The Importance of Being Slow - The Costs and Benefits of Phasing-In Regulatory Reforms*

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Abstract

We build a stylized dynamic equilibrium model with financial frictions to study how capital requirements affect the dynamics of aggregate bank lending. A sudden regulatory tightening may cause substantial issuance of bank capital, but also a severe credit crunch. When the regulatory tightening is introduced with a transition period, the anticipation of larger profits from lending induces banks to retain more earnings during the transition phase. This accelerates the accumulation of capital buffers and reduces the risk of a credit crunch. In line with recent empirical evidence, lending can increase in response to the announcement of a regulatory tightening.

Keywords: Bank capital requirements, credit crunch, systemic risk

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1 Introduction

The 2007–2009 financial crisis has provoked a remarkable sequence of events. Initially, large losses eroded the capital buffers of banking systems around the world. Many banks were recapitalized by a combination of government interventions (e.g. Laeven & Valencia, 2013), and capital injections by shareholders (e.g. Black et al., 2016; Homar & van Wijnbergen, 2017), which was followed by severe credit crunches and reductions in real economic activity (the "Great Recession") (e.g. Aiyar et al., 2016; De Jonghe et al., 2020; Fraisse et al., 2019). The Basel Committee has in turn introduced new regulatory guidelines (Basel III), which have been implemented over a transition phase of several years.[1]

Our paper studies the costs and benefits of phasing-in a regulatory tightening over a transition phase.[2] The common rationale for phasing-in a regulatory tightening is to allow banks to gradually build up capital buffers and thereby avoid the reduction in lending following an immediate implementation. We find that this strategy is so effective because banks adjust their dividend policies during the transition phase: Anticipating that capital will be more valuable after the implementation of the regulatory tightening, banks retain more of their earnings and convert them to book equity. This helps to achieve the financial stability gains from tighter regulation more quickly and to reduce the risk of a credit crunch. As the anticipation effect thus increases the value of being slow, regulators would significantly underestimate the welfare maximizing length of the transition phase if they did not properly take into account the anticipation-induced change in bank policies. Furthermore, the anticipation of lending being more restricted after the implementation of a regulatory tightening affects the risk-premium required by banks. Quite counter-intuitively, this may cause lending to increase during the transition phase. Our results, thus, help to explain the recent empirical findings that aggregate lending increased in response to the announcement of Basel III (see BIS, 2022, Chapter 8.1.3 and Annex A.18).

In order to analyze how the introduction of (tighter) capital requirements affects aggregate lending, interest rates and capital buffers, we endogenize the dynamics of bank capital in a stylized

[1] As documented in World Bank (2020), minimum capital requirements have steadily increased worldwide since the 2007-2008 financial crisis. The 2010 phase-in of the Basel III capital regulation established a transition period for its implementation. Most jurisdictions completed its implementation in 2019, while others are still working on completion (see Financial Stability Board, 2021, Annex 2), or Kiley and Sim (2014).

[2] Most established literature has studied the optimal level of capital requirements instead of the optimal timing (cf. e.g. Admati et al., 2013; Basel Committee on Banking Supervision, 2019; DeAngelo & Stulz, 2015; Gorton & Winton, 2017).

equilibrium model of banking. Banks finance risky loans to the real sector by issuing equity and liquid deposits, which households use for transactions.[3] The two main frictions in the model are: (i) households cannot invest directly in the real sector and (ii) banks incur a flotation cost when they issue equity. This implies that banks will retain earnings in order to cover future losses on their loan portfolios and save on refinancing costs. Banks are subject to a simple capital requirement, according to which a minimum fraction of loans has to be financed by equity. The motivation for capital requirements in our model is the risk of systemic crises, with socially costly bank failures.

The equilibrium loan rate decreases in aggregate bank capital. This reflects the fact that the aggregate supply of bank loans increases with aggregate bank capital and, as is standard, aggregate loan demand decreases in the loan rate. Lending, therefore, becomes less profitable when banks build up capital buffers by converting retained earnings into book equity. Since loans are partially financed by equity, the market-to-book ratio of equity capital must therefore also decrease in aggregate bank capital. This implies that even though the loan market is perfectly competitive, banks earn a strictly positive lending premium, akin to the risk-premium required by a risk-averse individual with decreasing marginal utility.

The interplay between regulatory restrictions, optimal bank policies and the equilibrium spread described above implies three key testable implications of the model: spreads and market-to-book ratios are negatively related to aggregate bank capital, and aggregate lending is positively related to aggregate bank capital. Using two large international databases for the 1990-2017 period, we find that these implications are broadly consistent with the data. Specifically, we find positive and significant correlations at a cross sectional level between aggregate bank equity and total lending, and a negative and significant correlation between aggregate bank equity, spreads and market-to-book ratios.

Bank strategies in our model resemble the optimal corporate policies in partial equilibrium models such as Bolton et al. (2011); Bolton et al. (2013), Décamps et al. (2011), or De Nicolò et al. (2014), where issuance costs in combination with (exogenous) costs to hold liquidity lead to a decreasing marginal value of cash (i.e. firms become effectively risk averse). However, in our model, the costs and benefits of accumulating equity are determined endogenously in a market equilibrium: When the level of aggregate capital and, thus, loan supply is sufficiently high, the equilibrium loan

[3] In contrast to existing models such as Brunnermeier and Sannikov (2014), in which financial firms themselves manage productive assets in the economy, we explicitly take into account the intermediation by banks.

rate is so high that the marginal loan is unprofitable. Hence, instead of further expanding their loan making businesses, banks optimally distribute any further earnings to their shareholders at that point. If, on the other hand, aggregate capital is sufficiently low, the aggregate loan supply is restricted by the binding capital requirement. This implies a high equilibrium loan rate, which makes lending to the real sector very profitable. Despite having to bear flotation costs, it is then optimal for banks to issue new equity claims to avoid a further deleveraging. Hence, bank policies follow a barrier strategy. At the lower, or issuance barrier, the market-to-book ratio reaches its maximum equal to the total marginal issuance costs. At the upper, or payout barrier, the market-to-book ratio reaches its minimum equal to shareholders' marginal utility of consuming the paid out earnings. In between the two barriers, all earnings are retained to build up capital buffers which, in turn, are used to cover losses on the asset side. Our theory of bank capital therefore emphasizes its loss-absorbing role instead of its incentive effects.[4]

With tighter capital requirements, equity capital becomes more valuable for banks (reflected by a higher market-to-book ratio) for two reasons. First, more capital is needed to finance a given level of lending. Second, lending is more profitable since aggregate supply of bank loans is restricted more severely.[5] With equity capital thus being more valuable under tighter capital requirements, banks optimally retain more earnings and are more inclined to issue new claims despite the issuance costs. That is, both the payout boundary and the issuance boundary are higher under tighter regulation. A regulatory tightening without a transition phase may therefore trigger an immediate recapitalization up to the new issuance boundary. However, it can also cause a severe credit crunch when the tighter regulation becomes binding upon implementation.

If a regulatory tightening is phased-in over a transition period, banks anticipate the announced regulatory tightening. The anticipation of loan supply being more restricted under tighter regulation is "priced-in" by banks if the banking sector is very well capitalized. This leads to a higher equilibrium loan rate and thus a lower level of aggregate lending. If the banking sector is poorly capitalized, however, aggregate lending is fully determined by the binding capital requirement and therefore unaffected by the announcement of a regulatory tightening. Hence, if aggregate capital increases, banks move closer towards the region where lending is more profitable since the anticipated

[4]This appears to capture regulators' motives to impose capital requirements which restrict banks' total equity (i.e. their capital structure). What matters for bank managers' risk-taking incentives, by contrast, is a bank's inside equity including compensation packages.

[5]A related point is made in Schliephake (2016).

scarcity of loans is priced-in. This implies that in the intermediate region, where the regulatory requirement becomes slack, the market-to-book ratio of equity decreases more slowly in aggregate capital. That is, just as with the utility function of a risk-averse individual becoming less concave, the lending premium required by banks decreases in response to the regulatory announcement in this region. As a result, lending will increase, even compared to the status-quo without a regulatory reform. This prediction is in line with the surprising findings by BIS (2022), that aggregate lending increased in response to the announcement of the tighter capital requirements of Basel III. Furthermore, as banks anticipate capital to be more valuable after the implementation of tighter regulation, they want to accumulate more capital during the transition phase already. More precisely, since the anticipated increase in the value of capital is reflected in its current value only for high levels of capital, the additional capital is generated by retaining more earnings instead of issuing more claims.

Finally, we study the welfare maximizing length of the transition phase. The cost of a slower implementation is that with a longer transition phase, it takes longer for the financial stability gains in terms of larger capital buffers to materialize. The benefit is that banks have more time to accumulate capital such that the tighter capital requirements are less likely to be binding when they are implemented. The fact that banks change their policies in response to the regulatory announcement tilts this trade-off towards a longer transition phase: Capital accumulation is accelerated as banks retain more earnings during the transition phase, which tends to reduce the costs of a slower introduction and increase the benefits. The anticipation effect discussed in this paper thus increases the value of being slow. We show that regulators would indeed severely underestimate the optimal transition phase if they did not properly take into account how banks adjust their policies if expectations about regulatory reforms change. Furthermore, the value of being slow is lower (the optimal transition phase shorter) if the social costs of bank failures are higher. Intuitively, with bank failures being more costly, the regulator becomes relatively more concerned about a faster realization of financial stability gains than the stabilization of aggregate lending. If banks have already accumulated larger equity buffers at the time of the regulatory announcement, by contrast, regulators are less concerned about a quick realization of financial stability gains. At the same time, however, they become also less concerned about the stabilization of lending, as the tighter capital requirement is less likely to become binding even if the regulatory change is introduced quickly.

We find the second effect to dominate in all considered examples, such that the regulatory tightening should optimally be introduced more quickly if banks have already accumulated larger capital buffers at the time it is announced.

Related Literature. First, our paper is related to the academic literature that views capital requirements as a way to trade off the expected social cost of bank failures (which is not internalized by bankers) and the welfare reduction due to the limitations on banks' deposits and lending activities. Admati et al. (2013) argue that the first effect dominates and that capital requirements should be much higher. On the contrary, DeAngelo and Stulz (2015); Diamond and Rajan (2000) emphasize the second effect and argue that high capital requirements impede banks' provision of liquidity. Van den Heuvel (2008) was the first to develop a dynamic general equilibrium model allowing a quantitative assessment of this trade off. By calibrating his model on US data, he finds that the social cost of capital requirements amounts to a permanent reduction of aggregate consumption of 0.1 to 1 percent. Martinez-Miera and Suarez (2014) also analyse the impact of capital requirements on bankers' risk-taking incentives in a quantitative dynamic general equilibrium model.[6]

Our model, however, emphasizes the loss absorbing role of bank capital, instead of its incentive effects. In practice, banks typically maintain equity ratios well in excess of regulatory capital requirements,[7] which can only be understood as precautionary buffers against a future need for a costly issuance of new capital. Milne and Whalley (2001) were the first to explore a simple dynamic model with this feature. They show that in the long run, capital requirements have no impact on banks' risk taking. Allen et al. (2011) elaborate on the role of capital buffers by showing how they allow banks to commit to monitoring loans, which ultimately benefits borrowers. Finally, also Corbae and D'Erasmo (2021) consider a model that emphasizes the loss absorbing role of bank capital and analyze the relation between regulation and industry structure.

Following He and Krishnamurthy (2012) and Brunnermeier and Sannikov (2014), a new strand of the literature has developed dynamic macroeconomic models with financial frictions and shown that the (endogenous) capitalization of the financial sector is a crucial factor for explaining the performance of the economy. In Kondor and Vayanos (2019) arbitrageurs offer risk management

[6] The incentive role of (inside) equity is studied in a large literature including for instance Hellmann et al. (2000); Morrison and White (2005); Repullo (2004).

[7] Fonseca and González (2010) show how these capital buffer vary across countries.

services to hedgers. In equilibrium, the wealth of arbitrageurs is a priced risk factor that influences market risk aversion. In Phelan (2016), banks cannot issue new equity but need to rely on retained earnings to increase their capital buffer. As a result, aggregate outcomes depend on the endogenously determined total equity of the banks. In Bolton et al. (2021), banks cannot perfectly control their deposit flows. Precautionary equity buffers are needed to limit the risk that sudden inflows of deposits may force a bank to issue costly equity. Banks' risk aversion is also endogenously determined. Our model exhibits similar features: banks' risk aversion and credit spreads are endogenous functions of the total capitalization of the banking sector.

Our paper is also related to the recent literature that has developed dynamic general equilibrium models to examine the long term impact of capital requirements, such as Begenau and Landvoigt (2021); Clerc et al. (2015); Davydiuk (2017). Begenau (2020) analyses the long term impact of capital requirements on bank lending in a quantitative general equilibrium model. She finds that imposing tighter capital requirements tends to reduce banks' demand for deposits, which drives down interest rates paid on deposits and, thus, banks' funding costs. As a result, bank lending can increase in the long-run.[8] In our paper, the long-run impact on lending is negative — albeit relatively small. Instead we stress that the anticipation of a future regulatory tightening lowers the equilibrium risk-premium and hence increases lending in the *short-run*. Our paper is also related to the recent literature that analyses the impact of new banking regulations such as liquidity requirements (Hugonnier & Morellec, 2017) or counter-cyclical capital requirements (Malherbe, 2020).

The remainder of the paper is organized as follows. Section 2 introduces the model and Section 3 solves for the competitive equilibrium. Section 4 discusses the properties of this equilibrium and presents empirical evidence for our model's implications. Section 5 studies the implementation of a regulatory tightening and 6 concludes. Omitted proofs are in Appendix A and Appendix B provides additional material.

[8] In Begenau and Landvoigt (2021), a similar mechanism leads to an increase in risk-taking by shadow banks when commercial banks face tighter capital requirements.

2 Model

Households, Banks, and Firms. We consider a stylized dynamic general equilibrium model that captures in the simplest possible way the role of banks in the economy. Banks provide payment services to households and extend loans to the real sector. Time is continuous with infinite horizon and there is a single physical good that can be consumed or invested. There is a continuum of risk-neutral households, a continuum of banks and a continuum of short lived, risk-neutral entrepreneurs, all of mass one. Deposits are fully insured and households enjoy a convenience yield of χ from holding deposits (see e.g. Stein, 2012). The deposit rate is thus given by

$$r = \rho - \chi, \tag{1}$$

which is lower than the common discount rate ρ.[9] Furthermore, banks issue equity to households. As is common for instance in the literature on corporate liquidity management (Bolton et al., 2011; Décamps et al., 2011), but also in dynamic models of banking, such as Hugonnier and Morellec (2017), we assume that the equity market is not perfectly efficient: when banks issue new equity, they incur a proportional flotation cost of γ, e.g. from brokerage commissions or underwriting fees.

Dynamics of Bank Capital. Banks can only invest in one risky asset, k_t, which represents loans to entrepreneurs. We denote the loan rate spread by R_t, which defined as the loan rate net of banks' borrowing costs r. In a slight abuse of language, we will sometimes refer to R_t simply as the loan rate. Hence, the instantaneous return on assets for a given bank is given by:

$$(R_t + r)dt - \sigma dZ_t - \phi dN_t. \tag{2}$$

Asset risk contains a continuous component represented by Brownian motion Z,[10] and a jump component represented by Poisson process N with intensity ζ.[11] The latter captures the risk of systemic banking crises, which occur at the jump times of N, denoted by $(t_k)_{k\geq 1}$. A systemic crisis

[9]We further assume, as for instance in Brunnermeier and Sannikov (2014), that households can consume positive as well as negative amounts. Negative consumption can be interpreted as providing funds generated by an alternative source of income, often referred to as a "backyard-technology." Thus, at a deposit rate that satisfies (1), the market for deposits clears.

[10] Capturing e.g. changes in total factor productivity (see Brunnermeier & Sannikov, 2014).

[11]For the sake of tractability, we neglect idiosyncratic shocks to banks' loan portfolios.

destroys fraction $\phi < 1$ of bank assets, sufficiently large to wipe out all its equity.[12] The deposit insurance agency covers the shortfall on deposits and t he social costs of a bank failure are given by $\theta(\phi k_{t_k} - e_{t_k})$, where the parameter $\theta \geq 1$ reflects the fact that the social costs of a bank failure may exceed the deposit insurance agency's direct costs.

A bank's total liabilities consist of equity capital e_t and deposits d_t, which has to equal its asset value:

$$k_t = d_t + e_t. \tag{3}$$

The book value of bank capital thus evolves according to

$$de_t = re_t dt + k_t\big(R_t dt - \sigma dZ_t - \phi dN_t\big) + di_t - dc_t, \tag{4}$$

where the first term refers to the avoided financing cost due to equity,[13] and the second term to the instantaneous net earnings from lending. Newly issued issued shares are denoted by di_t, and payouts to shareholders by dc_t. Bank capital absorbs losses on the bank's assets when the second term is negative. Summing up over all banks in the economy, we obtain the dynamics of aggregate bank capital E_t

$$dE_t = rE_t dt + K_t\big(R_t dt - \sigma dZ_t - \phi dN_t\big) + dI_t - dC_t, \tag{5}$$

where K_t, dI_t and dC_t stand, respectively, for the aggregate volumes of lending, equity issuance and payments to shareholders at time t. Banks are subject to a regulatory capital requirement Λ, such that

$$l_t := \frac{k_t}{e_t} \leq \Lambda. \tag{6}$$

That is, a bank's leverage (asset-to-equity ratio) l_t may not exceed Λ. Or, equivalently, at least fraction $1/\Lambda$ of a bank's assets has to be financed by equity. The motivation for capital requirements is to limit the social costs of bank failures in case of a systemic banking crisis (see e.g. Miles et al., 2013).

[12] With an equity value of zero, the bank enters a resolution process, which will lead to a sale to new shareholders once the systemic crisis is over.

[13] Note that these costs are smaller than the required return on equity, as $\rho > r$.

Demand for Bank Loans. Entrepreneurs finance production of the consumption good by bank loans. Due to technological or informational frictions, households cannot invest directly in the productive sector, but only save in deposits or invest in bank equity.[14] We postulate an iso-elastic aggregate loan demand given by

$$L(R) = \left(\frac{\widehat{R} - R}{\widehat{R}}\right)^{\beta} \widehat{L}. \tag{7}$$

Thus, the real sector's total demand for bank loans, $L(R)$, is strictly decreasing in the loan rate spread: $L'(R) < 0$. Furthermore, $\widehat{L} := L(0)$ refers to the maximum loan demand under a spread of zero. This corresponds to the level of aggregate lending obtained in the frictionless benchmark case.

3 Competitive Equilibrium

Equilibrium Conditions. A competitive equilibrium is defined by a map from shock histories $\{Z_s, N_s, s \in [0,t]\}$, to the loan rate and bank capital such that individual banks maximize their shareholder value and the loan market clears. Each individual bank's dynamic strategy consists of a lending policy, dividend distributions, and equity issuance as a function of the bank's individual capital e_t, and aggregate capital E_t. We focus on Markov equilibria, in which the equilibrium loan rate, R_t, is a deterministic function of aggregate bank capital. Furthermore, we assume that banks' shareholders form rational expectations and, in particular, correctly anticipate that the loan rate spread is a deterministic function of banks' aggregate equity.

A given bank's shareholder value maximization problem is given by

$$v(e_t, E_t) = \max_{k_t \in [0, \Lambda e_t],\ dc_t \geq 0,\ di_t \geq 0} \mathbb{E}\left[\int_t^{\tau} e^{-\rho(s-t)}\Big(dc_s - (1+\gamma)di_s\Big)\right]. \tag{8}$$

While aggregate bank policies K_t, dC_t, and dI_t, are determined as the sums of banks' individual policies k_t, dc_t, and di_t, banks are competitive and take all aggregate variables as given. Each bank is run until the stochastic default time $\tau := \inf\{t : e_t \leq 0\}$, which denotes the first time when the book value of its equity falls to or below zero. At the equilibrium of our model, banks only default if there is a systemic crisis. When flotation costs γ are not too high (which we will assume),

[14] See e.g. Freixas and Rochet (2008), Chapter 2.

shareholders find it indeed optimal to inject fresh equity before book equity falls to zero, such that

$$e_t > 0, \tag{9}$$

for all $t \in \{\mathbb{R}^+ \setminus (t_k)_{k\geq 1}\}$.

Definition 1. *A competitive Markov equilibrium is such that*

(i) the loan rate is a deterministic function of aggregate bank capital: $R_t = R(E_t)$*;*

(ii) banks maximize their shareholder value (8)*, taking all aggregate variables as given;*

(iii) individual and aggregate bank capital follow (4) *and* (5) *with initial conditions* e_0 *and* E_0*;*

(iv) the market for bank loans clears: $K_t = L(R_t)$.

From Itô's Lemma, the change of variables formula for jump processes, and the dynamics of capital in (4) and (5), it follows that banks' shareholder values have to satisfy the following Hamilton-Jacobi-Bellman equation:

$$\begin{aligned}\rho v = \max_{k_t\in[0,\Lambda e],\ dc\geq 0,\ di\geq 0} & \big[re + Rk + di - dc\big]v_e + \big[rE + RK + dI - dC\big]v_E \\ & + [dc - (1+\gamma)di] + \big(k^2 v_{ee} + K^2 v_{EE} + 2kKv_{eE}\big)\frac{\sigma^2}{2} - \zeta v,\end{aligned} \tag{10}$$

where we use sub-indices to denote partial derivatives and omit all function arguments for brevity. The left hand side of (10) reflects the fact that shareholders' required return equals ρ. The first line on the right hand side equals the change in shareholder value induced by a change in the bank's own capital, e, and aggregate capital, E, respectively. The first term of the second line represents payments to the bank's shareholders net of capital injections. The second term captures the value impact of the variance of (and the covariance between) individual and aggregate bank capital. The last term in the second line of (10) reflects the bank's failure in case of a systemic crisis.

Individual banks' optimization problems can be greatly simplified by observing that the shareholder value function $v(e, E)$ is homogeneous of degree one in individual bank capital e. That is, when we multiply the initial condition e_0 by some factor $n > 0$, it is clearly optimal for banks to follow a strategy that consists of equivalently scaled controls $ni, nc,$ and nk. Since both the feasible set of strategies and the objective function itself are thus homogeneous, the shareholder value in (8) satisfies

$$v(ne, E) = nv(e, E). \tag{11}$$

We define the scale-adjusted version of the bank's policies, for $n = 1/e$, which can be interpreted as the bank's seasoned offerings relative to outstanding equity di/e, its dividend-to-equity ratio dc/e, and its leverage (asset-to-equity ratio) $l = k/e$, which is restricted by the capital requirement (6). Likewise, by using (11), define the bank's scaled shareholder value, or market-to-book ratio of equity, as

$$u(E) := v(1, E) = \frac{v(e, E)}{e}. \tag{12}$$

The market-to-book ratio of equity $u(E)$ is the same for all banks and a deterministic function of aggregate bank equity E. Hence, we may indeed focus on equilibria where bank loans only depend on aggregate equity, and not on its distribution among banks. We thus consider only Markov equilibria in which the loan rate is a deterministic function of aggregate bank capital (see Definition 1). This is equivalent to considering a "representative bank," that solves a scaled version of the original stochastic control problem with aggregate bank capital E as the single state variable.

Recapitalization and Dividend Policies. To characterize banks' equity issuance and payout policies, we take first order conditions in equation (10) and use the fact that by (12), the marginal value of equity is equal to the market-to-book ratio: $v_e(e, E) = u(E)$. It then follows immediately that it is optimal to make payments to shareholders ($dc > 0$) if and only if $u(E) \leq 1$. That is, as long as the value of maintaining book equity inside the bank is higher than shareholders' marginal value of receiving a payout, banks retain profits to build up equity buffers. Similarly, raising new capital ($di > 0$) is optimal if and only if $u(E) \geq 1 + \gamma$. That is, as long as the value of an additional unit of book equity is smaller than the marginal costs of raising new capital, only the internally generated equity is used to absorb losses on a bank's loans.

These considerations give rise to "barrier-type" payout and recapitalization strategies: Banks issue new equity if aggregate capital reaches a lower bound $\underline{E}$, which satisfies

$$u\left(\underline{E}\right) = 1 + \gamma. \tag{13}$$

Similarly, banks pay out earnings to shareholders if aggregate capital reaches an upper bound $\overline{E}$, which is characterized by

$$u\left(\overline{E}\right) = 1. \tag{14}$$

Between the two boundaries, banks make no payments to shareholders and do not issue new capital.[15] The payout boundary is pinned down by a standard no-arbitrage condition,

$$v(e - dc, \overline{E} - dC) + dc = v(e, \overline{E}). \tag{15}$$

That is, the ex-dividend equity value plus the dividend payment must equal the cum-dividend equity value. Applying a Taylor expansion to the left-hand side of (15), while using the homotheticity property $v(e, E) = eu(E)$, yields:

$$u(\overline{E})dc + eu'(\overline{E})dC = dc.$$

Since $u\left(\overline{E}\right) = 1$ by (14) and book equity e must be strictly positive by (12), it follows that

$$u'(\overline{E}) = 0. \tag{16}$$

A similar no-arbitrage condition has to hold at the recapitalization boundary:

$$v(e + di, \underline{E} + dI) - (1 + \gamma)di = v(e, \underline{E}),$$

which, after applying a Taylor expansion, yields

$$u(\underline{E})di + eu'(\underline{E})dI = (1 + \gamma)di. \tag{17}$$

Since $u(\underline{E}) = 1 + \gamma$ by boundary condition (13) and book equity e must remain strictly positive by (12), it follows that

$$u'\left(\underline{E}\right) = 0. \tag{18}$$

[15] It is important to stress that, in contrast to the partial equilibrium models featuring similar barrier-type recapitalization and dividend policies (see e.g. Bolton et al., 2011; Hugonnier & Morellec, 2017), in our framework these boundaries are in terms of the aggregate, rather than the individual state.

Since banks are homothetic, they all follow the same strategy, such that aggregate payouts ($dC > 0$) cause aggregate capital to be reflected at $\overline{E}$. Likewise, banks' joint issuance strategies prevent aggregate capital from falling below $\underline{E}$. In between the two boundaries, there are no capital injections or payouts, i.e.,

$$dC(E) = dc(E) = dI(E) = di(E) = 0 \text{ for } E \in (\underline{E}, \overline{E}) \tag{19}$$

Intuitively, when the banking system is well capitalized, the aggregate loan supply is high, which makes lending relatively unprofitable. Hence, banks pay out profits to shareholders at $E \geq \overline{E}$ (*payout region*). Banks issue new equity at $E \leq \underline{E}$ (*external financing region*), when the banking system is poorly capitalized and aggregate loan supply is so low that lending is highly profitable. Between the boundaries, banks retain profits and convert them into book equity which is used to absorb losses and finance loans to the real sector (*internal financing region*).

Equilibrium Loan Rate Spread and Market-to-Book Value. Using homotheticity property (12) and the fact that banks follow a barrier strategy, we rewrite equation (10) for the *internal financing region* $(\underline{E}, \overline{E})$ as:

$$\begin{aligned}(\zeta + \chi)u(E) = \Big[rE + R(E)K(E)\Big]u'(E) + \frac{\sigma^2 K(E)^2}{2}u''(E) \\ + \max_{l \in [0,\Lambda]} l\Big[R(E)u(E) + \sigma^2 K(E)u'(E)\Big].\end{aligned} \tag{20}$$

In equilibrium, the market-to-book ratio of capital has to grow at rate $\zeta + \chi$ reflecting the risk of a systemic crisis and the reduction in financing costs due to depositors' convenience yield. Since (20) is a second order ODE, it requires two boundary conditions, (13) and (14), to pin down a solution. The two free boundaries, $\underline{E}$ and $\overline{E}$, are determined by the no-arbitrage conditions (16) and (18).

Now consider equilibrium leverage l (or, equivalently, bank lending $k = l \cdot e$). Optimal leverage is equal to Λ (i.e. the capital requirement (6) is binding), if the second line of (20) is positive. If the constraint binds for an individual bank, it binds also on the aggregate level due to homotheticity. The aggregate supply of loans is then given by $K(E) = \Lambda E$. For the loan market to clear, this has to equal entrepreneurs' aggregate demand for bank loans $L(R)$. Inverting this market clearing condition yields the equilibrium loan rate $R(E)$ for the region where the capital requirement is binding:

$$R(E) = L^{-1}(\Lambda E). \tag{21}$$

The term in square brackets in the second line of (20) can be interpreted as the "shadow costs" associated with the capital requirement. Notably, as the term in square brackets is positive, the binding constraint is associated with a *higher* market-to-book ratio. This seemingly counterintuitive result reflects the fact that a binding capital requirement restricts aggregate loan supply, such that the equilibrium loan rate increases.

For the equilibrium level of lending to be interior (i.e. a non-binding capital requirement), the shadow costs attributed to the capital requirement must be zero, such that banks are indifferent with respect to leverage $l \leq \Lambda$. Thus, setting to zero the term in square brackets in the second line of (20) yields

$$R(E) = -\frac{u'(E)}{u(E)}\sigma^2 K(E), \tag{22}$$

for the region where the capital requirement is slack.[16] Since the market-to-book ratio of bank capital is decreasing in aggregate bank capital (as we show in the Proof of Proposition 1), condition (22) implies that banks require a positive spread over their own borrowing costs r, i.e., $R(E) \geq 0$. This reflects the following mechanism: when an individual bank makes profits, all other banks also make profits. As the banking sector thus becomes better capitalized, aggregate loan supply increases, which lowers the equilibrium loan rate. With lower profits from lending, the market-to-book ratio $u(E)$ decreases. Likewise, when a bank makes losses, other banks make losses as well. As the capitalization of the banking system deteriorates, this reduces aggregate loan supply and the market-to-book ratio of equity increases. This effect is similar to a the decreasing marginal utility of consumption for a risk-averse individual.

Proposition 1. *There exists a unique competitive Markov equilibrium, in which the loan rate $R(E)$ is given by* (21) *in the region $E \in [\underline{E}, E^*)$, and by* (22) *in the region $E \in [E^*, \overline{E}]$. The critical capitalization E^*, at which capital requirement* (6) *becomes slack, is uniquely defined by continuity of $R(E)$. The equilibrium market-to-book ratio $u(E)$ satisfies HJB* (20) *subject to the boundary*

[16] Note that the equilibrium spread in (22) can also be interpreted as a "hedging premium," rather than a standard risk-premium: It contains the *cross-derivative* of the bank's shareholder value with respect to individual and aggregate equity, i.e., $v_{eE}(e, E)/v_e(e, E) = u'(E)/u(E)$, and not its second derivative, as in a measure for risk aversion. A similar mechanism drives the intertemporal hedging demand in the dynamic model of liquidity provision by Kondor and Vayanos (2019).

conditions (13)*,* (14)*,* (16)*, and* (18)*. Aggregate bank capital evolves according to*

$$dE_t = rE_t dt + L\big(R(E_t)\big)\big(R(E_t)dt - \sigma dZ_t - \phi dN_t\big), \tag{23}$$

for $E_t \in \big(\underline{E}, \overline{E}\big)$ *and it is reflected at* $\underline{E}$ *by* $dI_t > 0$ *and at* $\overline{E}$ *by* $dC_t > 0$;

Note that all equilibrium objects are indeed deterministic functions of aggregate bank capital E, the single state variable. Likewise, the market value of a unit of individual book equity (the market-to-book ratio) $u(E)$, is a deterministic function of aggregate capital as well. The equilibrium evolution of aggregate bank capital follows from banks' individual issuance and payout policies in (19), which in turn are individually optimal under the boundary conditions (13), (14), (16), and (18). That is, absence of arbitrage on the equity market implies that individual banks cannot increase their shareholder value by deviating from the barrier strategy to raise new equity ($di > 0$) at $\underline{E}$ and to distribute dividends ($dc > 0$) at $\overline{E}$.

4 Equilibrium Analysis

In Section 4.1 we first discuss the properties of the market equilibrium for a given level of capital regulation. Next, we show how the equilibrium outcome depends on the level of regulation. In Section 4.2, we confront the equilibrium relations with the data.

4.1 Market Outcome

Consider first, the equilibrium loan rate $R(E)$. When aggregate capital is low (i.e., for $E \leq E^*$), it is fully determined by the binding capital requirement ($R(E) = L^{-1}(\Lambda E)$) and, hence, strictly decreasing in aggregate equity. To show that the equilibrium loan rate decreases in aggregate equity also in the region where the constraint is slack, we derive the following auxiliary result.

Corollary 1. *In the equilibrium characterized in Proposition 1, the loan rate satisfies the following first-order differential equation:*

$$R'(E) = -\left(\frac{1}{\sigma^2}\right)\frac{2(\zeta+\chi)\sigma^2 + R(E)^2 + 2r\frac{E}{L(R(E))}R(E)}{L(R(E)) - L'(R(E))R(E)} < 0, \tag{24}$$

for $E \in [E^*, \overline{E}]$, subject to the boundary condition:

$$R(\overline{E}) = 0. \tag{25}$$

Proof. Expression (24), follows from substituting (22) in HJB (20) and eliminating the market-to-book ratio $u(E)$ and its derivatives. Equation (25) follows from (22) and boundary conditions (13) and (16). Since $R(E)$ is non-negative and aggregate loan demand is strictly decreasing, i.e., $L'(R) < 0$ for $R \in [0, \widehat{R}]$, the denominator of (24) is positive as well and thus, $R'(E) < 0$. □

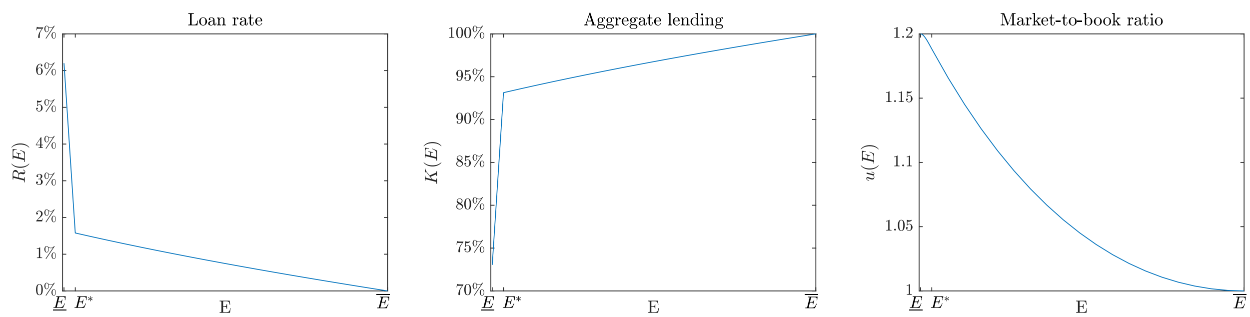

Figure 1: Illustrates the equilibrium loan rate, aggregate lending, and the market-to-book ratio of equity as a function of aggregate capital E, for a maximum leverage of $\Lambda = 25$ (which corresponds to a minimum regulatory capital ratio of 4%). Aggregate lending is expressed relative to the level of aggregate lending in the friction-less benchmark $\widehat{L}$. Parameter values are: $\rho = 0.05$, $\chi = 0.01$, $\zeta = 0.02$, $\beta = 1$, $\widehat{L} = 1$, $\widehat{R} = 0.23$, $\sigma = 0.1$, $\gamma = 0.2$.

Boundary condition (25) has an intuitive interpretation: At the payout barrier $\overline{E}$, aggregate loan supply is so high, that loan making becomes unprofitable at the margin. As is illustrated in the left panel of Figure 1, the loan rate is indeed highest in the external financing region $E \leq \underline{E}$ and achieves its minimum in the payout region $E \geq \overline{E}$. As is illustrated in the middle panel of Figure 1, aggregate lending is thus strictly increasing in E. Finally, as illustrated in the right panel of Figure 1, the market-to-book ratio of bank equity obtains its maximum of $1 + \gamma$ in the issuance region, $E \leq \underline{E}$, where lending is so profitable that banks raise new capital to avoid further unlevering in the face of a binding capital requirement. The market-to-book ratio is strictly decreasing in E and obtains its minimum in the payout region $E \geq \overline{E}$, where loan making becomes unprofitable at the margin. These testable implications are summarized in the following Corollary:

Corollary 2. *In the competitive Markov equilibrium characterized in Proposition 1, it holds that*

(i) the loan rate (spread), $R(E)$, is strictly decreasing in aggregate bank capital;

(ii) aggregate loan volume, $K(E)$, is strictly increasing in aggregate bank capital;

(iii) the market-to-book ratio of equity, $u(E)$, is strictly decreasing in aggregate bank capital.

Outcome under Different Regulatory Regimes. Now we compare the respective market outcomes under two different regulatory regimes. This exercise can be interpreted as an unanticipated regulatory tightening. We thus refer the milder capital requirements as "old regulatory regime" (Λ_{old}) and to the stricter requirements as "new regulatory regime" (Λ_{new}). For the issuance boundary $\underline{E}$, the effect is straightforward: Since the capital requirement binds at $E = \underline{E}_{old}$, a reduction of the maximum leverage reduces the total supply of loans and, thus, increases the equilibrium loan rate. This makes lending more profitable, which would, all else equal, increase the market-to-book ratio to a value $u_{new}(\underline{E}_{old}) > 1 + \gamma$. By continuity, the new payout boundary $\underline{E}_{new}$ thus has to be higher than $\underline{E}_{old}$, which is illustrated in Figure 2. The same reasoning applies to the critical level of aggregate capital where the capital requirement becomes slack, i.e., $E^*_{new} > E^*_{old}$. This implies that at any given level of capital, the tighter regulation Λ_{new} is more likely to *become* binding than the milder regulation Λ_{old}. This increases the market-to-book-ratio of equity also for levels of aggregate bank capital where the constraint is slack. Hence, we would have that $u_{new}(\overline{E}_{old}) > 1$, implying that the payout boundary has to increase (see Figure 2): $\overline{E}_{new} > \overline{E}_{old}$.[17] Figure 2 also shows that an unannounced regulatory tightening (i.e. one without a transition phase) may lead to a reduction in aggregate lending by up to 24.16%. Note that the most severe losses in lending occur at levels of capital where the mild regulation Λ_{old} is slack, but the tighter regulation Λ_{new} becomes binding if it is introduced (i.e. for $E \in [E^*_{old}, \underline{E}_{new}]$). At the same time, an unexpected regulatory tightening triggers an immediate capital injection of $(\underline{E}_{new} - E)$ if $E < \underline{E}_{new}$, which reduces the social costs in case of a banking crisis.

This discussion suggests that a transition phase might indeed improve the trade-off between financial stability and loses in lending. In Section 5, we therefore analyze how a (longer) transition phase affects the dynamics of capital accumulation and aggregate lending.

[17] Note that ceteris paribus, a tighter regulation would lead to more frequent recapitalization, which tends to decrease the market-to-book ratio at a given level of aggregate capital and, thus, decrease the payout and issuance boundaries. Our numerical analysis (available from the authors upon request), however, suggests that this countervailing effect may dominate only for excessively high levels of regulation under which the constraint is binding for all $E \in [\underline{E}, \overline{E}]$. In the region where the constraint is slack, however, this effect is off-set by the fact that a higher probability of recapitalization increases banks' implied risk-aversion and, hence, the required lending premium. A higher loan rate in turn lowers banks' exposure and, thus, reduces the frequency of recapitalization.

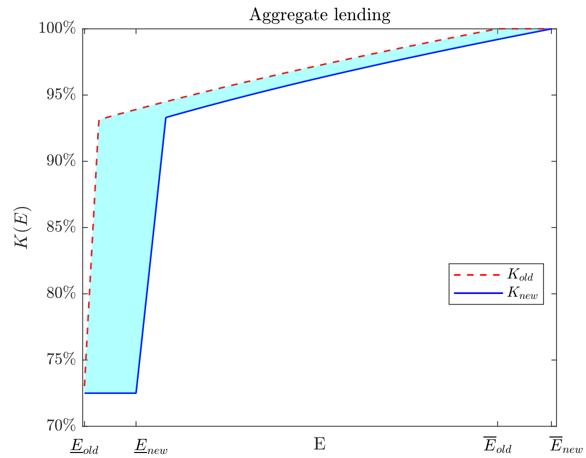

Figure 2: Illustrates aggregate lending relative to the level of aggregate lending in the friction-less benchmark $\widehat{L}$ for two regulatory regimes $\Lambda_{old} = 25$ and $\Lambda_{new} = 12.5$, which corresponds to minimum regulatory capital ratios of 4% and 8%, respectively. Parameter values are: $\rho = 0.05$, $\zeta = 0.02$, $\chi = 0.01$, $\beta = 1$, $\widehat{L} = 1$, $\widehat{R} = 0.23$, $\sigma = 0.1$, $\gamma = 0.2$

4.2 From the Model to the Data

Before we turn to the analysis of the transition phase, we empirically assess the implications for the dynamics of bank lending and bank capitalization in Corollary 2 with data from two large international panel datasets at an annual frequency covering publicly quoted banks (Dataset 1) and aggregate country data (Dataset 2).

Dataset 1 is taken from the Worldscope database, retrieved from Datastream, which contains consolidated accounts and market data for a large number of publicly quoted banks worldwide. Dataset 1 covers data for 1,316 banks in 39 countries during the period 1990-2017, including 629 U.S. Bank Holding Companies (BHCs), 304 European banks, 192 Asia (developed) banks, and 191 banks operating in countries classified as emerging. This panel dataset is unbalanced due to mergers and acquisitions, but all banks active in each period are included in the sample to avoid survivorship bias. The variables in Dataset 1 include the log of bank (common) equity (*lequity*), the log of *aggregate* bank equity (*lE*) by country, the log of bank loans (*lloans*), the interest spread on bank loans (*spread*), the bank market-to-book ratio (*mtb*), and the bank (common) equity-to-asset ratio (*ea*). The variables in levels are all expressed in US$. The interest rate spread on bank loans is computed as the difference between the loan rate and the cost of funding, where the cost of funding is the weighted average of the cost of deposits and market sources of funding. We use the bank (common) equity-to-asset ratio rather than a regulatory capital ratio, as bank coverage of the latter is very limited in this database.[18]

Dataset 2 is taken from the World Bank Financial Structure Database, which assembles financial and bank data from a wide array of international databases. Dataset 2 covers 120 countries during the period 1998-2017, including 47 high income countries and 73 middle-to-low income countries, as per the income classification of the World Bank. The variables in Dataset 2 include country aggregates of the log of bank regulatory capital (RC), the log of bank loans (L), the spread between lending and deposit rates ($SPREAD$), and the bank regulatory capital ratio (RCR), measured by the ratio of regulatory capital to risk weighted assets. As in Dataset 1, the variables in levels are all expressed in US$.[19]

We use these two large datasets to maximize the robustness of the empirical assessment of our model. Dataset 1 allows us to explore the implications of our model for market valuation. However, the banks included in this dataset do not represent the entire banking system in a country, although they capture a significant proportion of total assets of each country's banking system. Dataset 2 complements Dataset 1 by including data for entire banking systems, with a country coverage significantly larger than that of Dataset 1. Importantly, Dataset 2 includes medium-to-low income countries where banks are the predominant vehicles in the provision of credit, as in our model. Figure 3 shows correlations between the key variables in our model.

Our model predicts that aggregate bank equity is positively correlated with aggregate bank lending, and negatively correlated with loan spreads and market-to-book ratios (Corollary 2). Figure 3 illustrates scatter plots and correlations of the time series of the (log) difference of aggregate bank (common) equity with the (log) difference of total bank loans (as both series trend upward), and the correlations of aggregate bank equity with interest spreads and market-to-book ratios. The signs of these correlations, which are all statistically significant, match the predictions of our model.

[18] The country aggregate of the log of bank loans is denoted by lL. The country averages of bank spreads and market-to-book ratios are denoted by $SPREAD$ and MTB respectively. The data points in the database used to construct our variables are: total assets (WC02999), total loans (WC02771), total liabilities (WC03999-WC03501), common equity (WC03501), total deposits (WC03019), loan rate (WC01007/WC02271), total interest expenses (WC01075), and market capitalization (WC08001).

[19] The data points in the database used to construct our variables are: Bank regulatory capital to risk-weighted assets (GFDD.SI.05), the bank lending-deposit spread (GFDD.EI.02), Bank regulatory capital to total assets (GFDD.SI.03), Deposit money banks' assets to GDP (GFDD.DI.02), and GDP in current US$ (NY.GDP.MKTP.CD)

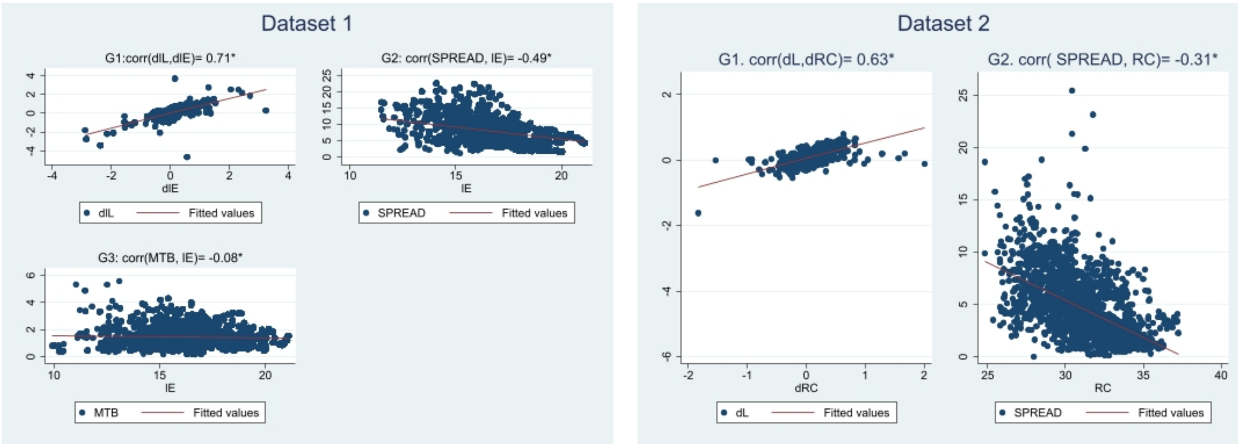

Figure 3: Correlations of the Δ(log) of aggregate bank (common) equity with Δ(log) of total bank loans (Dataset 1, Graph G1, and Dataset 2, Graph G1), and correlation of the (log) of aggregate bank (common) equity with the interest spread on bank loans (Dataset 1, Graph G2, and Dataset 2, Graph G2), and with the market-to-book ratio (Dataset 1, Graph G3). The * denotes significance at a 5% level.

5 Implementation of a Regulatory Tightening

We have seen in the last section that a sudden regulatory tightening may trigger immediate capital injections, but has the potential to cause a severe credit crunch. In line with common regulatory practice, we therefore now consider the introduction of the regulatory tightening over a transition phase.

5.1 Equilibrium in the Transition Phase

In $t = 0$, the regulator announces a new capital requirement Λ_{new}, to which all banks have to adhere after a transition period of T years. For the sake of tractability, we assume that during the transition phase, the old capital requirement Λ_{old} applies, and that the implementation of Λ_{new} is governed by a Poisson process N^{Λ} with intensity $\frac{1}{T}$. That is, banks anticipate a regulatory tightening after a transition phase of T years, but their optimization problems during that transition phase remains stationary. Banks' market-to-book ratio $u_{tr}(E)$ during the transition period satisfies the following Hamilton-Jacobi-Bellman equation

$$\begin{aligned}(\zeta+\chi)u_{tr}(E) = {} & \Big[rE + R_{tr}(E)K_{tr}(E)\Big]u'_{tr}(E) + \frac{\sigma^2 K_{tr}(E)^2}{2}u''_{tr}(E) \\ & + \max_{l\in[0,\Lambda_{old}]} l\Big[R_{tr}(E)u_{tr}(E) + \sigma^2 K_{tr}(E)u'_{tr}(E)\Big], \\ & + \frac{1}{T}\Big[u_{new}(E) - u_{tr}(E)\Big]\end{aligned} \tag{26}$$

subject to the usual boundary conditions

$$u_{tr}\left(\underline{E}_{tr}\right) - (1+\gamma) = u_{tr}\left(\overline{E}_{tr}\right) - 1 = u'_{tr}\left(\underline{E}_{tr}\right) = u'_{tr}\left(\overline{E}_{tr}\right) = 0.$$

Compared to (20), equation (26) contains an additional jump-term in the third line which reflects the anticipation of the tighter regulation being implemented with Poisson intensity $1/T$. The market-to-book ratio then jumps to $u_{new}(E)$, which refers to the solution to (20) under $\Lambda = \Lambda_{new}$.

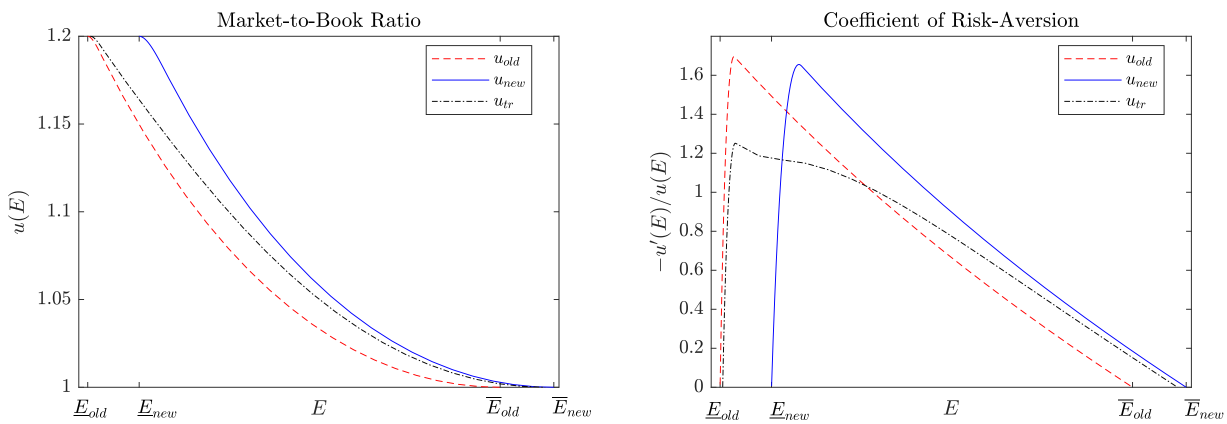

Figure 4: Illustrates the market-to-book ratio function and banks' implied coefficient of risk-aversion before, during and after the transition period for a regulatory change from $\Lambda_{old} = 25$ to $\Lambda_{new} = 12.5$. The length of the transition phase equals one year. Parameter values are: $\rho = 0.05$, $\chi = 0.01$, $\zeta = 0.02$, $\beta = 1$, $\widehat{L} = 1$, $\widehat{R} = 0.23$, $\sigma = 0.1$, $\gamma = 0.2$.

The left panel of Figure 4 shows that for high levels of aggregate capital, the market-to-book ratio u_{tr} (the black dash-dotted line) approaches u_{new} (the blue line). Hence, the anticipated increase in the profitability of lending is fully "priced in." For low levels of aggregate capital, by contrast, the market-to-book ratio is almost unaffected by the regulatory announcement, as in this region, the market outcome is determined by the binding capital requirement. Hence, as the market-to-book ratio $u_{tr}(E)$ is kept artificially low in this region, it falls more slowly in aggregate capital, in the region where the constraint becomes slack. Intuitively, as aggregate capital increases, the system moves closer to the states of the world, in which capital is already more valuable due to the anticipated regulatory tightening. The right panel of Figure 4 shows that this decrease in $|u'_{tr}(E)|$ implies that banks become less "risk-averse" in response to the regulatory announcement. That is, their implied coefficient of risk-aversion $-u'(E)/u(E)$ (the black dash-dotted line) decreases, which in turn leads to a smaller equilibrium loan rate $R_{tr}(E)$, as specified in (22). Quite surprisingly, the anticipation of a future *scarcity* of loans therefore *reduces* the current loan rate.

The effect of a regulatory announcement on the aggregate lending function $K(E)$ is illustrated in Figure 5. For low values of aggregate capital, aggregate lending K_{tr} is completely determined by the binding regulatory constraint and thus equal to K_{old}. Under the relatively short transition phase of one year, considered in the left panel, the anticipated regulatory change is fully priced-in for high levels of capital. Hence, K_{tr} approaches K_{new} in this region, such that the anticipated decrease in loan supply leads to lower levels of lending today. For intermediate levels of aggregate capital, however, the anticipation of the regulatory tightening has the opposite effect. As a reduction in banks' implied coefficient of risk-aversion (see Figure 4) decreases the equilibrium loan rate, lending increases above K_{old} in that region. This is in line with recent empirical evidence that aggregate bank lending increased in response to the announcement of Basel III (see BIS, 2022, Chapter 8.1.3 and Annex A.18). The right panel shows that when the transition phase is extended to 5 years, the anticipation effect is less pronounced (formally, the jump-term in HJB (26) enters with a lower intensity). As a result, both the increase of K_{tr} above K_{old} for intermediate levels of capital is smaller, but also the decrease towards K_{new} for high levels of capital is smaller. Hence, the total effect on aggregate lending depends on the dynamics of aggregate capital, to which we turn next.

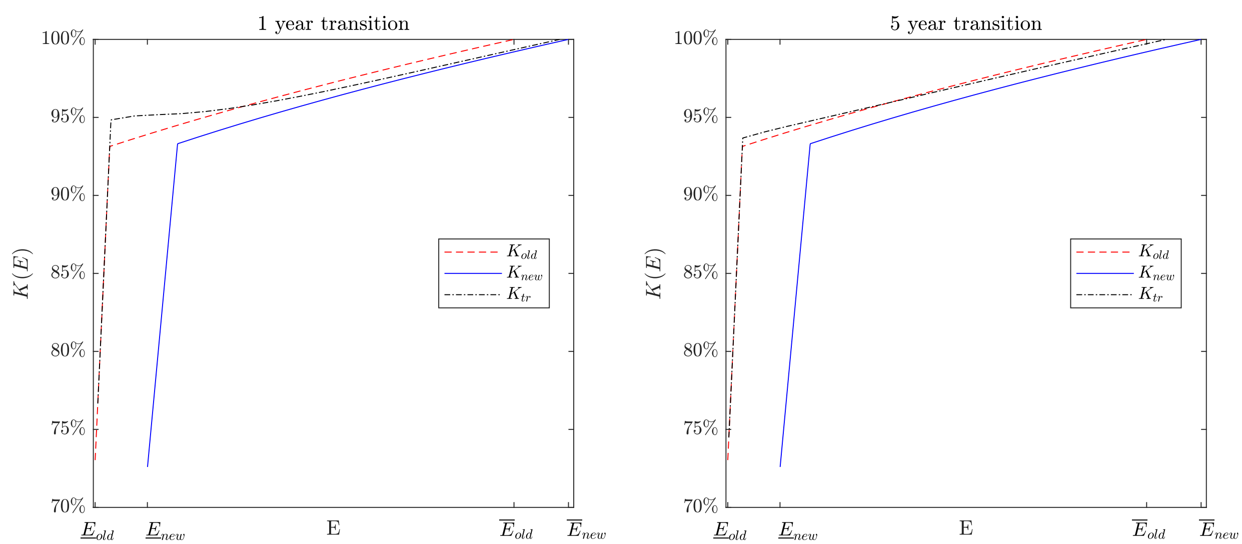

Figure 5: Illustrates the aggregate lending function before, during and after the transition phase for a regulatory change from $\Lambda_{old} = 25$ to $\Lambda_{new} = 12.5$. The length of the transition phase equals 1 year in the left panel and 5 years in the right panel. Aggregate lending is expressed relative to aggregate lending in first-best, $\widehat{L}$. Parameter values are: $\rho = 0.05$, $\chi = 0.01$, $\zeta = 0.02$, $\beta = 1$, $\widehat{L} = 1$, $\widehat{R} = 0.23$, $\sigma = 0.1$, $\gamma = 0.2$.

Figure 6 illustrates the accumulation of bank capital during the transition phase. The paths are simulated under the assumption that the regulatory change is announced in $t = 0$, and phased-in over a transition phase of one year (blue line), and five years (red dashed line), respectively. The black dash dotted line represents the status-quo without a regulatory change. Recall that the anticipation of tighter regulation increases the value of bank capital already in the transition period, but only so for high levels of capital. Hence, the payout boundary $\overline{E}$ increase upon the regulatory announcement and a substantial part of the total increase in capital materializes already during the transition phase. In the example with a 5 year transition phase (red dashed line), this happened when after 3.5 years aggregate capital is reflected at $\overline{E}_{tr}$ for the first time. In the example with a shorter transition phase of one year (blue line), however, the payout boundary increases even more in response to the regulatory announcement. However, this has no effect on capital accumulation as, in this particular path, $\overline{E}_{tr}$ is not hit during the short transition phase. Instead, as aggregate capital is still relatively low at the end of the one year transition phase, the implementation of the regulatory tightening triggers a capital injection (and credit crunch) right after the transition phase in this example. These examples illustrate that with a longer transition phase, additional capital is more likely to come from retained earnings, while with a shorter transition phase, it is more likely to come from additional capital injections. The latter allows for a quicker accumulation of capital, but at the costs of losses in lending when the tighter regulation becomes binding.

5.2 Welfare Analysis

We now ask whether there is a welfare maximizing length for the transition phase. Note that the only welfare relevant figures are the social costs of financial crises, $\theta\big(\phi K_{t_k} - E_{t_k}\big)$ for $k \geq 1$, the convenience yield enjoyed by depositors, χD^+, and the output produced by the real sector which, from aggregate loan demand (7), is given by:[20]

$$Y(R) = L(R)\left(\frac{\widehat{R} - R}{1+\beta} + r + R\right). \tag{27}$$

Increasing the length of the transition phase has a non-trivial effect on aggregate capital and aggregate lending during the transition phase (see Section 5.1).[21] To assess the welfare implications

[20]See Appendix Appendix B for a derivation of aggregate output Y.

[21]While a full calibration of the equilibrium is beyond the scope of this paper, our numerical analysis offers some qualitative properties and comparative statics of the optimal length of the transition phase.

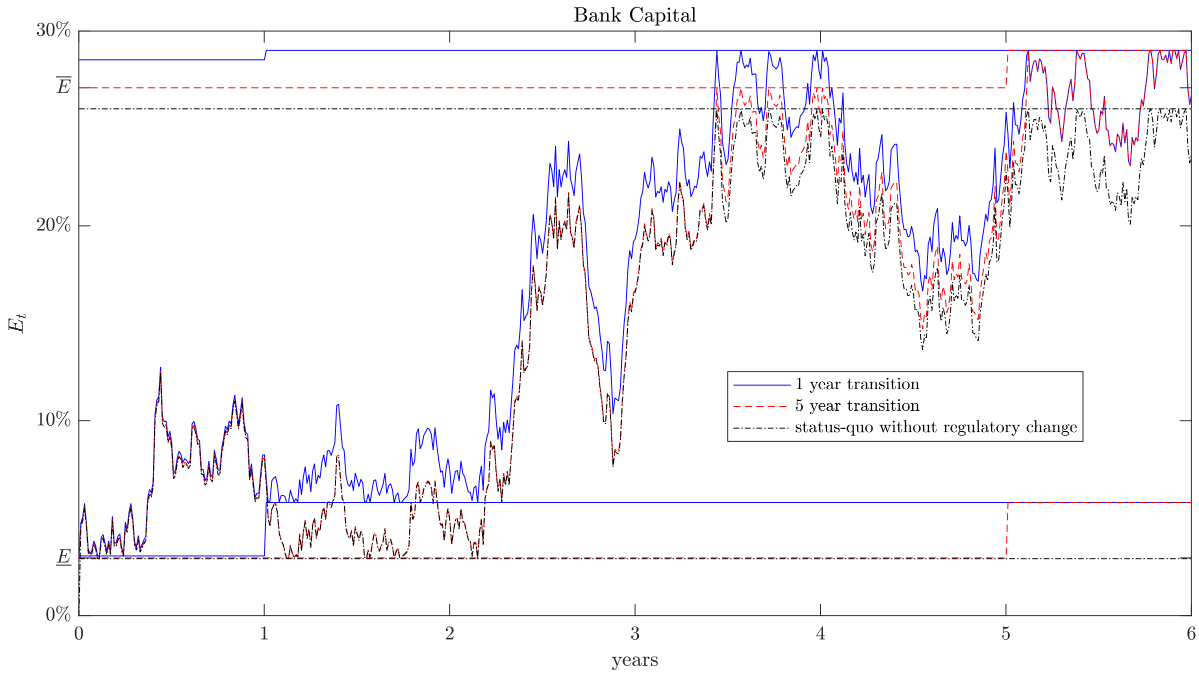

Figure 6: Illustrates sample paths for aggregate capital, E_t, as well as the evolution of the payout and issuance boundary, $\overline{E}$ and $\underline{E}$. We consider the introduction of the regulatory change from $\Lambda_{old} = 25$ to $\Lambda_{new} = 12.5$ over a one year and a five year transition phase. Aggregate capital is expressed relative to the level of aggregate lending in the friction-less benchmark $\widehat{L}$. Parameter values are: $\rho = 0.05$, $\chi = 0.01$, $\zeta = 0.02$, $\beta = 1$, $\widehat{L} = 1$, $\widehat{R} = 0.23$, $\sigma = 0.1$, $\gamma = 0.2$.

of a longer transition phase, we consider the present value for each of the three welfare components, e.g. for the case of output Y:

$$PV(Y) = \mathbf{E}\left[\int_0^\infty \mathrm{e}^{-\rho t} Y_t dt \middle| E_0\right]. \tag{28}$$

We compute the expectation using Monte Carlo simulations with 5.000 paths over a period of 100 years. Total welfare is then computed by adding up the three present values:

$$W := PV\big(Y\big) + PV\big(\chi D^+\big) - PV\big(\theta(\phi K_{t_k} - E_{t_k})\big). \tag{29}$$

Figure 7 plots welfare and its components as functions of the length of the transition phase T. The upper left panel illustrates the costs of tighter regulation: Despite the initial increase due to the anticipation effect, the present value of output (blue line) decreases compared to the status-quo without a regulatory change (red dashed line). However, the loss in output can be reduced

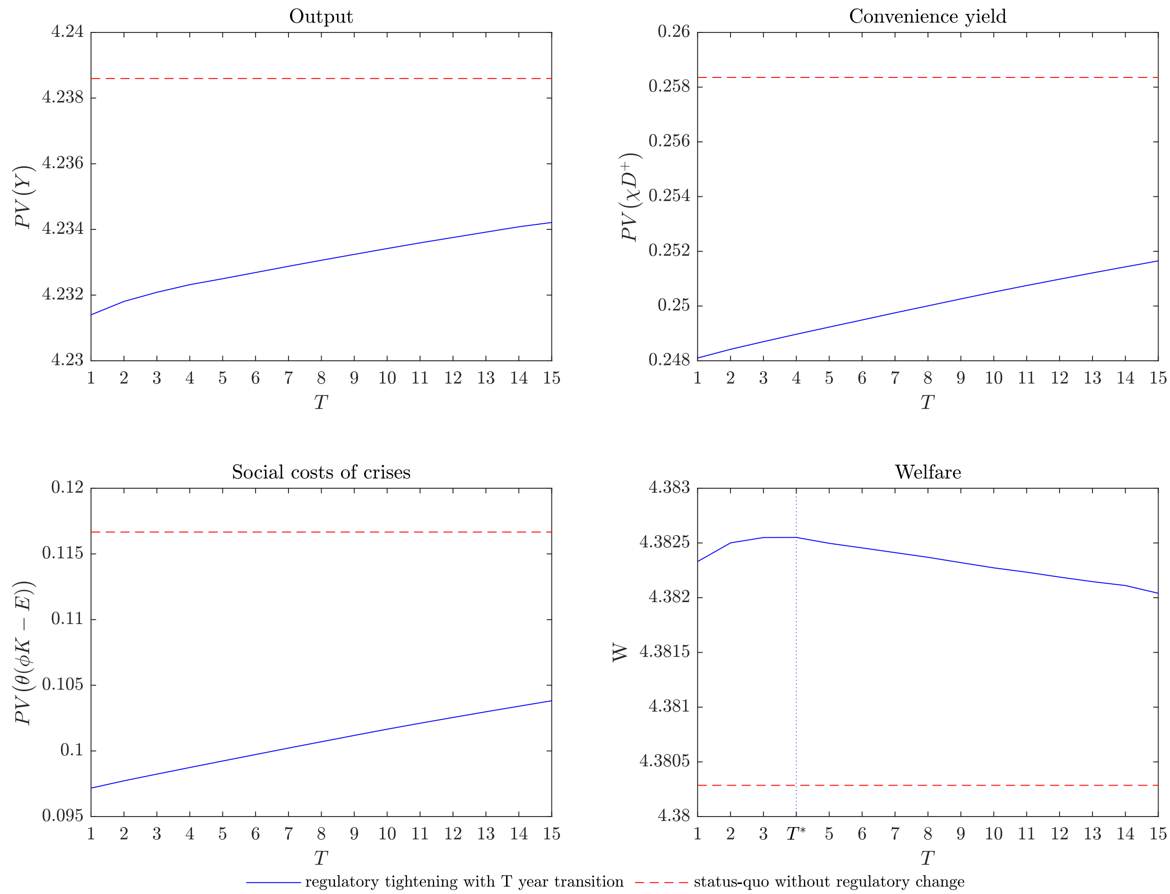

Figure 7: The blue lines illustrate the present values of aggregate output (upper left panel), depositors' convenience yield (upper right panel), the social costs of financial crises (lower left panel), and total welfare (lower right panel). All are expressed as functions of the length of the transition phase T, over which a regulatory change from $\Lambda_{old} = 25$ to $\Lambda_{new} = 12.5$ is introduced and the level of bank capital in $t = 0$ is assumed to be $E_0 = \underline{E}_{old}$. The red dashed lines refer to the status-quo without a regulatory change. Parameter values are: $\rho = 0.05$, $\chi = 0.01$, $\zeta = 0.02$, $\beta = 1$, $\widehat{L} = 1$, $\widehat{R} = 0.23$, $\sigma = 0.1$, $\gamma = 0.2$, $\phi = 1/3$, $\theta = 1.05$.

by implementing the regulatory change more slowly. Similarly, the convenience yield enjoyed by depositors in the upper right panel is smaller than in the status-quo without a regulatory change, but again, the reduction is less severe if the regulatory change is introduced more slowly. This implies that also the present value of aggregate bank lending and, thus, losses incurred in a systemic crisis increase in T. Together with the slower capital accumulation illustrated in Figure 6, this implies that the present value of social costs from financial crises also increase in the length of the transition phase (see lower left panel of Figure 7). A regulatory tightening thus, reduces the expected social costs of financial crises most significantly if it is implemented in $T = 1$ already. Intuitively, poorly capitalized banks will (have to) issue new equity when the regulatory tightening is implemented after a shorter transition phase. This leads to the highest improvement in financial stability, but comes at the highest costs in terms of lower aggregate lending and output. A longer transition phase, by contrast, allows banks to accumulate capital in the form of retained earnings and thereby avoid drastic deleveraging when it is implemented. As shown in the lower right panel of Figure 7, this trade-off gives rise to a welfare maximizing transition phase of $T^* = 4$ in the considered example.

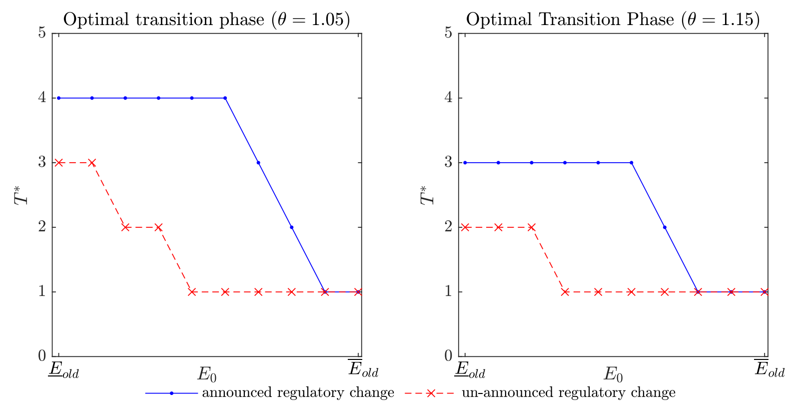

Figure 8: Illustrates the welfare maximizing transition phase as a function of E_0. Parameter values are: $\rho = 0.05$, $\chi = 0.01$, $\zeta = 0.02$, $\beta = 1$, $\widehat{L} = 1$, $\widehat{R} = 0.23$, $\sigma = 0.1$, $\gamma = 0.2$, $\phi = 1/3$, $\theta = 1.05$.

To understand the role of the anticipation effect for the optimal transition phase, we now consider a benchmark case in which regulators decide in $t = 0$ that the regulatory change will be implemented after T years, but do not communicate this decision to the market. Hence, no anticipation effect is present in this benchmark. As illustrated in Figure Figure 8, the optimal transition phase in that benchmark (red dashed line) is generally smaller than if the anticipation effect is properly taken into account (blue line). The intuition for this result is that the announcement of a regulatory tightening induces banks to retain a larger part of their earnings during the transition phase as $\overline{E}_{tr} > \overline{E}_{old}$. This allows regulators to increase the transition phase and thereby stabilize lending, while achieving the same rate of capital accumulation as in the benchmark with un-announced regulatory tightening.[22]

Furthermore, we vary E_0 in Figure 8 as, so far, we have assumed that the regulatory change is announced right after banks have suffered significant losses and received a capital injection ($E_0 = \underline{E}_{old}$). An increase in E_0 has two countervailing effects on the optimal transition phase. First, the tighter regulation is less likely to be binding, even if it is implemented immediately. This effect, which tends to decrease the optimal transition phase, is isolated in the benchmark case without

[22]Intuitively, this trade-off can potentially be improved by introducing the regulatory change in several sub-steps.

regulatory announcement (red dashed line). Second, with a higher E_0, it is more likely that the change in banks' payout policies becomes relevant during the transition period, i.e. that aggregate capital is reflected at $\overline{E}_{tr} > \overline{E}_{old}$. Hence, the optimal transition phase with announcement (blue line) increases compared to the benchmark without announcement (red dashed line). For very high levels of E_0, however, the probability of hitting the (higher) payout boundary before T approaches one. As only the first effect remains, the optimal transition phase decreases towards its minimum, which coincides with that in the benchmark without announcement. Hence, the anticipation effect is most important for intermediate levels of E_0. If it is not properly taken into account, the optimal transition phase can be underestimated by a factor of four in this case. In the right panel of Figure 8, we consider the case where bank failures are more costly (as captured by the parameter θ). Intuitively, as the regulator becomes more concerned about bank failures, the optimal transition phase becomes shorter for all considered cases.

6 Conclusion

Regulators are often criticized for being slow in implementing post-crisis regulatory reforms. Basel III was published in 2010 and while most jurisdictions completed its implementation in 2019, others are still working on completion (see Financial Stability Board, 2021, Annex 2). This paper is the first to offer a rigorous analysis of the timing of regulatory reforms and it shows the virtue of being slow.

To this end, we develop a stylized dynamic general equilibrium model in which banks finance risky loans to the real sector by equity and liquid deposits. Banks incur flotation costs when issuing equity and, thus, build up capital buffers from retained earnings. Furthermore, despite being perfectly competitive, banks require a strictly positive lending premium because of the following equilibrium mechanism: If banks retain earnings, aggregate capital increases, which leads to a larger aggregate loan supply and, thus, a lower equilibrium loan rate. With lower profits from lending, the market-to-book ratio of of any bank's equity decreases. Hence, banks become effectively risk-averse, as their shareholder value resembles a concave utility function. This yields three key testable implications of our model: aggregate lending increases in aggregate capital, while the equilibrium loan rate and the market-to-book ratio of equity decrease in aggregate capital. Using two large international databases, we find that these implications are broadly consistent with the data in the cross-section.

After the implementation of a regulatory reform, the tighter capital requirements restrict aggregate loan supply more severely. This increases the profitability of banks' loan making business and raises the market-to-book ratio of equity. The announcement of such a regulatory tightening changes banks' financing policies: Anticipating a higher market-to-book ratio, banks retain more of their earnings during the transition phase, which has the benefit that capital is accumulated more quickly, such that the tighter regulation is less likely to be binding when it is implemented. A longer transition phase, hence, lowers the risk of a credit crunch. Furthermore, we find that the additional bank capital is more likely to be generated internally from retained earnings instead of externally by issuing new equity. The adjustment of bank behavior in anticipation of the tighter regulation is crucial to assess the timing of a regulatory reform. In our numerical examples, the optimal transition phase is underestimated by a factor of up to four if regulators do not take into account the change in bank policies due to the anticipation effect.

However, the anticipation effect does not only affect banks' financing policies, but also their implied risk-aversion and, hence, aggregate lending in the transition phase. Quite surprisingly, we find that the announcement of a regulatory tightening can lead to an increase in aggregate lending. The reason for this finding is the following: For low levels of aggregate capital, aggregate lending is determined by the binding capital requirement and therefore unaffected by the regulatory announcement. For high levels of aggregate capital, the future regulatory tightening is "priced-in" in banks' optimization problems. Hence, when aggregate capital grows into the region where the regulatory constraint is slack, banks become effectively less risk-averse because with any further increase in capital, the system moves closer to the region where the anticipation effect has increased the profitability of lending. This is in line with recent empirical evidence that aggregate lending increased after the respective jurisdictional announcement dates of the Basel III reforms (BIS, 2022, Chapter 8.1.3 and Annex A.18).

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Appendix A Omitted Proofs

Proof of Proposition 1. In the region where the capital requirement is binding, we denote the market-to-book ratio and the equilibrium interest rate by $u_b(\cdot)$ and $R_b(\cdot)$, respectively, and in the region where it is slack by $u_s(\cdot)$ and $R_s(\cdot)$. For future reference, define

$$\begin{aligned} A(E) &:= -\frac{u_b'(E)}{u_b(E)}, \text{ and} \\ B(E) &:= \frac{L^{-1}(\Lambda E)}{\Lambda E \sigma^2}. \end{aligned} \tag{A.1}$$

We first establish the properties of $u(E)$ and $R(E)$ for a region in which the constraint (6) is slack.

Lemma A.1. *Assume that the constraint is slack, i.e., $A(E) \geq B(E)$, over a region $E \in [E^*, \overline{E}]$ with $E^* > \underline{E}$, then it holds for $E \in [E^*, \overline{E})$ that*

(i) $u_s'(E) < 0$,

(ii) $R_s(E) > 0$,

(iii) $u_s''(E) > 0$.

Proof. Note first that substituting boundary conditions (14) and (16) into HJB (20) implies that u_s'' is positive at the top:

$$u_s''(\overline{E}) = \frac{2}{\sigma^2 \widehat{L}^2} \zeta > 0. \tag{A.2}$$

By continuity, it must therefore hold that $u_s'(\overline{E} - \epsilon) < 0$ for a small ϵ. Now assume that u_s' changes sign and let $\hat{E} := \sup\{E < \overline{E} : u_s'(E) > 0\}$. By continuity it holds that $u_s'(\hat{E}) = 0$ and $u_s''(\hat{E}) < 0$, implying that $u_s(\hat{E}) = \frac{\sigma^2 L^2}{2(\zeta+\chi)} u_s''(\hat{E}) < 0$. This is a contradiction since $u_s(\overline{E}) = 1$ and $u_s'(E) < 0$ for $E \in (\hat{E}, \overline{E})$. The first claim of Lemma A.1 thus follows. The second claim follows then immediately from (22). The third claim follows immediately from (20) together with the first two claims. □

We next establish an auxiliary result analogous to Lemma A.1 for a region where the constraint is binding.

Lemma A.2. *Consider the region $E \in [\underline{E}, \widetilde{E}]$ with $\widetilde{E} \leq \overline{E}$, such that $A(E) < B(E)$, i.e. constraint (6) is binding.*

(i) It then holds that $R_b(E) > 0$ *for* $E \in [\underline{E}, \widetilde{E}]$*.*

(ii) If $\widetilde{E} < \overline{E}$*, it holds that* $u_b'(E) < 0$ *for* $E \in [\underline{E}, \widetilde{E}]$*. If* $\widetilde{E} = \overline{E}$ *it holds that* $u_b'(E) < 0$ *for* $E \in [\underline{E}, \widetilde{E})$ *and* $u_b'(\widetilde{E}) = 0$*.*

Proof. To prove the first claim, we establish that

$$R_b(E) = L^{-1}(\Lambda E) > 0 \; \forall E \in [\underline{E}, \widetilde{E}]. \tag{A.3}$$

This follows from $\partial L^{-1}(\Lambda E)/\partial E < 0$ together with the fact that $R_b(\widetilde{E}) > 0$. That is, if the constraint binds globally, i.e., $\widetilde{E} = \overline{E}$, then $R_b(\widetilde{E}) = L^{-1}(\Lambda \overline{E}) > 0$ since $u_b'(\overline{E}) = 0$ by boundary condition (16). If the constraint becomes slack at $\widetilde{E} = E^*$, it follows from Lemma A.1 that $R_b(\widetilde{E}) = R_s(E^*) > 0$.

In order to prove the second claim, we first establish that

$$u_b''(\underline{E}) < 0 < u_b''(\widetilde{E}). \tag{A.4}$$

If the constraint binds only over $E \in [\underline{E}, \widetilde{E}]$ with $\widetilde{E} = E^* < \overline{E}$, condition (A.4) follows immediately from Lemma A.1. If the constraint binds globally, i.e., $\widetilde{E} = \overline{E}$, assume to the contrary that $u_b''(\underline{E}) > 0$, which implies that $u_b'(\underline{E} + \epsilon) > 0$ and, thus, $u_b(\underline{E} + \epsilon) > 1 + \gamma$, a contradiction. Similarly, $u_b''(\overline{E}) < 0$ would imply that $u_b'(\overline{E} - \epsilon) > 0$ and, thus, $u_b(\overline{E} - \epsilon) < 1$, a contradiction.

We can now establish that $u'(E) < 0$ for $E \in (\underline{E}, \widetilde{E})$. Assume to the contrary that there exists a $\hat{E} < \widetilde{E}$ such that $u_b'(\hat{E}) \geq 0$. Since $u_b'(\underline{E}) = 0$, this would imply that there exists $\hat{E}_1 < \hat{E}$, at which $u_b''(E)$ becomes positive. But from (A.4) and the fact that $u_b'(\widetilde{E}) \leq 0$, there has to exist a $\hat{E}_2 > \hat{E}$ where $u_b''(E)$ turns negative. Furthermore, as $u_b''(\widetilde{E}) > 0$ by (A.4), there would exist another critical level $\hat{E}_3 > \hat{E}_2$, where $u_b''(E)$ becomes positive again. Evaluating equation (20) in $\hat{E}_2$ and $\hat{E}_3$ yields

$$\begin{aligned} \left(\zeta + \chi - \Lambda R_b(\hat{E}_2)\right) u_b(\hat{E}_2) &= \left[r\hat{E}_2 + R_b(\hat{E}_2)\Lambda \hat{E}_2 + \sigma^2 \Lambda \hat{E}_2\right] u_b'(\hat{E}_2), \\ \text{and } \left(\zeta + \chi - \Lambda R_b(\hat{E}_3)\right) u_b(\hat{E}_3) &= \left[r\hat{E}_3 + R_b(\hat{E}_3)\Lambda \hat{E}_3 + \sigma^2 \Lambda \hat{E}_3\right] u_b'(\hat{E}_3), \end{aligned} \tag{A.5}$$

respectively. From (A.3), the terms in square brackets on the RHS of (A.5) are strictly positive.

Since $u_b'(\hat{E}_2) > 0 > u_b'(\hat{E}_3)$, we would, thus, have that

$$\left(\zeta + \chi - \Lambda R_b(\hat{E}_2)\right) > 0 > \left(\zeta + \chi - \Lambda R_b(\hat{E}_3)\right),$$

which is a contradiction as $R_b'(E) = \partial L^{-1}(\Lambda E)/\partial E < 0$ and $\hat{E}_2 < \hat{E}_3$. Hence, it must hold that $u_b'(E) < 0$ for $E \in (\underline{E}, \widetilde{E})$ and the second claim follows. □

Finally, we are ready to piece together the two regions characterized in Lemma A.1 and Lemma A.2 and show that there can exist at most two regions, i.e., if the constraint becomes slack at some E^*, it is slack for all $E \in [E^*, \overline{E}]$. Note that E^*, the lowest point at which the constraint is not strictly binding, is characterized by $A(E^*) = B(E^*)$. Note further that in E^* it has to hold that $R_s = R_b$, i.e., using (21) and (22),

$$\frac{\Lambda E}{L^{-1}(\Lambda E)} = -\frac{1}{\sigma^2}\frac{u_s(E)}{u_s'(E)}. \tag{A.6}$$

Now note that for the constraint to become binding again at some $E_1^* > E^*$, (A.6) would have to hold with equality at E_1^* as well. Differentiating the LHS of (A.6) yields

$$\Lambda \frac{L^{-1}(\Lambda E) - \Lambda E \frac{\partial L^{-1}(\Lambda E)}{\partial E}}{(L^{-1}(\Lambda E))^2} > 0,$$

which follows from $L'(R) < 0$ and (A.3). Differentiating the RHS of (A.6) yields

$$-\frac{u_s'(E)^2 - u_s'(E)u_s''(E)}{(\sigma u_s'(E))^2} < 0,$$

which follows from Lemma A.1. Hence, (A.6) cannot be satisfied for any other value $E_1^* \neq E^*$.

After having established the above regularities, we now show how to construct the equilibrium. To solve for the equilibrium couple $u(E)$ and $R(E)$ in this case, we first consider a candidate value for the recapitalization barrier, $\underline{E}_c$, and solve (20) subject to boundary conditions (13) and (18), i.e.,

$$u_b(\underline{E}_c; \underline{E}_c) - (1+\gamma) = \frac{\partial}{\partial E} u_b(\underline{E}_c; \underline{E}_c) = 0.$$

Here, we adopt the notation $u_b(E; \underline{E}_c)$ to emphasize that the market-to-book ratio is a function of E and parameterized by the candidate value $\underline{E}_c$, for which the remaining boundary conditions are not necessarily satisfied. We can then determine, also parameterized by $\underline{E}_c$, the critical level of aggregate equity,

$$E_c^* := E^*(\underline{E}_c),$$

at which the constraint imposed by (6) becomes slack, i.e.,

$$-\frac{\frac{\partial}{\partial E} u_b(E_c^*; \underline{E}_c)}{u_b(E_c^*; \underline{E}_c)} = B(E_c^*). \tag{A.7}$$

Note that through the respective boundaries, also the equilibrium spread is parameterized by the candidate value $\underline{E}_c$:

$$R_b\left(E; \underline{E}_c\right) = L^{-1}\left(\Lambda E\right), \; E \in \left[\underline{E}_c, E_c^*\right]. \tag{A.8}$$

Turning next to the region where the regulatory constraint is slack, we can determine the equilibrium spread $R_s(E; \underline{E}_c)$ by solving (24) subject to the following boundary condition

$$R_s(E_c^*; \underline{E}_c) = R_b(E_c^*; \underline{E}_c), \tag{A.9}$$

which ensures continuity of the spread at the point E_c^*. By substituting $R_s(E; \underline{E}_c)$ into the first order condition for banks' leverage (22), we can compute the the market-to-book ratio in the region where the constraint is slack:[23]

$$u_s(E; \underline{E}_c) = u_s(E_c^*; \underline{E}_c) \times \exp\Big(- \int_{E_c^*}^{E} \frac{R_s(q; \underline{E}_c)}{\sigma^2 L\big(R_s(q; \underline{E}_c)\big)} dq \Big), \tag{A.10}$$

It is important to stress that $u_s(E; \underline{E}_c)$ is parameterized by the candidate $\underline{E}_c$ first, through the spread from (A.9) and, second, by imposing value-matching at E_c^* in (A.10), i.e.,

$$u_s(E_c^*; \underline{E}_c) = u_b(E_c^*; \underline{E}_c).$$

Next, we determine — also parameterized by the candidate $\underline{E}_c$ — the dividend boundary $\overline{E}(\underline{E}_c)$ by

[23] Note that this expression allows us to explicitly derive the dynamics of $R(E)$ in the benchmark case without capital regulation by setting $E_c^* = \underline{E} = 0$ and $E = \overline{E}$, such that $u_s(E; \underline{E}_c) = 1$ and $u_s(E_c^*; \underline{E}_c) = 1 + \gamma$.

using the boundary condition (16):

$$\frac{\partial}{\partial E} u_b\big(\overline{E}(\underline{E}_c); \underline{E}_c\big) = 0.$$

Finally, note that we have constructed a continuous, piece-wise function

$$u(E; \underline{E}_c) = \begin{cases} u_b(E; \underline{E}_c) & \text{if } E \leq E_c^*, \\ u_s(E; \underline{E}_c) & \text{if } E > E_c^*. \end{cases}$$

The same applies to $R(\cdot)$. Since all endogenous objects are parameterized by the candidate value $\underline{E}_c$, it remains to pin down $\underline{E}$ by the remaining boundary condition (14):

$$u(\overline{E}(\underline{E}_c); \underline{E}_c) = 1.$$

□

Appendix B Aggregate Loan Demand and Output

The aggregate loan demand in (7) stems from the assumption that the productive sector is populated by a continuum of firms that differ with respect to their productivity x. That is, at any time t, a firm with productivity x can covert one unit of the consumption good in $1 + xdt$ units of the consumption good at time $t + dt$. Firms' productivity is distributed according to a continuous distribution with density function

$$f(x) = \frac{\beta(r + \widehat{R} - x)^{\beta-1}}{\widehat{R}^{\beta}} \widehat{L}, \tag{B.1}$$

for $x \in [r, r + \widehat{R}]$ and zero otherwise. The rate of return on investment for a firm with productivity x is always equal to its productivity minus the rental rate of capital. Hence, a firm asks for a bank loan and invests if and only if its productivity exceeds the prevailing rental rate of capital, i.e., if

$x > R + r$. Hence, aggregate demand for bank loans equals

$$\begin{aligned} L(R) &= \int_{r+R}^{r+\widehat{R}} f(x)dx \\ &= \left(\frac{\widehat{R} - R}{\widehat{R}}\right)^{\beta} \widehat{L}, \end{aligned} \tag{B.2}$$

which is equation (7). Next, integration by parts implies that total output is given by

$$\begin{aligned} Y(R) &= \int_{r+R}^{r+\widehat{R}} xf(x)dx \\ &= L(R)\left(\frac{\widehat{R} - R}{1+\beta} + r + R\right). \end{aligned} \tag{B.3}$$