^5$, $Q = 1$, $m = (N + A)W = 10^{13}$, $\sigma_v = 0.2m$, $\sigma_x = 0.3m$, $\sigma_q = 0.2$, $\alpha = 0.8$, $\bar{c} = 0.96$, and $k = 5.9 \times 10^6$. 

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Figure 1: **Equilibrium for assets and asset management.** Illustration of the equilibrium determination of the number of active investors $I$ (among all investors $N$) and the number of asset managers $M$. Each investor decides whether to search for an asset manager or be passive depending on the actions $(I, M)$ of everyone else, and, similarly, managers decide whether or not to pay the information cost to enter the asset management industry. The right-most crossing of the indifference conditions is a stable equilibrium.

The economy in Figure 1 has two equilibria. One equilibrium is that there is no asset management: $(I, M) = (0, 0)$. In this equilibrium, no investor searches for asset managers as there is no one to be found, and no asset manager sets up operation because there are no investors. We naturally focus on the more interesting equilibrium with $I > 0$ and $M > 0$.

Figure 1 also helps illustrate the set of equilibria more generally. As we state more formally below, there are three general classes of equilibria. First, if the search and information frictions $c$ and $k$ are strong enough, then the blue line is initially steeper than the red line and the two lines only cross at $(I, M) = (0, 0)$, meaning that this equilibrium is unique. Second, if frictions $c$ and $k$ are mild enough, then the blue line ends up below the red line at the right-hand side of the graph with $I = N$. In this case, all investors being active is an equilibrium. Lastly, when frictions are intermediate — as in Figure 1 — the largest equilib-
rium is an interior equilibrium. While Figure 1 has only a single interior equilibrium, more interior equilibria may exist for other specifications of the search cost function (e.g., because the investor indifference condition starts above the origin, or because it can in principle “wiggle” enough to create additional crossings of the two lines).

In the interest of being specific, in particular in the comparative statics that follow, we focus on the largest equilibrium, that is, the equilibrium with the highest levels of $I$ and $M$. As seen in Proposition 1, this is the equilibrium in which the asset market is most efficient, and it is stable.\footnote{As is standard, we denote an equilibrium as stable (unstable) if a deviation in $I$ or $M$ from the equilibrium amounts results in incentives for agents to change their behavior towards (away from) the behavior required by the equilibrium.} The concept of largest equilibrium is well defined due to the results in Proposition 2.

**Proposition 2 (General equilibrium)** There always exists a general equilibrium of masses $(I, M)$ of active investors and asset managers, a linear asset-market equilibrium $p$, and fee $f$. In case of multiple equilibria, $I$ and $M$ are positively related across equilibria, and the largest equilibrium can be characterized as follows: (i) If frictions $k$ and $c$ are sufficiently large, the unique equilibrium features zero asset managers and active investors, $M = I = 0$. (ii) If frictions are sufficiently low, all investors search for asset managers in the largest equilibrium, $I = N$. (iii) Otherwise the largest equilibrium is an interior equilibrium, $0 < I < N$.

### 3 Equilibrium Properties

We now turn to our central results on how the frictions in the market for money management interact with the efficiency of the asset market. Our results use the fact that the asset-market efficiency is determined by the number of active investors $I$ in equilibrium, as shown in Proposition 1. We say that the asset price is fully efficient if $\eta = 0$, meaning that the price fully reflects the signal (which is never the case in equilibrium, but it can happen as a limit). We say that the asset price is constrained efficient if $\eta$ is given by (12) with $I = N$, meaning that the price reflects as much information as it can when all investors are active.
Finally, *efficiently inefficient* simply refers to the equilibrium efficiency given the frictions.

We start by considering some basic properties of performance in efficiently inefficient markets in the benchmark model. We use the term outperformance to mean that an informed investor’s performance yields a higher expected utility than that of an uninformed, and vice versa for underperformance.

**Proposition 3 (Performance)** In a general equilibrium for assets and asset management:

(i) Informed asset managers outperform passive investing before and after fees, $u_i - f > u_u$.

(ii) Uninformed asset managers underperform after fees.

(iii) Searching investors’ outperformance net of fees just compensates their search costs in an interior equilibrium, $u_i - f - c = u_u$.

(iv) Larger equilibrium search frictions means higher net outperformance for informed managers.

(v) The value-weighted average manager (or, equivalently, the value-weighted average investor) outperforms after fees if the number of noise allocators $A$ is smaller than the number of optimizing active investors $I$ and underperforms if $A > I$.

These results follow from the fact that investors must have an incentive to incur search costs to find an asset manager and pay the asset-management fees. Investors who have incurred a search cost can effectively predict manager performance. Interestingly, this performance predictability is larger in an asset management market with larger search costs.

To the extent the search costs are larger for hedge funds than mutual funds, larger for international equity funds than domestic ones, larger for insurance products than mutual funds, and larger for private equity than public equity funds, this result can explain why the former asset management funds may deliver larger outperformance and why the markets they invest in are less efficient.

Next, we consider the other effects of investors’ cost $c$ of searching for asset managers.
Proposition 4 (Search for asset management)

(i) Consider two search cost functions, $c_1$ and $c_2$, with $c_1 > c_2$ and the corresponding largest equilibria. In the equilibrium with the lower search costs $c_2$, the number of active investors $I$ is larger, the number of managers $M$ may be higher or lower, the asset price is more efficient, the asset management fee $f$ is lower, and the total fee revenue $fI$ may be either higher or lower.

(ii) If $\{c_j\}_{j=1,2,3,...}$ is a decreasing series of cost functions that converges to zero at every point, then $I = N$ when the cost is sufficiently low, that is, the asset price becomes constrained efficient. If the number of investors $\{N_j\}$ increases towards infinity as $j$ goes to infinity, then $\eta$ goes to zero (full price efficiency in the limit), the asset management fee $f$ goes to zero, the number of asset managers $M$ goes to zero, the number of investors per manager goes to infinity, and the total fee revenue of all asset managers $fI$ goes to zero.

This proposition provides several intuitive results, which we illustrate in Figure 2. As seen in the figure, a lower search costs means that the investor indifference curve moves down, leading to a larger number of active investors in equilibrium. This result is natural, since investors have stronger incentives to enter when their cost of doing so is lower.

The number of asset managers can increase or decrease (as in the figure), depending on the location of the hump in the manager indifference curve. This ambiguous change in $M$ is due to two countervailing effects. On the one hand, a larger number of active investors increases the total management revenue that can be earned given the fee. On the other hand, more active investors means more efficient asset markets, leading to lower asset management fees. When the search cost is low enough, the latter effect dominates and the number of managers starts falling as seen in part (ii) of Proposition 4.

As search costs continue to fall, the asset-management industry becomes increasingly concentrated, with fewer and fewer asset managers managing the money of more and more investors. This leads to an increasingly efficient asset market and market for asset management. Specifically, the asset-management fee and the total fee revenue decrease toward zero,
Figure 2: **Equilibrium effect of lower investor search costs.** The figure illustrates that lower costs of finding asset managers implies more active investors in equilibrium and, hence, increased asset-market efficiency.

and increasingly fewer resources are spent on information collection as only a few managers incur the cost $k$, but invest on behalf of an increasing number of investors.

We next consider the effect of changing the cost of acquiring information.

**Proposition 5 (Information cost)** As the cost of information $k$ decreases, the largest equilibrium changes as follows: The number of active investors $I$ increases, the number of asset managers $M$ increases, the asset-price efficiency increases, the asset-management fee $f$ goes down, while the total fee revenue $fI$ increases for large values of $k$ and decreases for the other ones. If $k$ is sufficiently small, all investors are active and the asset price is constrained efficient.

The results of this proposition are illustrated in Figure 3. As seen in the figure, a lower information cost for asset managers moves their indifference curve out. This leads to a higher number of asset managers and active investors in equilibrium, which increases the asset-price efficiency. Naturally, less “complex” assets — assets with lower $k$ — are priced
more efficiently than more complex ones, and the more complex ones have fewer managers, higher fees, and fewer investors.

We also consider the importance of fundamental asset risk and noise trader risk in the determination of the equilibrium.

**Proposition 6 (Risk)** *An increase in the fundamental volatility* $\sigma_v$ *or in the noise-trading volatility* $\sigma_q$ *leads to more active investors* $I$, *more asset managers* $M$, *and higher total fee revenue* $fI$. *The effect on the efficiency of asset prices and the asset-management fee* $f$ *is ambiguous. The same results obtain with a proportional increase in* $(\sigma_v, \sigma_e)$ *or in all risks* $(\sigma_v, \sigma_e, \sigma_q)$.

An increase in risk increases the disadvantage of investing uninformed, which attracts more investors and more managers to service them. Interestingly, the asset-market efficiency may increase or decrease. For instance, if the search cost depends only on the number of investors searching, then new investor entry will mitigate the disadvantage of being unin-
formed only partially — so as to justify the higher search cost. On the other hand, if it depends only on the number of managers, then the higher number of managers decreases the search cost and investors enter until the market efficiency exceeds the original level.

4 Further Applications of the Framework

4.1 Small and Large Investors: Equilibrium Fee Structure

So far, we have considered an economy in which all investors are identical ex ante, but, in the real world, investors differ in their wealth and financial sophistication. Should large asset owners such as high-net-worth families, pension funds, or insurance companies invest differently than small retail investors? If so, how does the decision to be active depend on the amount of capital invested and the financial sophistication, including the access to useful financial advice? How do fees depend on the size of the investment?

We address these issues in the following subsections by extending the model to allow for heterogeneous investors. In particular, each investor \( i \in [0, N] \) has an investor-specific cost \( c_i \) of finding an informed asset manager, where a smaller search cost corresponds to a more sophisticated investor. Further, we assume that investor \( i \) has a wealth \( W_i \) and relative risk aversion \( \gamma_{iR} \), corresponding to an absolute risk aversion of \( \gamma_i = \gamma_{iR} W_i \).

We solve the model as before, but now investors have different portfolio choices, asset management fees, and optimal search decisions. In terms of portfolio choice, any investor invests an amount in the risky asset that is proportional to the ratio of the expected excess return to the variance of the return given the information set (informed or uninformed), \(^{11}\) where the factor of proportionality is \( 1/\gamma_i = W_i/\gamma_{iR} \). Hence, an investor with twice the wealth buys twice the number of shares. Likewise, an investor with twice the relative risk aversion buys half the number of shares.

We assume (without loss of generality) that each asset manager runs a fund that invests based on a relative risk aversion of \( \tilde{\gamma}_i \) (where we can think of \( \tilde{\gamma}_i \) as the typical risk aversion, \(^{11}\) Said differently, the investment size in terms of risk is the Sharpe ratio multiplied by \( 1/\gamma_i = W_i/\gamma_{iR} \).

\(^{11}\) Said differently, the investment size in terms of risk is the Sharpe ratio multiplied by \( 1/\gamma_i = W_i/\gamma_{iR} \).
although this is not important). Therefore, investors with relative risk aversion $\bar{\gamma}^R$ optimally invest their entire wealth with the manager. For such investors, it is straightforward to define the percentage fee $f\%$, namely as the fee in dollars divided by the investment, $f\% := f_i/W_i$. More broadly, an investor with relative risk aversion $\gamma^R_i$ invests $\bar{\gamma}^R/\gamma^R_i$ times his wealth in the fund so his percentage fee is

$$f_i^\% := \frac{f_i}{\bar{\gamma}^R_i W_i}. \quad (25)$$

Each investor’s equilibrium asset management fee is determined through the same bargaining process as before and, in fact, equation (13) continues to hold. We obtain the following result.

**Proposition 7 (Fee structure)** Each agent $i$ pays an equilibrium dollar fee of

$$f_i = \frac{\eta W_i}{2 \bar{\gamma}^R_i}, \quad (26)$$

corresponding to a percentage fee that is the same for all investors,

$$f^\% = \frac{\eta}{2 \bar{\gamma}^R_i}. \quad (27)$$

We see that all investors pay the same proportional asset management fee. This result corresponds well to the fee structure observed in mutual funds and the stated fees in most other forms of investment management. While an endogenously proportional fee is a nice and realistic result, we note that large institutional investors in managed accounts and hedge funds may sometimes get discounts relative to the proportional stated fees (according to practitioners we talked to, although we are not aware of a study of the relation between size and fees actually paid for large institutions). Such fee differences are not captured by this version of our model, but could be obtained if investors also differ in their costs of passive investment (as seen below) or if asset managers have a fixed cost for each investor.
4.2 Small and Large Investors: Who Should be Passive vs. Active?

The equilibrium proportional fees derived above imply that large investors do not have a fee advantage in active investing. Large investors have another advantage in being active, however: Their cost of finding and vetting an asset manager is a smaller fraction of their investment. Adapting in a straightforward way the earlier arguments, we see that investor $i$ optimally searches for an asset manager if

$$
\gamma_i c_i \leq \frac{1}{2} \eta.
$$

(28)

Hence, we have the following characterization of the optimal allocation policy by investors.

**Proposition 8 (Who should be active/passive)** An investor $i$ should invest with an active manager if the investor has a large wealth $W_i$, low relative risk aversion $\gamma_i R$, or low cost $c_i$ of finding and assessing the manager, all relative to the asset-market inefficiency $\eta$. Specifically, the investor should allocate to active management if

$$
\frac{\gamma_i R c_i}{W_i} \leq \frac{1}{2} \eta,
$$

(29)

and otherwise should be passive.

This result appears intuitive and is consistent with the idea that the active investors should be those who have a comparative advantage in asset allocation.

4.3 Understanding the Economic Magnitude

To illustrate the economic magnitudes of some of the interesting properties of the model in a simple way, it is helpful to write our predictions is relative terms. Specifically, as seen in Sections 4.1–4.2, investors’ preferences can be written in terms of the relative risk aversion $\gamma R$ and wealth $W$ such that $\gamma = \gamma R / W$. Further, the asset management fee can be viewed as a fixed proportion of the investment size and we define the proportional fee as $f\% = f / W$. 

$27$
With these definitions, we get the following predictions on the economic magnitude of the market inefficiency, asset management fee, and improvement in gross Sharpe ratio (i.e., before fees and search costs) for investors allocating to informed managers relative to uninformed managers.\textsuperscript{12}

**Proposition 9 (Economic magnitude)** The market inefficiency $\eta$ is linked to the proportional asset management fee and relative risk aversion,

$$\eta = 2 f^\% \gamma^R, \quad (30)$$

and can be characterized by the difference in squared gross Sharpe ratios attainable by informed ($SR_i$) vs. uninformed ($SR_u$) investors using a log-linear approximation:

$$\eta \approx \frac{1}{2} \left( E(SR_i^2) - E(SR_u^2) \right). \quad (31)$$

To illustrate these results, suppose that all investors have relative risk aversion of $\gamma^R = 3$ and that the equilibrium percentage asset management fee is $f^\% = 1\%$. Then we have the following relation for the asset market inefficiency $\eta$ based on (30):

$$\eta = 2 f^\% \gamma^R = 2 \cdot 1\% \cdot 3 = 6\%. \quad (32)$$

In other words, the standard deviation of the true asset value from the perspective of a trader who knows the signal is $e^{-6\%} \approx 94\%$ of that of a trader who only observes the price. Further, we see that the Sharpe ratios must satisfy

$$E(SR_i^2) - E(SR_u^2) = 4 f^\% \gamma^R = 4 \cdot 1\% \cdot 3 = 0.12.$$  

Hence, if uninformed investing yields an expected squared Sharpe ratio of 0.4\textsuperscript{2} (similar to that of the market portfolio), informed investing must yield an expected Sharpe ratio around

\textsuperscript{12}Since each type of investor $a = i, u$ chooses a position of $x = \frac{E_a(v) - p}{\gamma Var_a(v)}$, the investor’s conditional Sharpe ratio is $SR_a = \frac{|E_a(v) - p|}{\sqrt{Var_a(v)}}$ (where $E_a$ and $Var_a$ are the mean and variance conditional on $a$’s information).
0.53^2 (i.e., 0.53^2 - 0.4^2 = 0.12). Hence, at this realistic fee level, the implied difference in Sharpe ratios between informed and uninformed managers is relatively small and hard to detect empirically.

4.4 Welfare and Market Liquidity

It is interesting to consider welfare implications of the model, although many welfare effects are non-monotonic and ambiguous as is often the case in welfare analysis. We consider a welfare function that is simply the sum of all agents’ utilities:

\[
\text{welfare} = I(u_i - c - f) + (N - I)u_u + A(u_u - f) + M \left( \frac{fI}{M} - k \right) + Af + \nu, \tag{33}
\]

namely the utilities of the \(I\) active investors, the \(N - I\) passive investors, the \(A\) noise allocators, the \(M\) informed asset managers, the uninformed asset managers who earn the fees from the noise allocators, and the utility of the noise traders \(\nu\). To define the utility of the noise traders, we proceed in the spirit of Leland (1992) and endow them with risk-neutral preferences over their proceeds, \(\nu = \mathbb{E}[(q - Q)p]\). We could also include the utility of the original securities owners, but for simplicity we set the supply of shares to be \(Q = 0\).

In the real world, the welfare benefits of efficient markets also derive from a better allocation of resources due to real investment decisions, better labor market allocations, improved incentives of corporate officers, and many other effects not captured by our model. A complete study of all such welfare effects is beyond the scope of this paper so we limit ourself to showing that even this limited welfare function yields complex results.\(^{13}\)

We can simplify the welfare function as follows:

\[
\text{welfare} = (N + A)u_u + I(u_i - u_u) - Ic - Mk + \nu, \tag{34}
\]

which makes clear that the central welfare costs are the resources spent on search, \(Ic\), and

\(^{13}\)The complexity of the welfare analysis in noisy-REE frameworks is apparent, for instance, in Leland (1992), who studies the desirability of banning insider trading.
the resources spent on information collection, $Mk$. These costs are offset by the investment benefits ($u_i$, $u_u$, and $\nu$) of resources spent on information and matching.

In an interior equilibrium, investors are indifferent towards being active vs. passive and managers break even. These two observations allow us to further simplify the welfare function as

$$\text{welfare} = (N + A)u_u + \nu,$$

that is, the welfare is the same as if all agents receive the utility they would have as uninformed agents in a market characterized exogenously by the equilibrium market efficiency $\eta$. (Achieving this efficiency level endogenously, of course, requires that a certain number of agents be active.)

Interestingly, the utility of the noise traders, $\nu$, is closely linked to the equilibrium market liquidity. To see this link, we define market illiquidity as the equivalent of Kyle’s lambda in our model,

$$\lambda \equiv -\frac{dp}{dq} = \theta_s \theta_q,$$

where $\theta_s$ and $\theta_q$ are given in Equations (8)–(9). In other words, $\lambda$ measures the market impact of trading. Since noise traders move prices against themselves despite their lack of information, a higher market illiquidity is associated with lower utility:

$$\nu = -\lambda \sigma_q^2.$$

We see that a higher market illiquidity $\lambda$ and more noise trading $\sigma_q^2$ both lower the utility of the noise traders.

We are interested in the dependence of welfare on the search cost. While the overall welfare depends on search costs in a complex way, the model yields some nice results regarding liquidity and noise trader utility. Indeed, as we have seen, noise trader utility depends on the market liquidity, but $\lambda$ is not monotonic in the search cost or the number of active investors.
in general. However, under certain conditions, market liquidity is at its highest when search costs are low, as the following proposition states.

**Proposition 10 (Welfare and liquidity)** When the search cost $c$ and $A$ are small enough, a decrease in $c$ reduces Kyle’s lambda $\lambda$, and this improvement in market liquidity increases the welfare of the noise traders. The total welfare can increase or decrease as a result of the lower $c$ depending on the parameters.

### 4.5 The Cost of Passive Investing

In the benchmark model, investors had to choose between incurring a search cost to find an active manager and using passive investing for free. In the real world, however, passive investing also comes at a cost. Indeed, buying a diversified portfolio takes time and is associated with transaction costs. The costs of passive investing has come down over time due to the introduction and adoption of discount brokers, low-cost index funds, and exchange traded funds (ETFs), e.g., those run by Vanguard. It is interesting to consider how these costs of passive investing and their reduction affects the market for active asset management and the security markets.

We augment the benchmark with the assumption that investors (who are ex ante identical as in the benchmark model) must pay a cost $c_u$ for passive investing (i.e., to put on the portfolio $x_u(p)$). For simplicity, we assume that investors are committed to passive or active investing (e.g., because leaving the money under the mattress is associated with lower expected utility).

Solving this generalized model requires only to note that the cost $c_u$ modifies the gains from trade between a matched investor-manager pair from $\frac{2}{\gamma}$ to $\frac{2}{\gamma} + c_u$. These gains from trade feature both in the ex-ante decisions of the investor and manager, and in the determination of the fee.

Specifically, the Nash bargaining problem becomes to maximize

$$
\left(\frac{n}{\gamma} - f + c_u\right)(f),
$$

(38)
with solution

\[ f = \frac{c^u}{2} + \frac{\eta}{2\gamma}, \]  

(39)

while the indifference conditions (18)–(19) are modified to

\[ \frac{\eta}{2\gamma} + \frac{c^u}{2} = c(M,I), \]  

(40)

\[
M = \frac{I}{k} \left( \frac{\eta}{2\gamma} + \frac{c^u}{2} \right). \]  

(41)

Based on these revised indifference conditions, we can characterize how the general equilibrium for assets and asset management depends on the cost of passive investing.

**Proposition 11 (Cost of passive investing)** As the cost of passive investing \( c^u \) decreases, the largest equilibrium changes as follows. The number of active investors \( I \) is lower, the number of informed active managers \( M \) is lower, the asset price is less efficient, and the total active fee revenue \( fI \) is lower. The asset management fee \( f \) may increase or decrease.

As seen in the proposition, we would expect that lower costs of passive investing due to index funds and ETFs should drive down the relative attractiveness of active investing and therefore reduce the amount of active investing, rendering the asset market less efficient. The supply of informed managers catering to active investors declines. The search costs, too, react to the changes in the numbers of investors in managers, to the effect that the relative gains from investing with an informed manager, as well as the fee, may either increase or decrease.

### 4.6 Managers with Different Signals

In this section we describe briefly how the model would change if different managers received different signals. We adopt the formulation of Hellwig (1980), in that manager \( j \) receives signal

\[ s^j = v + \varepsilon^j, \]  

(42)
where \( \varepsilon^j \) are i.i.d. conditional on \( v \).

When every agent invests with only one manager, then the asset-market equilibrium is characterized by

\[
p = \hat{\theta}_0 + \hat{\theta}_v (v - \hat{\theta}_q (q - Q)),
\]

where \( \hat{\theta}_0, \hat{\theta}_v, \) and \( \hat{\theta}_q \) are constant and computed by matching coefficients in the market-clearing condition.

**Proposition 12** An equilibrium exists in which the price takes the form (43). The money-management-market equilibrium continues to be characterized by (18)–(19), with

\[
\eta = \gamma (u_i - u_u)
\]

\[
= \frac{1}{2} \left( \log(\text{var}(v|p)) - \log(\text{var}(v|p, s^j)) \right)
\]

\[
= -\frac{1}{2} \log \left( 1 - \frac{\gamma^2 \sigma^2 \sigma^2_q \sigma^2_v}{I^2 + \gamma^2 \sigma^2 \sigma^2_v + \gamma^2 \sigma^4 \sigma^2_q \sigma^2_v - 2} \right).
\]

We note that the equilibrium is qualitatively the same as in the base-case model, which can be seen by comparing equations (12) and (44). The dependence of \( \eta \) on the parameters (risk aversion and variances), in the two cases, is the same.

5 Empirical Implications

In this section, we lay out some of our model’s testable empirical implications for asset markets, asset management, and their interaction. The model has implications both for the cross-section of assets and asset managers — e.g., in cross-country comparisons or across different asset classes or market segments — and the time series, e.g., studying secular trends

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\(^{14}\)Proposition 12 is stated under the assumption that agents may interact with only one manager. This assumption is, however, not necessary. In the appendix we show that, if investing with a second manager must be done at the fee negotiated by all the other agents, then the agent would be strictly losing by paying the meeting cost \( c \) and fee \( f \) to receive the — smaller — marginal benefits of investing with another manager.
as access to information changes. We consider both empirical predictions that correspond to existing evidence as well as new predictions that are yet to be tested.

A. Search frictions and asset managers. Search frictions in the asset management fund industry are documented by Sirri and Tufano (1998), Jain and Wu (2000), and Hortaçsu and Syverson (2004) and, consistent with our model, proxies for lower search costs are associated with more investors. In a cross-country study, Khorana, Servaes, and Tufano (2008) find that mutual fund “fees are lower in wealthier countries with more educated populations,” which may be related to lower search frictions for well-educated investors. If search costs have gone down over time due to improvements in information technology (e.g., the internet), our model predicts an increasing allocation to managers and a downward trend in fees, somewhat consistent with the result of French (2008), although the fees may not have come down as much as our model might predict. Appendix A describes the real-world search and due diligence process in some detail.

B. Performance of asset managers. Larger search frictions for asset managers should lead to less efficient asset markets and larger outperformance net of fees, which could help explain why international equity funds perform better than domestic equity funds (Dyck, Lins, and Pomorski (2013)) and why hedge funds may add more value than mutual funds (references cited in the introduction). The unimpressive performance of the average mutual fund may also be related to a larger number of noise allocators in that sector.

C. Predicting manager performance. In our model, investors who collect information about asset managers can achieve some ability to predict their performance, consistent with the evidence cited in the introduction. Given that noise allocators invest randomly and searching investors allocate to informed managers, the model implies that informed managers receive more funds than uninformed managers, on average, as found empirically by Berk and Binsbergen (2012). However, as emphasized by Berk and Green
(2004), this effect may be mitigated by decreasing returns to scale in asset management, e.g., due to larger transaction costs for large managers. In our model, individual managers do not face decreasing returns to scale, but the asset management industry does because a larger total investment \(I\) leads to more efficient markets (lower \(\eta\)), reducing manager performance, consistent with the evidence of Pastor, Stambaugh, and Taylor (2015).\(^{15}\)

D. Asset management fees. We predict that asset-management fees should be larger for managers of more inefficient assets and in more inefficient asset management markets. For instance, if search costs for managers are large, this leads to less active investing and higher management fees. Note that the higher management fee in this example is not driven by higher information costs for managers, but, rather, by the equilibrium dynamics between the markets for the asset and asset management. This may help explain why hedge funds have historically charged higher fees than mutual funds. Also, markets for more complex assets that are costly to study should be more inefficient and have higher management fees. This can help explain why equity funds tend to have higher fees than bond funds and why global equity funds have higher fees than domestic ones.

E. Efficiently inefficient markets. While the efficient market hypothesis is a powerful theory, it can nevertheless be difficult to test because of the so-called “joint hypothesis” problem. However, the existence of deviations from the Law of One Price (securities with the same cash flows that trade at different prices) is a clear rejection of fully efficient asset markets. The theory of efficiently inefficient markets is not the entire complement to fully efficient markets, but, rather, it should be viewed as an equally well-defined null hypothesis. Efficiently inefficient markets means that the marginal investor should be indifferent between passive investing and searching for asset man-

\(^{15}\)While the overall asset management industry clearly has decreasing returns to scale for reasons described in our model, there might also be effects related to the size of each individual firm in the real world. Anecdotally, small asset managers face increasing returns to scale (due to fixed costs of trading infrastructure, worse commissions and other terms from brokers for small managers, etc.) while very large managers face decreasing returns to scale due to market impact.
agers, where the latter should deliver an expected outperformance balanced by asset management fees and search costs, consistent with the findings of Gerakos, Linnainmaa, and Morse (2014) for professional asset managers. The average manager might not deliver this outperformance due to noise allocators, but investors should be able to collect sufficient information to achieve an outperformance that compensates their costs in an efficiently inefficient market.

F. Anomalies. In an efficiently inefficient market, anomalies are more likely to arise the more resources a manager needs to trade against them (higher $k$) and the more difficult it is for investors to build trust in such managers (higher $c$). For instance, while convertible bond arbitrage is a relatively straightforward trade for an asset manager (low $k$), it might have performed well for a long time because it is difficult for investors to assess (high $c$).

G. Fraction of active investors: the size of the asset management industry. Our model also has several implications for the size of the asset management industry. The asset management industry grows when investors search cost diminish or when asset managers’ information costs go down, leading to more efficient asset markets, consistent with the evidence of Pastor, Stambaugh, and Taylor (2015). Other important models that speak to the size of the asset management industry include Berk and Green (2004), Garcia and Vanden (2009), and Pastor and Stambaugh (2012).

H. Number and size of asset managers: industrial organization of asset management. When investors’ search costs go down, our model predicts that the number of managers will fall, but the remaining managers will be larger (in fact so much larger that the total size of the asset management industry grows as mentioned above). Such consolidation of the asset management industry is discussed in the press, but we are not aware of a direct test of this model prediction.

I. Private equity and venture capital. We can also think of our model as a description of the markets for private equity and venture capital, where investors search for
asset managers who in turn examine private (and public) companies. Our model’s predictions may help explain the puzzling performance persistence documented by Kaplan and Schoar (2005), given the complications of finding an informed manager (Korteweg and Sorensen (2014)).

J. Investment consulting firms, investment advisors, and funds of funds. Lastly, investors’ search frictions in our model are consistent with the demand for investment consulting services and funds of funds who may essentially help lower these frictions. Counter to our model’s predictions, Bergstresser, Chalmers, and Tufano (2009) find no evidence that U.S. mutual fund brokers add value and, similarly, Jenkinson, Jones, and Martinez (2015) find no evidence that consultants add value in selecting U.S. equity funds (unless they cater to noise allocators), but, consistent with our predictions, Cain, McKeon, and Solomon (2015) find that top intermediaries have predictive power of private equity returns and affect flows.

6 Conclusion

We propose a model in which investors looking for informed asset managers examine whether managers have acquired information about the securities they trade, rather than only focusing on past performance. This broader focus corresponds to real world investors’ who carefully examine an asset manager’s investment process, the performance attribution (i.e., the trades and strategies that drive the past returns), the number of employees, their turnover, and their pedigree (education and experience), whether the manager operates a trading desk 24/7, co-location on major trading venues, costly information sources, risk management, valuation methods, financial auditors, and so on. This vetting process is challenging and time consuming, captured by our search model. At the same time, managers spend significant resources on making informed investments, captured by embedding the asset managers in the Grossman-Stiglitz information model.

We find that asset managers can increase asset price efficiency by letting investors essen-
tially share information costs, but their ability to do so is limited by the search frictions in the asset-management industry. Therefore, the efficiency of asset markets is fundamentally connected to the efficiency of the asset management market.

Our model shows how lower search frictions in asset management leads to improved asset price efficiency, lower asset management fees, less outperformance by asset managers before and after fees, fewer and larger asset managers (i.e., a consolidation of the asset management industry), and potential welfare improvements. Further, we find that large sophisticated investors should search for informed active managers, while smaller investors are better served by passive investing as the search and due diligence costs outweigh the potential gains from improved performance of a small portfolio. The model predictions help explain a number of existing empirical facts that are puzzling in light of existing models and lay the ground for further tests.
A  Real-World Search and Due Diligence of Asset Managers

Here we briefly summarize some of the main real-world issues related to finding and vetting an asset manager. While the search process involves a lot of details, the main point that we model theoretically is that the process is time consuming and costly. For instance, there exist more mutual funds than stocks in the U.S. Many of these mutual funds might be charging high fees while investing with little or no real information, just like the uninformed funds in our model (e.g., high-fee index funds, or so-called “closet indexers” who claim to be active, but in fact track the benchmark, or funds investing more in marketing than their investment process). Therefore, finding a suitable mutual fund is not easy for investors (just like finding a cheap stock is not easy for asset managers).

We first consider the search and due diligence process of institutional investors such as pension funds, insurance companies, endowments, foundations, funds of funds, family offices, and banks. Such institutional investors invite certain specific asset managers to visit their offices and also travel to meet asset managers at their premises. If the institutional investor is sufficiently interested in investing with the manager, the investor often asks the manager to fill out a so-called due diligence questionnaire (DDQ), which provides a starting point for the due diligence process. Here we provide a schematic overview of the process to illustrate the significant time and cost related to the search process of finding an asset manager and doing due diligence, but a detailed description of each of these items is beyond the scope of the paper.\footnote{Standard DDQs are available online, e.g., from the Managed Funds Association (http://www.managedfunds.org/wp-content/uploads/2011/06/Due-Diligence-Questionnaire.pdf) or the Institutional Limited Partner Association (http://ilpa.org/wp-content/publicmedia/ILPA_Due_Diligence_Questionnaire_Tool.docx). See also “Best Practices in Alternative Investments: Due Diligence,” Greenwich Roundtable, 2010 (www.greenwichroundtable.org/system/files/BP-2010.pdf), the CFA Institute’s “Model RFP: A standardized process for selecting money managers” (http://www.cfainstitute.org/ethics/topics/Pages/model_rfp.aspx), and “Best Practices for the Hedge Fund Industry,” Report of the Asset Managers’ Committee to the President’s working group on financial markets, 2009 (http://www.cftc.gov/ucm/groups/public/swaps/documents/file/bestpractices.pdf). We are grateful for helpful discussions with Stephen Mellas and Jim Riccobono at AQR Capital Management.}

- **Finding the asset manager:** the initial meeting.
  - **Search.** Institutional investors often have employees in charge of external managers. These employees search for asset managers and often build up knowledge of a large network of asset managers whom they can contact. Similarly, asset managers employ business development staff who maintain relationships with investors they know and try to connect with other asset owners, although hedge funds are subject to non-solicitation regulation preventing them from randomly contacting potential investors and advertising. This two-way search process involves a significant amount of phone calls, emails, and repeated personal meetings, often starting with meetings between the staff members dedicated to this search...
process and later with meetings between the asset manager’s high-level portfolio managers and the asset owner’s chief investment officer and board.

- **Request for Proposal.** Another way for an institutional investor to find and asset managers is to issue a request for proposal (RFP), which is a document that invites asset managers to “bid” for an asset management mandate. The RFP may describe the mandate in question (e.g., $100 million of long-only U.S. large-cap equities) and all the information about the asset manager that is required.

- **Capital introduction.** Investment banks sometimes have capital introduction (“cap intro”) teams as part of their prime brokerage. A cap intro team introduces institutional investors to asset managers (e.g., hedge funds) that use the bank’s prime brokerage.

- **Consultants, investment advisors, and placement agents.** Institutional investors often use consultants and investment advisors to find and vet investment managers that meet their needs. On the flip side, asset managers (e.g., private equity funds) sometimes use placement agents to find investors.

- **Databases.** Institutional investors also get ideas for which asset managers to meet by looking at databases that may contain performance numbers and overall characteristics of the covered asset managers.

**• Evaluating the asset management firm.**

- **Assets, funds, and investors.** Institutional investors often consider an asset manager’s overall assets under management, the distribution of assets across fund types, client types, and location.

- **People.** Key personnel, overall headcount information, headcount by major departments, stability of senior people.

- **Client servicing.** Services and information disclosed to investors, ongoing performance attribution, market updates, etc.

- **History, culture, and ownership.** When was the asset management firm founded, how has it evolved, general investment culture, ownership of the asset management firm, and do the portfolio managers invest in their own funds.

**• Evaluating the specific fund.**

- **Terms.** Fund structure (e.g., master-feeder), investment minimum, fees, high water marks, hurdle rate, other fees (e.g., operating expenses, audit fees, administrative fees, fund organizational expenses, legal fees, sales fees, salaries), transparency of positions and exposures.

- **Redemption terms.** Any fees payable, lock-ups, gating provisions, can the investment manager suspend redemptions or pay redemption proceeds in-kind, and other restrictions.
- **Asset and investors.** Net asset value, number of investors, do any investors in the fund experience fee or redemption terms that differ materially from the standard ones?

- **Evaluating the investment process.**
  - **Track record.** Past performance numbers and possible performance attribution.
  - **Instruments.** The securities traded and geographical regions.
  - **Team.** Investment personnel, experience, education, turnover.
  - **Investment thesis and economic reasoning.** What is the underlying source of profit, i.e., why should the investment strategy be expected to be profitable? Who takes the other side of the trade and why? Has the strategy worked historically?
  - **Investment process.** The analysis of the investment thesis and process is naturally one of the most important parts of finding an asset managers. Investors analyze what drives the asset manager’s decisions to buy and sell, the investment process, what data is used, how is information gathered and analyzed, what systems are used, etc.
  - **Portfolio characteristics.** Leverage, turnover, liquidity, typical number of positions and position limits.
  - **Examples of past trades.** What motivated these trades, how do they reflect the general investment process, how were positions adjusted as events evolved.
  - **Portfolio construction methodology.** How is the portfolio constructed, how are positions adjusted over time, how is risk measured, position limits, etc.
  - **Trading methodology.** Connections to broker/dealers, staffing of trading desk and is it operating 24/7, possibly co-location on major exchanges, use of internal or external broker algorithms, etc.
  - **Financing of trades.** Prime brokers relations, leverage.

- **Evaluating the risk management.**
  - **Risk management team.** Team members, independence, and authority.
  - **Risk measures.** Risk measures calculated, risk reports to investors, stress testing.
  - **Risk management.** How is risk managed, what actions are taken when risk limits are breached and who makes the decision.

- **Due diligence of operational issues and back office.**
  - **Operations overview.** Teams, functions, and segregation of duties.
- **Lifecycle of a trade.** The different steps a trade makes as it flows through the asset manager’s systems.
- **Cash management.** Who can move cash, how, and controls around this process.
- **Valuation.** Independent pricing sources, what level of PM input is there, what controls and policies ensure accurate pricing, who monitors this internally and externally.
- **Reconciliation.** How frequency and granularly are cash and positions reconciled.
- **Client service.** Reporting frequency, transparency levels, and other client services and reporting.
- **Service providers.** The main service providers used and any major changes (recent or planned).
- **Systems.** What are the major homegrown or vendor systems with possible live system demos.
- **Counterparties.** Who are the main ones, how are they selected, how is counterparty risk managed and by whom.
- **Asset verification.** Some large investors (and/or their consultants) will ask to speak directly to the asset manager’s administrator in order to independently verify that assets are valued correctly.

  - **Due diligence of compliance, corporate governance, and regulatory issues.**
    - **Overview.** Teams, functions, independence.
    - **Regulators and regulatory reporting.** Who are the regulators for the fund, summary of recent visits/interactions, frequency of reporting.
    - **Corporate governance.** Summary of policies and oversight.
    - **Employee training.** Code of ethics and training.
    - **Personal trading.** Policy, frequency, recent violations and the associated penalties for breach.
    - **Litigation.** What litigation the firm has been involved with.

  - **Due diligence of business continuity plan (BCP) and disaster recovery plan.**
    - **Plan overview.** Policy, staffing, and backup facilities.
    - **Testing.** Frequency of tests and intensity.
    - **Cybersecurity.** How IT systems and networks are defended and tested.

The search process for finding an asset manager is very different for retail investors. Clearly, there is no standard structure for the search process for retail investors, but here are some considerations:
Retail investors searching for an asset manager.

- **Online search.** Some retail investors can search for useful information about investing online and they can make their investment online. However, finding the right websites may require a significant search effort and, once located, finding and understanding the right information within the website can be difficult as discussed further below.

- **Walking into a local branch of a financial institution.** Retail investors may prefer to invest in person, e.g., by walking into the local branch of a financial institution such as a bank, insurance provider, or investment firm. Visiting multiple financial institutions can be time consuming and confusing for retail investors.

- **Brokers and intermediaries.** Bergstresser, Chalmers, and Tufano (2009) report that a large fraction of mutual funds are sold via brokers and study the characteristics of these fund flows.

- **Choosing from pension system menu.** Lastly, retail investors get exposure to asset management through their pension systems. In defined contributions pension schemes, retail investors must search through a menu of options for their preferred fund.

Searching for the relevant information.

- **Fees.** Choi, Laibson, and Madrian (2010) find experimental evidence that “search costs for fees matter.” In particular, their study “asked 730 experimental subjects to allocate $10,000 among four real S&P 500 index funds. All subjects received the funds prospectuses. To make choices incentive-compatible, subjects expected payments depended on the actual returns of their portfolios over a specified time period after the experimental session. ... In one treatment condition, we gave subjects a one-page ‘cheat sheet’ that summarized the funds front-end loads and expense ratios. ... We find that eliminating search costs for fees improved portfolio allocations.”

- **Fund objective and skill.** Choi, Laibson, and Madrian (2010) also find evidence that investors face search costs associated with respect to the funds’ objectives such as the meaning of an index fund. “In a second treatment condition, we distributed one page of answers to frequently asked questions (FAQs) about S&P 500 index funds. ... When we explained what S&P 500 index funds are in the FAQ treatment, portfolio fees dropped modestly, but the statistical significance of this drop is marginal.”

- **Price and net asset value.** In some countries, retail investors buy and sell a mutual fund shares as listed shares on an exchange. In this case, a central piece of information is the relation between the share price and mutual fund’s net asset value, but investors must search for these pieces of information on different websites and are often they not synchronous.
• Understanding the relevant information.
  
  – Financial literacy. In their study on the choice of index funds, Choi, Laibson, and Madrian (2010) find that “fees paid decrease with financial literacy.” Simply understanding the relevant information and, in particular, the (lack of) importance of past returns is an important part of the issue.

  – Opportunity costs. Even for financially literate investors, the non-trivial amount of time it takes to search for a good asset manager may be viewed as a significant opportunity cost given that people have other productive uses of their time and value leisure time.
B Proofs

Proof of Proposition 1. This result is effectively provided, and proved, in Grossman and Stiglitz (1980). In the interest of being self-contained, we include here a sketch.

An agent having conditional expectation of liquidating value $\mu$ and variance $V$ optimally demands

$$x = \frac{\mu - p}{\gamma V}. \tag{B.1}$$

Conjecturing the form (6) for the price, we have

$$E[v|p] = E[v|v + \varepsilon + \theta_q(q - Q)] = m + \beta_{v,p}(p - m) = m + \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2 + \theta^2_\varphi \sigma_q^2}(p - m) \tag{B.2}$$

$$E[v|s] = E[v|v + \varepsilon] = m + \beta_{v,s}(s - m) = m + \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2}(s - m) \tag{B.3}$$

$$\text{var}(v|p) = \sigma_v^2 - \beta_{v,p}^2 \sigma_p^2 = \sigma_v^2 \left(1 - \beta_{v,p}^2\right) = \frac{\sigma_v^2 \left(\sigma_v^2 + \theta^2_\varphi \sigma_q^2\right)}{\sigma_v^2 + \sigma_\varepsilon^2 + \theta^2_\varphi \sigma_q^2} \tag{B.4}$$

$$\text{var}(v|s) = \sigma_v^2 - \beta_{v,s}^2 \sigma_s^2 = \frac{\sigma_v^2 \sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\varepsilon^2}. \tag{B.5}$$

The price is now computed from the market-clearing condition, which shows that a linear price (in $q$ and $s$) exists and leads to equations determining its coefficients:

$$q = I \frac{E[v|s] - p}{\text{var}(v|s)} + (N - I + A) \frac{E[v|p] - p}{\text{var}(v|p)}. \tag{B.6}$$

Direct computation establishes (7)–(9).

To compute the relative utility, we start by noting that, with $a \in \{u, i\}$,

$$e^{-\gamma a u} = E \left[e^{-\frac{1}{2} \frac{(\mu_a - p n)^2}{V_a}}\right], \tag{B.7}$$

where $\mu_a$ and $V_a$ are the conditional mean and variance of $v$ for an investor of type $a$. To complete the proof, one uses the fact that, if $z \sim \mathcal{N}(\mu_z, V_z)$, then

$$E \left[e^{-\frac{1}{2} z^2}\right] = (1 + V_z)^{-\frac{1}{2}} e^{-\frac{1}{2} \frac{\mu_z^2}{1 + V_z}}.$$
and performs the necessary calculations giving

\[ u_i = \frac{1}{\gamma} \log \left( \frac{\sigma_{v-p}}{\sigma_{v|s}} \right) + \frac{1}{2\gamma} \frac{(m-\theta_0)^2}{\sigma_{v-p}^2} \]  
\[ u_i = \frac{1}{\gamma} \log \left( \frac{\sigma_{v-p}}{\sigma_{v|s}} \right) + \frac{1}{2\gamma} \frac{(m-\theta_0)^2}{\sigma_{v-p}^2}. \]  

(B.8)

(We note that the last term, \( \frac{1}{2\gamma} \frac{(m-\theta_0)^2}{\sigma_{v-p}^2} \), represents the utility attainable by an agent who cannot condition on the price.)

The proofs below make use of the following result.

**Lemma 1** The function of \( I \) given by \( I\eta \) increases up to a point \( \bar{I} \) and then decreases, converging to zero.

**Proof of Lemma 1.** The function of interest is a constant multiple of

\[ h(x) := x \log \left( \frac{a + x^2}{b + x^2} \right), \]  
(B.10)

with \( a > b > 0 \). Its derivative equals

\[ h'(x) = \log \left( \frac{a + x^2}{b + x^2} \right) + \frac{b + x^2}{a + x^2} \frac{2x(b + x^2) - 2x(a + x^2)}{(b + x^2)^2} \]

\[ = \log \left( \frac{a + x^2}{b + x^2} \right) - \frac{2(a-b)x^2}{(a + x^2)(b + x^2)}. \]

For \( x = 0 \), the first term is clearly higher: \( h'(0) > 0 \). For \( x \to \infty \), the second is larger, so that \( \lim h'(x) < 0 \). Finally, letting \( y = x^2 \) and differentiating \( h'(y) \) with respect to \( y \) one sees that \( h''(y) = 0 \) when \( y \) satisfies the quadratic

\[ y^2 - (a+b)y - 3ab = 0, \]  
(B.11)

which clearly has a root of each sign. Thus, since \( y = x^2 \) is always positive, \( h''(x) \) changes sign only once. Given that \( h'(x) \) starts positive and ends negative and its derivative changes sign only once, we see that \( h' \) itself must change sign exactly once. This result means that \( h \) is hump-shaped. Finally, we can apply L'Hôpital's rule to \( h(x) = \log \left( \frac{a+x^2}{b+x^2} \right) / (1/x) \) to conclude that \( \lim_{x \to \infty} h(x) = 0 \).  

**Proof of Proposition 2.** Let \( M \) denote the function that assigns to a value of \( I \) the corresponding value \( M \) at which managers are indifferent between entering and being passive — the red line in Figure 1. Likewise, let \( I \) give the equilibrium number of investors as a function of \( M \); note that the blue line in the figure graphs the function \( I^{-1} : I \mapsto M \), which is clearly increasing as seen from (18).
The first observation is that \((0, 0)\) is always an asset-management-market equilibrium. Second, this is the only equilibrium as long as the cost \(c\) is high enough, since the benefit to entering, \(\eta/(2\gamma)\), is bounded above (independent of cost functions).

For part (ii), the statement can be rephrased as \(c(M(N), N) \leq \frac{\eta}{2\gamma}\), which is a particular way of requiring that \(c\) or \(k\) be small enough. Explicitly, the condition is that

\[
e_c \left( \frac{\eta}{2k\gamma} N, N \right) \leq \frac{\eta}{2\gamma}.
\]

Finally, given that \(I\) is increasing, \(I\) and \(M\) are positively related across equilibria. We also note that the largest equilibrium is stable. This owes to the fact that, by virtue of the lemma, the function \(M\) increases in \(I\) and then decreases to approach zero as \(I\) increases without bound, while \(I^{-1}\) is increasing. It follows that the graph of \(M\) crosses that of \(I^{-1}\) the last time from above, for large enough \(I\). If \(I = N\) is an equilibrium, on the other hand, then \(M(N) > I^{-1}(N)\) and it is a stable equilibrium.

**Proof of Proposition 3.** Part (i) is a restatement of the fact that investors matched with good managers rationally choose to pay the fee and invest with the manager rather than invest as uninformed. Part (ii) is a juxtaposition of the facts that uninformed managers do not provide any investment value and that the fee is strictly positive. Part (iii) is literally the indifference condition for the active investors. For part (iv), we note that the outperformance \(u_i - f - u_a = c\) is clearly larger if the equilibrium \(c\) is larger. Finally, part (v) follows from expressing the aggregate outperformance as

\[
I (u_i - f - u_a) + A(-f) = I \left( \frac{\eta}{\gamma} - f \right) - Af
\]

using that \(u_i - u_a = \frac{\eta}{\gamma}\). This outperformance is positive if and only if \(A \leq I \left( \frac{2}{\gamma} - f \right) / f = I\), where we used \(f = \frac{\eta}{2\gamma}\).

**Proof of Proposition 4.** (i) We offer an informal, graphical argument. As \(c\) decreases, the blue line \(M(I)\) shifts to the right. Given that the blue line crosses the red one from below, the number of investors \(I\) unambiguously increases, while the number of managers \(M\) increases if and only if the red curve, thus \(I^{-1}(I)\), increases at the equilibrium point. The increase in \(I\) translates into a lower \(\eta\) because of (12) and a lower \(f\) because of (14). Further, from (17) we see that \(fI = Mk\) so that the total fee revenue behaves is proportional to \(M\), thus unimodal.

(ii) Fixing \(N\), \(j\) can be taken high enough to ensure that

\[
c_j (M(N), N) = c_j \left( \frac{\eta}{2k\gamma} N, N \right) \leq \frac{\eta}{2\gamma}.
\]

Thus, for any level \(N\), for \(j\) high enough \(I_j \geq N\), so that \(I_j\) tends to infinity, \(\eta_j\) tends to
zero, and $f_j$ decreases to zero. In addition, from Lemma 1 we know that $M_j = \frac{I_j \eta_j}{2 \gamma k} \to 0$ and so $f_j I_j = M_j k \to 0$.

**Proof of Proposition 5.** A decrease in $k$ causes the red line $\mathcal{M}(I)$ to shift upwards — i.e., $M$ increases for every level of $I$.

**Proof of Proposition 6.** Letting $x$ denote either $\sigma_v^2$ or $\sigma_q^2$, we note that $\frac{\partial M}{\partial x} > 0$ — graphically, this translates in an upward shift in the red curve.

We rewrite (18)–(19) abstractly as

\[
0 = -\frac{1}{2} \eta + \gamma \mathcal{C}(M, I) \equiv g^I(I, M) = g^I(\mathcal{I}(M), M) \quad (B.15)
\]

\[
0 = -\frac{1}{2} \eta + \gamma M \frac{M}{I} \equiv g^M(I, M) = g^M(I, \mathcal{M}(I)), \quad (B.16)
\]

and note that the fact that $\mathcal{M}$ crosses $\mathcal{I}^{-1}$ from above means that $\mathcal{M}'(I) < (\mathcal{I}^{-1})'(I)$, which, using subscripts to indicate partial derivatives, translates into

\[
-\frac{g^I_M}{g^M_M} < -\frac{g^I_I}{g^M_M}, \quad (B.17)
\]

which is equivalent to

\[
g^I_M g^M_I < g^I_I g^M_M \quad (B.18)
\]

because $g^I_M < 0$ and $g^M_M > 0$.

The dependence of $I$ and $M$ on $x$ is given as a solution to

\[
\begin{pmatrix} g^I_I & g^I_M \\ g^M_I & g^M_M \end{pmatrix} \begin{pmatrix} I_x \\ M_x \end{pmatrix} = \begin{pmatrix} 1 \frac{\partial \eta}{\partial x} \\ 2 \frac{\partial \eta}{\partial x} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad (B.19)
\]

which is given by

\[
\begin{pmatrix} I_x \\ M_x \end{pmatrix} = \frac{1}{g^I_M g^M_M - g^I_I g^M_M} \left( g^I_M - g^M_M \right) \left( g^I_I - g^M_I \right) \begin{pmatrix} 1 \frac{\partial \eta}{\partial x} \\ 2 \frac{\partial \eta}{\partial x} \end{pmatrix}. \quad (B.20)
\]

We note that $g^I_M - g^M_M < 0$ and $g^I_I - g^M_I < 0$, while the determinant $g^I_M g^M_I - g^I_I g^M_M$ is negative because the function $\mathcal{M}(I)$ crosses $\mathcal{I}^{-1}(I)$ from above. Thus, at an interior stable equilibrium both $I$ and $M$ increase as $\sigma_v^2$ or $\sigma_q^2$ increases. At a corner equilibrium, given by $I = N < \mathcal{M}^{-1}(M)$, $M$ increases with $\sigma_v^2$ or $\sigma_q^2$, while $I$ is constant.

The total amount of fees $fI = kM$ increases with $M$.

The effect on the efficiency of the asset market, on the other hand, is not determined.
To see this clearly, differentiate (18) to get

\[ \frac{1}{2} \frac{d\eta}{dx} = \gamma (c_M M_x + c_I I_x) \]  

(B.21)

and remember that $c_M \leq 0$ and $c_I \geq 0$. Since $M_x > 0$ and $I_x > 0$, by setting one of the partial derivatives $c_M$ and $c_I$ to zero and keeping the other non-zero, the sign of $\frac{d\eta}{dx}$ can be made either positive or negative. Consequently the efficiency may increase as well as decrease, a conclusion that translates to the fee $f$.

Exactly the same argument works when increasing $(\sigma_v, \sigma_\varepsilon)$ or $(\sigma_v, \sigma_\varepsilon, \sigma_q)$ proportionally.

Proof of Propositions 7–8. These propositions follow from the observation that the derivation of the fee and indifference condition continues to hold, where the risk-aversion and search cost are made investor specific. In particular, Equations (14) and (16) with $\gamma$ replaced by $\gamma_i = \gamma RW_t^{-1}$ and $c$ by $c_i$ give the propositions.

Proof of Proposition 9. We have

\[
\gamma \eta = \log \left( E \left[ e^{-\frac{1}{2}(v-p) \frac{\var(v|p)}{\var(v|s)}} \right] \right) - \log \left( E \left[ e^{-\frac{1}{2}(v-p) \frac{\var(v|p)}{\var(v|s)}} \right] \right) 
\]

(B.22)

\[
\approx \frac{1}{2} \left( E \left[ (v-p) \frac{\var(v|s, p)}{\var(v|s)} \right] - E \left[ (v-p) \frac{\var(v|p)}{\var(v|p)} \right] \right) 
\]

(B.23)

\[
= \frac{1}{2} \left( E[SR_i^2] - E[SR_u^2] \right), 
\]

(B.24)

where the second line follows from linear approximations to the exponential and logarithmic functions, and the third owes to the fact that the conditional variances are constant.

Proof of Proposition 10. We can write the (il)liquidity as a function of $I$ given by

\[
\lambda(I) = \frac{\gamma \sigma_v^2 \sigma_\varepsilon^2 + (N - I + A) \frac{\var(v|s)}{\var(v|p)} \frac{\Theta}{\sigma_v^2 + \sigma_\varepsilon^2 + \Theta}}{I + (N - I + A) \frac{\var(v|s)}{\var(v|p)}}. 
\]

(B.25)

Suppose first that $A = 0$ and note that the numerator of $\lambda$ is minimized by $I = N$, while the denominator is increasing in $I$ because $\var(v|s) < \var(v|p)$ and $\var(v|p)$ decreases with $I$. Consequently, $\lambda$ is minimal at $I = N$. Given that (B.25) is a smooth function, we also infer that $\lambda$ decreases for $I$ in a neighborhood of $N$ — and therefore also if $A$ lies in a suitable neighborhood of $N$. Thus, if $A$ and $c$ are low enough that $I$ is sufficiently close to $N$, then $\lambda$ decreases as $c$ decreases further.

Once $c$ is low enough that $I = N - A$, then $\lambda$ is trivially constant.

Proof of Proposition 11. The effect of the cost $c^u$ is to increase the gain from trade between an investor and the manager — from $\eta$ to $\eta + \frac{c^u}{\gamma}$. Following the same line of reasoning
as in the proof of Proposition ??, we find
\[
\left( \begin{array}{c} I_{c\alpha} \\ M_{c\alpha} \end{array} \right) = \frac{1}{g^I_M g^M_I - g^I_I g^M_M} \left( g^M_M - g^M_I \right) \left( \frac{1}{2} \frac{\partial (\eta + c^n)}{\partial c^n} \right),
\]
(B.26)
which is positive given the proof of Proposition ?? and \( \frac{\partial (\eta + c^n)}{\partial c^n} = 1 > 0 \).

The other results follow from the facts that \( \eta \) decreases with \( I \) and that \( f I = k M \).

The effect on the fee \( f \) is ambiguous because \( f = \frac{1}{2} \left( \frac{2}{\gamma} + c^n \right) = c \) may either increase or decrease, as one can see by considering examples for \( c \) such as \( c(M, I) = \frac{c}{M} \) and \( c(M, I) = \frac{c^2}{M} \).

**Proof of Proposition 12.** We omit the details of the derivation, which is standard. The proof uses the well-known fact (B.1) to calculate the demands:
\[
x^j_i = \frac{\mathbb{E}[v | v - \hat{\theta}_q (q - Q), s^j] - p}{\gamma \text{var}(v | v - \hat{\theta}_q (q - Q), s^j)} \tag{B.27}
\]
\[
x^u = \frac{\mathbb{E}[v | v - \hat{\theta}_q (q - Q)] - p}{\gamma \text{var}(v | v - \hat{\theta}_q (q - Q))} \tag{B.28}
\]

We note that, in computing the optimal demands, the following quantities are helpful:
\[
\text{var}(v | p)^{-1} = \sigma_v^{-2} + \hat{\theta}_q^{-2} \sigma_q^{-2} \tag{B.29}
\]
\[
\text{var}(v | p, s^j)^{-1} = \sigma_v^{-2} + \sigma_\varepsilon^{-2} + \hat{\theta}_q^{-2} \sigma_q^{-2}. \tag{B.30}
\]

Furthermore, given the relation between \( \eta \) and the ratio of these two variances, using \( \hat{\theta}_q = \frac{\gamma^2}{\gamma^2 + \sigma_\varepsilon^2 \sigma_q^2 (\sigma_v^2 + 1)} \), we derive
\[
\eta = -\frac{1}{2} \log \left( 1 - \frac{\gamma^2 \sigma_v^2 \sigma_q^2}{\gamma^2 + \sigma_\varepsilon^2 \sigma_q^2 (\sigma_v^2 + 1)} \right). \tag{B.31}
\]

**Remark:** No agent would choose to search and invest with a second manager if the cost and fee that she would have to pay were the same. Intuitively, this result is due to the diminishing marginal value of information. Precisely, we have
\[
\text{var}(v | p, s^{j_1}, s^{j_2})^{-1} = \sigma_v^{-2} + 2 \sigma_\varepsilon^{-2} + \hat{\theta}_q^{-2} \sigma_q^{-2} \tag{B.32}
\]
and the utility gain
\[
\gamma (u_{2i} - u_i) = \frac{1}{2} \log \left( \frac{\text{var}(v | p, s^j)}{\text{var}(v | p, s^{j_1}, s^{j_2})} \right) < \frac{1}{2} \log \left( \frac{\text{var}(v | p)}{\text{var}(v | p, s^j)} \right) = \gamma (u_i - u_u). \tag{B.33}
\]
References


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