Cash Flow Multipliers and Optimal Investment Decisions

Holger Kraft[1] Eduardo S. Schwartz[2]

[1]Goethe University Frankfurt

[2]UCLA Anderson School

Agenda

1. Contributions
2. Model
3. Optimal Cash-Flow Multiplier
4. Panel Regressions
5. Robustness Checks
6. Value of the Option to Invest
7. Conclusion

Agenda

1. Contributions
2. Model
3. Optimal Cash-Flow Multiplier
4. Panel Regressions
5. Robustness Checks
6. Value of the Option to Invest
7. Conclusion

Contributions

- We develop a **theoretical discounted cash flow valuation model**, determine the optimal investment policy and calculate the ratio of the current value of the firm and the current cash flow which we call the "cash flow multiplier".
- The model provides a **link between the cash flow multiplier and the optimal investment policy**.
- Using a very **extensive data set** comprised of more than 16,500 firms over 38 years we examine the determinants of the cash flow multiplier.
- We include as explanatory variables **macro and firm specific variables** suggested by the theoretical model.
- We find **strong support for the variables** suggested by the model.
- Perhaps the most interesting aspect of the paper is the formulation of a parsimonious **empirical asset pricing model**.

Related Literature

- Discounting of stochastic cash flows and stock valuation
  - Ang and Liu (2001, 2004, and 2007)
- Conditional expected returns when there are growth options
  - Theoretical: Berk, Green and Naik (1999) and Carlson, Fisher and Giammarino (2004)
  - Empirical: Titman, Wei and Xie (2004), Anderson and Garcia (2005), and Li and Zhang (2009)
- Real options and competitive markets
  - Real options: Brennan and Schwartz (1985) and McDonald and Siegel (1986)
  - Competitive markets: Grenadier (2002) and Aguerrevere (2009)
- Multipliers in Accounting
  - Boatsman and Baskin (1981), Alford (1992), Baker and Ruback (1999), Nissim and Thomas (2001), Liu, Nissim and Thomas (2002, 2007), Bhojraj and Ng (2007)

Agenda

1. Contributions
2. Model
3. Optimal Cash-Flow Multiplier
4. Panel Regressions
5. Robustness Checks
6. Value of the Option to Invest
7. Conclusion

Model

We consider a firm with **cash flow** dynamics (before investment)

$$dC = C[\mu(\pi, X)dt + \sigma(\pi, X)dW], \quad C(0) = c,$$

where

- X is a state process,
- π is the percentage of the firm's cash flow reinvested,
- W is a Brownian motion.

Important: Firm can control its cash flow stream by investing!

Firm Value with Endogenous Investment

The firm value is given by

$$V(c, x) = \max_{\pi} \mathrm{E}\Big[\int_0^\infty e^{-\int_0^s R(X_u)\,du}(C_s - I_s)ds\Big],$$

where $I = \pi C$.

Linearity and Cash Flow Multiplier

Proposition: Linearity of Firm Value

Firm value is linear in the cash flow, i.e.

$$V(c, x) = f(x)c,$$

where $f(x) = V(1, x)$.

The result leads to the following definition.

Definition: Cash Flow Multiplier

In our model, the function f is said to be the cash flow multiplier.

Interpretation: This is the multiple by which the current cash flow must be multiplied to obtain the firm value.

Examples: Specifications of Risk-adjusted Discount Rates

Two-factor model

Stochastic riskfree rate r, stochastic beta β

$$R = r + \beta\lambda,$$

where $\lambda = \overline{\lambda} + \lambda^r r$ is the risk premium

In this talk: One-factor model

$$R = \varphi + \psi r$$

where φ and ψ are constants.

Possible interpretation: $\varphi = \beta\overline{\lambda}$ and $\psi = 1 + \beta\lambda^r$

Endogenous Expected Growth and Volatility

- Dividend-discount model: Cash flow multiplier beyond the control of the firm (exogenous).
- In contrast, we explicitly model the firm's opportunity to change its risk-return tradeoff.
- More precisely, we allow the firm to control the expected growth rate and the volatility of the cash flow stream by its investment policy.

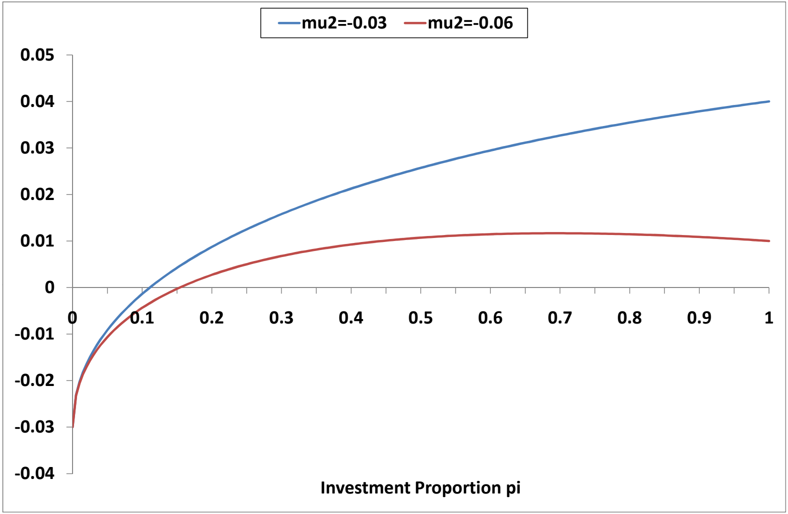
Benchmark Specification

Expected cash flow growth and volatility are given by the concave functions

$$\mu(\pi, r) = \mu_0(r) + \mu_1\sqrt{\pi} + \mu_2\pi, \quad \sigma(\pi) = \sigma_0 + \sigma_1\sqrt{\pi} + \sigma_2\pi,$$

where $\mu_0(r) = \overline{\mu}_0 + \widehat{\mu}_0 r$.

Endogenous Expected Growth



We assume $\mu_0 = -0.03$ and $\mu_1 = 0.1$. For the upper curve, we have $\mu_2 = -0.03$ and for the lower one $\mu_2 = -0.06$.

Agenda

1. Contributions
2. Model
3. Optimal Cash-Flow Multiplier
4. Panel Regressions
5. Robustness Checks
6. Value of the Option to Invest
7. Conclusion

HJB and Optimal Investment Policy

We assume the **short rate** to have **Vasicek** dynamics

$$dr = (\theta - \kappa r)dt + \eta dW_r,$$

where W_r is a Brownian motion with $d < W, W_r >= \rho dt$.

The **optimal cash flow multiplier** satisfies the **HJB-equation**

$$\begin{aligned} 0 &= \max_{\pi}\{(\overline{\mu}_0 + \widehat{\mu}_0 r + \mu_1\sqrt{\pi} + \mu_2\pi)f + 1 - \pi - (\varphi + \psi r)f \\ &\quad +(\theta - \kappa r)f_r + 0.5\eta^2 f_{rr} + \rho\eta(\sigma_0 + \sigma_1\sqrt{\pi} + \sigma_2\pi)f_r\}. \end{aligned}$$

Optimal Investment Proportion

The first-order condition yields the firm's optimal investment

$$\pi^* = \left(\frac{\mu_1 f + \rho\eta\sigma_1 f_r}{2(1 - \mu_2 f - \rho\eta\sigma_2 f_r)}\right)^2.$$

Optimal Cash Flow Multiplier

Substituting π^* into HJB equation yields

$$0 = (\widehat{\varphi}+\widehat{\psi} r) f + 1 + (\theta + \rho\eta\sigma_0 - \kappa r) f_r + 0.5\eta^2 f_{rr} + \frac{(\mu_1 f + \rho\eta\sigma_1 f_r)^2}{4(1 - \mu_2 f - \rho\sigma_2\eta f_r)}.$$

Proposition: Optimal Cash Flow Multiplier

The optimal cash flow multiplier has the stochastic representation

$$f(r) = \underbrace{\int_0^\infty e^{A(s)-B(s)r}\, ds}_{\text{CFM without Invest.}} + \underbrace{\mathcal{O}(r; f)}_{\text{Growth Opport.}} ,$$

where A and B are known deterministic functions and

$$\mathcal{O}(r; f) \equiv \int_0^\infty \widehat{\mathrm{E}}\left[e^{\int_0^s \widehat{\varphi}+\widehat{\psi} r_u\, du} \frac{(\mu_1 f(r_s) + \rho\eta\sigma_1 f_r(r_s))^2}{4(1 - \mu_2 f(r_s) - \rho\sigma_2\eta f_r(r_s))} \right] ds$$

captures the firm's growth opportunities.

Special Case: Constant Interest

We assume that the risk-adjusted discount rate is the sum of a constant short rate and spread, i.e.

$$R = r + \lambda = const.$$

Then the cash flow multiplier has the representation

$$f = \int_0^\infty e^{(\mu_0 - r - \lambda)s}\, ds + \underbrace{\int_0^\infty e^{(\mu_0 - r - \lambda)s} \frac{(\mu_1 f)^2}{4(1-\mu_2 f)}\, ds}_{=\mathcal{O}(f)},$$

which can be solved explicitly.

Optimal Cash Flow Multiplier under Constant Interest

If $\mu_1^2/4 - \mu_2(\mu_0 - r - \lambda) < 0$, then the optimal cash flow multiplier is uniquely given as the positive root of the quadratic equation

$$0 = \left[\mu_1^2/4 - \mu_2(\mu_0 - r - \lambda)\right] f^2 + (\mu_0 - r - \lambda - \mu_2) f + 1.$$

Dependence on Pi

The cash flow multiplier has the representation

$$f = \int_0^\infty e^{(\mu_0 - r - \lambda)s}\, ds + \int_0^\infty e^{(\mu_0 - r - \lambda)s} \frac{(\mu_1 f)^2}{4(1 - \mu_2 f)}\, ds,$$

which can be rewritten

$$f = f_0 + f_0(1 - \mu_2 f)\pi^*.$$

Solving for f and taking logarithms yields

$$\begin{aligned} \ln f &= \ln f_0 + \ln(1 + \pi^*) - \ln(1 + f_0 \mu_2 \pi^*) \\ &\approx \ln f_0 + \beta_1 \pi^* + \beta_2 (\pi^*)^2 \end{aligned}$$

where β_1 is positive and β_2 is negative (diminishing marginal returns on capital).

Special Case: Constant Interest and No Growth Options

Recall from the last slide:

$$0 = \left[\mu_1^2/4 - \mu_2(\mu_0 - r - \lambda)\right] f^2 + (\mu_0 - r - \lambda - \mu_2)f + 1.$$

If the firm has no control over the expected growth rate of its cash flow stream ($\mu_1 = \mu_2 = 0$), then we obtain

Cash Flow Multiplier of Gordon Growth Model

$$f = \frac{1}{r + \lambda - \mu_0}$$

Numerical Example

1st Example. $\mu_0 = -0.03$, $\mu_1 = 0.1$, $\mu_2 = -0.03$, $R = 0.07$

- Cash flow multiplier with optimal investment: 13.06
- Cash flow multiplier without investment: 10
- Growth option $\mathcal{O} = 3.06$
- Opportunity to invest increases cash flow multiplier by 30%

2nd Example. $\mu_0 = -0.05, \ldots$

- Cash flow multiplier with optimal investment: 9.91
- Cash flow multiplier without investment: 8.33
- Growth option $\mathcal{O} = 1.58$
- Opportunity to invest increases cash flow multiplier by 18%

This patterns hold in general!

Cash Flow Multiplier and Growth Options

Net Present Value of Growth Opportunities

If $\mu_1^2/4 - \mu_2(\mu_0 - r - \lambda) < 0$ and $\mu_0 - r - \lambda - \mu_2 < 0$ hold, then the optimal cash flow multiplier f, the option value $\mathcal{O}$, and the ratio $\mathcal{O}/f$ are increasing in μ_0.

- This result puts some of the classical results on real options into perspective.
- If the firm is forced to invest for instance because competitors do the same, then the option to invest loses (part of) its value.
- Hence, the cash flow multiplier decreases.

One State Variable: Stochastic Interest Rates

Optimal Cash Flow Multiplier under Stochastic Interest Rates

The cash flow multiplier has the following series representation

$$f(r) = \sum_{n=0}^{\infty} \sum_{i=0}^{\infty} a_i^{(n)} \left(r - \frac{\theta}{\kappa} \right)^i \eta^n$$

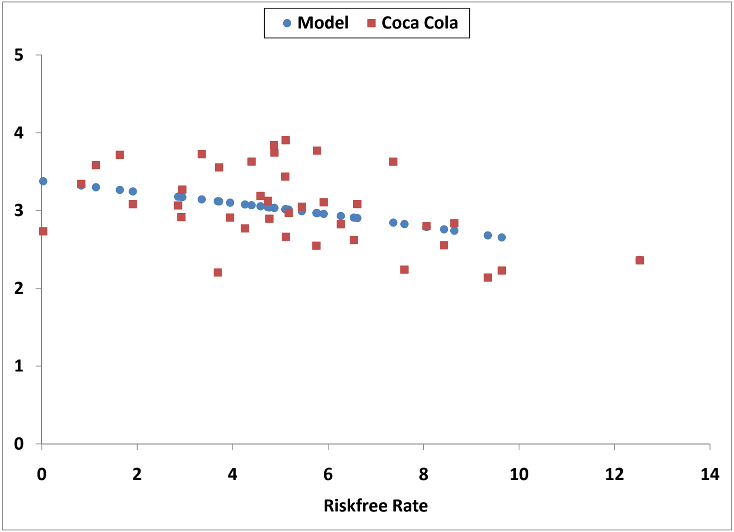
where the coefficients $a_i^{(n)}$ are given by an explicit recursion.

- It is convenient to expand at zero for the interest rate volatility η and at the short rate's mean reversion level θ/κ.
- To see the advantage, substitute $\eta = 0$ and $r = \theta/\kappa$ into the expansion to obtain $f = a_0^{(0)}$.
- This choice is equivalent to assuming that the short rate is constant and equal to the mean reversion level θ/κ.
- Consequently, our expansion is an expansion around the cash flow multiplier for constant interest rates.

Calibration Exercise: Coca Cola

- **Sample period:** 1971-2008, i.e. 38 observations of Coca Cola's cash flow multiplier on December 31
- Parameters of the **riskfree short rate** process: $\kappa = 0.08$, $\eta = 0.015$, and $\theta = 0.004$.
- This implies that the mean reversion level $\theta/\kappa = 0.05$ is close to the sample average of the one-month Fama-French riskfree rate as reported by CRSP.
- **Firm value** $\equiv$ book value + market value equity - book value equity - deferred taxes
- **Free cash flows** (before investment) $\equiv$ EBITDA $-$ taxes $-$ Δworking capital $+$ asset sales
- Least-square fit of our model.

Log Cash Flow Multiplier of Coca Cola (1971-2008)



Almost linear relationship

Agenda

1. Contributions
2. Model
3. Optimal Cash-Flow Multiplier
4. Panel Regressions
5. Robustness Checks
6. Value of the Option to Invest
7. Conclusion

Hypotheses about the Coefficients

- **Macro variables**
  - **Model** (discount rate):
    Real riskfree $(-)$, Slope $(-)$, Spread $(-)$
  - **Controls** (state of economy):
    Inflation $(-)$, S&P 500 $(+)$, Vol_sp $(-)$
- **Firm specific variables**
  - **Model** (investment policy):
    Pi $(+)$, Pi^2 $(-)$
  - **Controls**:
    Size $(+)$, Leverage $(-/+)$, Dividend dummy $(-)$

Firm Data

- Sample period covers 38 years ranging from 1971 to 2008.
- Firm data from Compustat
- Free cash flows (before investment)
  $\equiv$ EBITDA $-$ taxes $-$ Δworking capital $+$ asset sales
- We have 108,443 observations from 16,567 firms where the cash flow multiplier is positive.
- As a robustness test we introduce another measure of free cash flows which comes from the cash flow statement (available in Compustat since 1988).

Free Cash Flows from Income Statement

Accounting Figure	Compustat Name	Item
EBITDA	Operating Income before Deprec.	oibdp
$-$ Taxes	Income Taxes - Total	txt
$+$ Δ Deferred Taxes and Tax Credit	do.	Δ txditc
$-$ Δ Net Working Capital	Working Capital Change - Total	wcapch
$+$ Asset Sales	Sale of Property	sppe
$=$ Free Cash Flows (before Invest.)		

Summary Statistics: Firm Data for Free Cash Flows

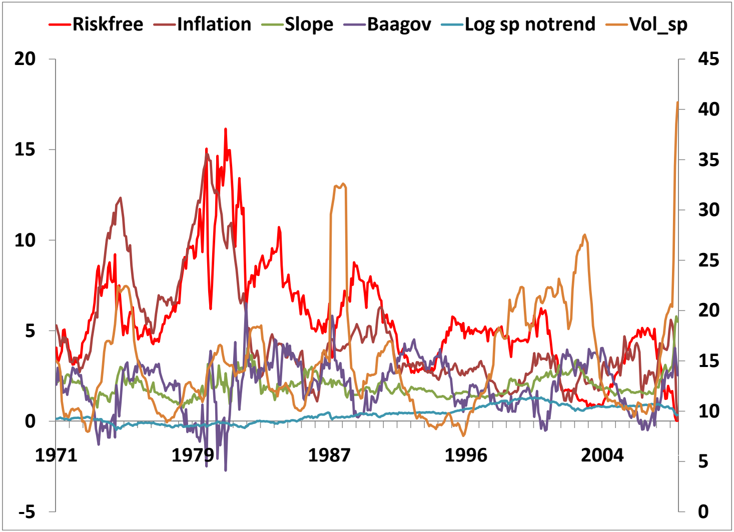
	Mean	Std. Dev.	Min.	Max.	Median
Log_ratio	2.656	1.078	-5.062	23.827	2.472
Pi	1.021	1.588	0	10.523	0.600
Log_real_size	-0.37	2.234	-12.968	7.927	-0.483
Leverage	0.177	0.172	0	0.952	0.135

- Log_ratio is the log of the cash flow multiplier.
- Pi is the fraction of the cash flow invested (winsorized at 1% level).
- Size $\equiv$ # shares outstanding $\times$ share price
- Real_size $\equiv$ Size / CPI
- Leverage $\equiv$ Debt / (Debt + Size)
- Debt: book value, Size: market value

Macro Data

- Macro data from CRSP and from Global Financial Data.
- One-month Fama-French riskfree rate from CRSP
- Inflation is the annual growth of CPI
- Slope of Treasury yield curve
  $\equiv$ 14y Treasury yield minus riskfree rate
- 14y Baa corporate bond spread (as reported by Moody's)
- Log of detrended S&P 500
- Historical volatility of the stock market from the value weighted S&P 500 index as reported in CRSP calculated over last 250 trading days.

Macro Data



The y-axis on the right-hand side applies to Vol_sp only.

Summary Statistics: Macro Data

	Mean	Std. Dev.	Min.	Max.	Median
Riskfree	5.565	2.856	0.03	16.15	5.107
Real_riskfree	0.912	2.42	-8.184	6.641	1.014
Inflation	4.653	2.992	0.091	14.756	3.645
Slope	1.958	1.421	-2.726	6.511	2.086
Baagov	1.956	0.628	0.754	5.788	1.818
Log_sp_notrend	0.374	0.458	-0.473	1.319	0.369
Vol_sp	14.839	5.575	7.547	40.697	13.426

Correlation Matrix of Explanatory Variables

	Log_rat	Real_rf	Infl	Slope	Baag	Log_sp	Vol	Pi	Log_rs	Lev
Log_ratio	1.000									
Real_riskfree	0.037	1.000								
Inflation	-0.096	-0.477	1.000							
Slope	-0.009	-0.119	-0.262	1.000						
Baagov	-0.076	-0.128	-0.112	0.181	1.000					
Log_sp_notrend	0.111	0.208	-0.674	-0.194	-0.063	1.000				
Vol_sp	-0.067	-0.129	-0.147	0.096	0.708	0.148	1.000			
Pi	0.572	0.026	0.024	-0.011	-0.018	-0.033	-0.024	1.000		
Log_real_size	0.118	-0.039	-0.099	-0.033	-0.009	0.140	-0.026	-0.005	1.000	
Leverage	-0.287	-0.059	0.144	-0.003	0.035	-0.155	0.026	-0.014	-0.092	1.000

Benchmark Panel Regressions

	(1)	(2)
Real_riskfree	-0.015*	-0.016
	(-2.43)	(-1.92)
Inflation	-0.020*	-0.020*
	(-2.50)	(-2.39)
Slope	-0.002	-0.003
	(-0.22)	(-0.35)
Baagov	-0.048*	-0.038
	(-2.10)	(-1.71)
Log_sp_notrend	-0.040	0.056
	(-0.86)	(1.05)
Vol_sp	-0.007**	-0.008**
	(-2.85)	(-2.84)
Pi	0.414***	0.399***
	(51.14)	(53.72)
Log_real_size	0.195***	0.067***
	(13.55)	(9.92)
Leverage	-0.619***	-1.296***
	(-13.56)	(-29.02)
Div_dummy	-0.156***	-0.158***
	(-13.89)	(-13.20)
Intercept	2.811***	2.881***
	(32.28)	(27.17)
R^2	0.505	0.478
Firm Fixed effects	yes	no
FF industry dummies	no	yes

Benchmark Panel Regressions

- All **firm specific** are very **significant** and have the expected signs.
- In particular, the investment policy is positively related with the cash flow multiplier.
- Both **macro variables**, real riskfree rate and the Baa spread, have **negatively significant** effects on the cash flow multiplier.
- The macro controls, inflation and volatility, are also negatively significant.
- Robust version of the Hausman test: Null hypothesis of no firm fixed-effects is rejected at all levels.
- In the following, we thus report the results of **firm fixed-effects** regressions (unless otherwise stated).

Excluding Variables

	(1)	(3)	(4)	(5)	(6)	(7)
Real_riskfree	-0.015*	-0.014*	-0.014*	-0.004		
	(-2.43)	(-2.06)	(-2.22)	(-0.46)		
Inflation	-0.020*	-0.022*	-0.016	-0.020		
	(-2.50)	(-2.22)	(-1.93)	(-1.58)		
Slope	-0.002	-0.004	-0.001	-0.004		
	(-0.22)	(-0.42)	(-0.06)	(-0.30)		
Baagov	-0.048*	-0.091***		-0.056		
	(-2.10)	(-3.68)		(-1.70)		
Log_sp_notrend	-0.040	-0.070	-0.009	0.011		
	(-0.86)	(-1.24)	(-0.19)	(0.15)		
Vol_sp	-0.007**		-0.011***	-0.013**		
	(-2.85)		(-4.04)	(-3.05)		
Pi	0.414***	0.414***	0.415***		0.414***	0.421***
	(51.14)	(51.56)	(50.89)		(50.48)	(50.45)
Log_real_size	0.195***	0.197***	0.194***		0.203***	
	(13.55)	(13.65)	(13.15)		(18.70)	
Leverage	-0.619***	-0.636***	-0.627***		-0.694***	
	(-13.56)	(-12.81)	(-14.33)		(-14.27)	
Div_dummy	-0.156***	-0.156***	-0.158***		-0.168***	
	(-13.89)	(-13.15)	(-13.90)		(-10.04)	
Intercept	2.811***	2.821***	2.738***	3.055***	2.506***	2.225***
	(32.28)	(27.55)	(30.39)	(22.40)	(103.31)	(96.42)
R^2	0.505	0.504	0.504	0.016	0.499	0.450

Regressions with Pi^2

	(1)	(8)	(9)
Real_riskfree	-0.015*	-0.019**	
	(-2.43)	(-2.67)	
Inflation	-0.020*	-0.023**	
	(-2.50)	(-3.04)	
Slope	-0.002	0.000	
	(-0.22)	(0.02)	
Baagov	-0.048*	-0.041	
	(-2.10)	(-1.89)	
Log_sp_notrend	-0.040	0.012	
	(-0.86)	(0.24)	
Vol_sp	-0.007**	-0.007**	
	(-2.85)	(-2.82)	
Pi	0.414***	0.729***	0.749***
	(51.11)	(23.25)	(22.58)
Pi^2		-0.035***	-0.037***
		(-12.95)	(-12.73)
Log_real_size	0.195***	0.180***	
	(13.57)	(12.73)	
Leverage	-0.619***	-0.596***	
	(-13.53)	(-14.03)	
Div_dummy	-0.156***	-0.148***	
	(-13.89)	(-13.09)	
Intercept	2.811***	2.578***	2.022***
	(32.29)	(32.26)	(75.81)
R^2	0.505	0.538	0.486

Agenda

1 Contributions

2 Model

3 Optimal Cash-Flow Multiplier

4 Panel Regressions

5 Robustness Checks

6 Value of the Option to Invest

7 Conclusion

Robustness Checks

- Now, we consider several robustness checks.
- The tests include
  - standard errors,
  - different ways to winsorize Pi,
  - different definitions of investment policy,
  - exclusion of firms with few observations,
  - alternative measure of cash flows.

Standard Errors

	(2)	(10)	(11)
Real_riskfree	-0.016	-0.016**	-0.013***
	(-1.92)	(-3.25)	(-8.75)
Inflation	-0.020*	-0.020**	-0.019***
	(-2.39)	(-3.13)	(-11.39)
Slope	-0.003	-0.003	-0.000
	(-0.35)	(-0.38)	(-0.16)
Baagov	-0.038	-0.038*	-0.044***
	(-1.71)	(-2.31)	(-10.16)
Log_sp_notrend	0.056	0.056	0.007
	(1.05)	(1.59)	(0.69)
Vol_sp	-0.008**	-0.008***	-0.008***
	(-2.84)	(-5.15)	(-15.23)
Pi	0.399***	0.399***	0.411***
	(53.72)	(103.61)	(144.70)
Log_real_size	0.067***	0.067***	0.129***
	(9.92)	(16.89)	(42.98)
Leverage	-1.296***	-1.296***	-0.935***
	(-29.02)	(-32.51)	(-36.49)
Div_dummy	-0.158***	-0.158***	-0.172***
	(-13.20)	(-14.99)	(-19.79)
Intercept	2.881***	2.881***	2.887***
	(27.17)	(34.35)	(44.20)

(2) Driscoll-Kraay, (10) clustering by firm and year, (11) clustering by firm (all with Fama-French industry dummies)

Different Ways to Winsorize Pi

	(1)	(12)	(13)
Real_riskfree	-0.015*	-0.020**	-0.019*
	(-2.43)	(-2.58)	(-2.19)
Inflation	-0.020*	-0.024***	-0.025**
	(-2.50)	(-3.29)	(-3.08)
Slope	-0.002	0.003	0.006
	(-0.22)	(0.41)	(0.66)
Baagov	-0.048*	-0.037	-0.026
	(-2.10)	(-1.68)	(-1.16)
Log_sp_notrend	-0.040	0.050	0.066
	(-0.86)	(1.01)	(1.19)
Vol_sp	-0.007**	-0.007**	-0.009**
	(-2.85)	(-2.82)	(-3.12)
Pi	0.414***		
	(51.11)		
Pi5		0.963***	
		(34.44)	
Pi$<$1			1.768***
			(28.98)
Log_real_size	0.195***	0.173***	0.179***
	(13.57)	(12.04)	(12.83)
Leverage	-0.619***	-0.556***	-0.390***
	(-13.53)	(-14.28)	(-9.31)
Div_dummy	-0.156***	-0.153***	-0.192***
	(-13.89)	(-10.89)	(-11.78)
Intercept	2.811***	2.394***	2.092***
	(32.29)	(29.76)	(24.31)
R^2	0.5051	0.4835	0.3476

In (12), Pi is winsorized at the 5% level. In (13), Pi is set to one if it is above one.

Definition of Investments

- Our proxy for investments are **capital expenditures** that do **not include R&D expenses**.
- The main reason for using this proxy is that we would have lost about 50% of our observations since Item46 is often **missing** in Compustat.
- Therefore, we consider **alternative ways to measure investments**:
  - Adding Capex and R&D together if R&D not missing
  - Defining two investment ratios (Capex and R&D)
- In the second case, we run two regressions (one where R&D is set to zero if it is missing and one where the observation is disregarded)
- Our results are **robust**.

Definition of Investment

	(1)	(14)	(15)	(16)
Real_riskfree	-0.015*	-0.010	-0.013*	-0.011
	(-2.43)	(-1.74)	(-2.26)	(-1.63)
Inflation	-0.020*	-0.015	-0.018*	-0.013
	(-2.50)	(-1.74)	(-2.24)	(-1.44)
Slope	-0.002	-0.002	-0.002	-0.003
	(-0.22)	(-0.22)	(-0.20)	(-0.24)
Baagov	-0.048*	-0.050*	-0.049*	-0.063*
	(-2.10)	(-2.22)	(-2.15)	(-2.57)
Log_sp_notrend	-0.040	-0.087	-0.056	-0.092
	(-0.86)	(-1.88)	(-1.21)	(-1.89)
Vol_sp	-0.007**	-0.007**	-0.007**	-0.006*
	(-2.85)	(-2.67)	(-2.76)	(-2.51)
Pi	0.414***		0.365***	0.315***
	(51.11)		(88.90)	(68.50)
Pi_total		0.252***		
		(140.86)		
Pi_rd			0.124***	0.148***
			(22.94)	(32.49)
Log_real_size	0.195***	0.222***	0.205***	0.256***
	(13.57)	(15.05)	(14.12)	(12.78)
Leverage	-0.619***	-0.529***	-0.570***	-0.516***
	(-13.53)	(-11.39)	(-12.66)	(-8.68)
Div_dummy	-0.156***	-0.162***	-0.153***	-0.162***
	(-13.89)	(-12.36)	(-12.83)	(-9.90)
Intercept	2.811***	2.832***	2.787***	2.859***
	(32.29)	(30.37)	(31.42)	(29.38)
R^2	0.505	0.521	0.5338	0.601

(16) is based on 53,887 observations, whereas the rest is based on 108,443 ob.

Exclusion of Firms with Few Observations

	(1)	(17)	(18)
Real_riskfree	-0.015*	-0.017*	-0.019*
	(-2.43)	(-2.47)	(-2.27)
Inflation	-0.020*	-0.024**	-0.028**
	(-2.50)	(-2.67)	(-2.94)
Slope	-0.002	-0.006	-0.011
	(-0.22)	(-0.53)	(-0.97)
Baagov	-0.048*	-0.030	-0.019
	(-2.10)	(-1.30)	(-0.87)
Log_sp_notrend	-0.040	-0.052	-0.061
	(-0.86)	(-0.95)	(-0.98)
Vol_sp	-0.007**	-0.006*	-0.006*
	(-2.85)	(-2.41)	(-2.31)
Pi	0.414***	0.422***	0.430***
	(51.11)	(49.33)	(51.41)
Log_real_size	0.195***	0.172***	0.166***
	(13.57)	(12.95)	(12.63)
Leverage	-0.619***	-0.681***	-0.612***
	(-13.53)	(-15.48)	(-9.53)
Div_dummy	-0.156***	-0.139***	-0.141***
	(-13.89)	(-10.47)	(-7.58)
Intercept	2.811***	2.711***	2.653***
	(32.29)	(26.35)	(22.35)
R^2	0.505	0.525	0.542
# ob. included	108,443	62,095	22,450
# firms included	16,567	3,842	879

Regressions include firms that have at least 1, 10, 20 full observations.

Alternative Definition of Cash Flows

Accounting Figure	Compustat Name	Item
Net Cash Flow from Operations	Operating Activities – Net Cash Flow	oancf
+ Interest Rate Expense after Tax	Interest and Related Expense – Total	xint
= Free Cash Flows (before Invest.)		

We use two versions of this measure:

- one without considering taxes and
- another assuming a tax rate of 30%

Panel Regressions with Alternative Cash Flow Definition

	(1')	(19)	(20)	(2')	(21)	(22)
Real_riskfree	-0.040***	-0.004	-0.000	-0.074***	-0.036***	-0.032***
	(-3.85)	(-0.48)	(-0.03)	(-7.58)	(-4.78)	(-4.32)
Inflation	-0.029*	-0.014	-0.011	-0.051***	-0.038***	-0.036***
	(-2.51)	(-1.50)	(-1.16)	(-4.12)	(-3.45)	(-3.42)
Slope	-0.009	0.006	0.011	-0.046***	-0.033***	-0.029***
	(-0.91)	(0.63)	(1.09)	(-4.28)	(-3.80)	(-3.42)
Baagov	-0.143***	-0.065**	-0.064**	-0.180***	-0.088***	-0.081***
	(-5.65)	(-2.92)	(-2.69)	(-9.19)	(-4.09)	(-3.72)
Log_sp_notrend	-0.079***	0.039	0.046	-0.053	0.056	0.063*
	(-3.40)	(1.53)	(1.85)	(-1.64)	(1.73)	(2.01)
Vol_sp	-0.000	-0.004	-0.004	0.002	-0.003	-0.004
	(-0.28)	(-1.95)	(-1.92)	(1.04)	(-1.30)	(-1.68)
Pi	0.433***	0.374***	0.352***	0.413***	0.353***	0.332***
	(55.17)	(62.32)	(90.89)	(50.29)	(48.91)	(52.89)
Log_real_size	0.223***	0.173***	0.173***	0.073***	0.064***	0.065***
	(10.60)	(10.93)	(12.00)	(9.38)	(7.79)	(7.87)
Leverage	-0.488***	-0.347***	-0.209***	-1.257***	-0.818***	-0.650***
	(-7.29)	(-7.31)	(-4.89)	(-25.15)	(-21.25)	(-18.41)
Div_dummy	-0.156***	-0.079***	-0.081***	-0.151***	-0.196***	-0.195***
	(-12.45)	(-6.51)	(-6.50)	(-7.59)	(-13.78)	(-13.17)
Intercept	2.983***	2.688***	2.686***	3.364***	3.120***	3.132***
	(33.30)	(32.86)	(34.34)	(28.46)	(29.37)	(29.75)
R^2	0.509	0.465	0.464	0.477	0.426	0.417
Fixed effects	yes	yes	yes	no	no	no
FF industry dummies	no	no	no	yes	yes	yes

Agenda

1 Contributions

2 Model

3 Optimal Cash-Flow Multiplier

4 Panel Regressions

5 Robustness Checks

6 Value of the Option to Invest

7 Conclusion

Value of the Option to Invest

- The **cash flow multiplier** consists of **two parts**.
- Whereas the first part is **exogenous**, the second part is **endogenous** and captures the firm's real option to invest.
- We have shown that the option value is increasing with μ_0.
- This parameter equals the expected cash flow growth if the firm does not invest at all.
- We **expect** μ_0 to be **on average smaller when** the firm operates in an industry that is **more investment intensive**.
- Investment intensity is **measured by** the average fraction of cash flows that is reinvested, i.e. by the **average** π of a particular industry.
- To test this hypothesis, we run regressions where this **average** is included as an **additional explanatory variable**.
- We have seen that the **cash flow multiplier increases with** π.
- Following our line of argument, the **opposite should be true for the mean** of the industry.

Value of the Option to Invest

- There are **two ways** of **calculating** an **industry mean**.
- Firstly, one can calculate the mean over the whole sample period leading to a constant. Secondly, one can compute the mean for every year of the sample period, which provides us with 48 time series of means for the 48 Fama-French industries.
- In the first case, it clearly makes no sense to include firm dummies or fixed effects since otherwise the coefficients of the average π cannot be identified.
- But also in the second case dummies would absorb a lot of the variability that we expect to be captured by the industry means of π.
- For this reason, we run four **pooled regressions without dummies**.

Panel Regressions with Average Investment Proportions

	(23)	(24)	(25)
Real_riskfree	-0.018*	-0.015	-0.014
	(-2.20)	(-1.84)	(-1.85)
Inflation	-0.020*	-0.019*	-0.014
	(-2.40)	(-2.20)	(-1.60)
Slope	-0.003	-0.001	-0.009
	(-0.33)	(-0.14)	(-0.95)
Baagov	-0.049*	-0.046*	-0.057**
	(-2.24)	(-2.01)	(-2.87)
Log_sp_notrend	0.069	0.061	-0.008
	(1.33)	(1.13)	(-0.19)
Vol_sp	-0.007*	-0.008**	-0.008**
	(-2.56)	(-2.81)	(-3.19)
Pi	0.384***	0.393***	0.395***
	(54.34)	(55.29)	(54.96)
Log_real_size	0.056***	0.065***	0.061***
	(7.40)	(8.84)	(8.17)
Leverage	-1.593***	-1.300***	-1.452***
	(-26.97)	(-28.46)	(-26.74)
Div_dummy	-0.160***	-0.132***	-0.147***
	(-11.27)	(-9.97)	(-11.04)
Av_pi		-1.097***	
		(-20.90)	
Av_pi_annual			-0.504***
			(-8.93)
Intercept	2.929***	3.417***	3.189***
	(30.11)	(31.91)	(35.23)
R^2	0.429	0.463	0.445

(23) same explanatory var. as benchmark (1), but no fixed effects or industry dummies.

Agenda

1 Contributions

2 Model

3 Optimal Cash-Flow Multiplier

4 Panel Regressions

5 Robustness Checks

6 Value of the Option to Invest

7 Conclusion

Summary of Results

- We develop a **simple** discounted cash flow valuation **model with optimal investment**.
- The model predicts a
  - **positive relation** between the cash flow multiplier and a firm's **investment policy** that is **nonlinear**.
  - **negative relation** between the multiplier and **discount rates**.
- These predictions are **confirmed** in our empirical analysis where we include additional macro and firm specific control variables.
- We **decompose** the **multiplier** into two parts: the first part reflects the firm value without investment, whereas the second part captures the option to invest optimally in the future.
- We provide empirical evidence that the cash flow multiplier is strongly **negatively related to the average investment policy** of the particular industry.

Implications

- Since the cash flow multiplier depends on observable and relatively easily obtainable variables, the approach taken in this paper can be considered as an **alternative valuation framework**.
- Even though it is based on a discounted cash flow model it does **not require** the **estimation of expected future cash flow** and an appropriate risk adjusted discount rate.
- Potentially then, the approach **could be used to value non-traded firms** and to determine under and over priced firms.