COLLATERAL REALLOCATION*

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PRELIMINARY

Abstract

Assets pledged as collateral for secured debt cannot be sold unless the debt is paid or otherwise renegotiated. We develop a model of this role of collateralization. We find that debt market frictions (alone) can cause the asset market to fail. Asymmetric information about debt values frustrates efficient asset sales despite perfect information about asset values and freedom to renegotiate. Still, borrowers issue secured debt in equilibrium, but only as a last resort, when net worth is low, as in recessions. Our theory provides an explanation of why secured borrowing is countercyclical and asset reallocation procyclical.

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1 Introduction

Aggregate economic efficiency requires assets to be allocated to their best users. Some estimates ascribe as much as a 30% loss in aggregate productivity to asset misallocation (e.g., Hsieh and Klenow (2009)), raising the question of what causes the asset market to fail: What prevents an asset's owner from selling it to a better user?

One potential impediment to the reallocation of assets is their use as collateral: Assets pledged as collateral for secured debt cannot legally be sold unless the debt is paid or otherwise renegotiated. In this paper, we explore how this role of collateralization can impede efficient sales, causing asset misallocation.

In a model focused on this role of collateral, we find that asymmetric information about debt values frustrates efficient asset sales despite perfect information about asset values and freedom to renegotiate. Thus, debt market frictions (alone) can cause the asset market to fail. This collateral channel of misallocation could be sizeable, as many assets are pledged as collateral; e.g., in the US, 80% of household debt and 65% of small business debt is secured by assets as collateral.[1]

We also show that borrowers use more secured debt when net worth is low, as in recessions. The collateral channel can thus connect stylized facts about debt and asset markets over the business cycle: In recessions, the secured debt share is high (Benmelech, Kumar, and Rajan (2019)) and asset reallocation low, despite high potential gains from reallocation (Eisfeldt and Rampini (2006) and Lanteri (2018)).

Model preview. A borrower B borrows from a creditor C to finance an investment in a productive asset. After one period, B can either sell the asset for price p or use it to produce cash flow y after another period. To focus on the case in which misallocation could arise, we assume that $p > y$; i.e. selling the asset after one period is efficient.

B's debt to C can be unsecured or secured by the asset as collateral, in which case B is not free to sell the asset unless the secured debt is paid (or otherwise renegotiated). In legal parlance, the transfer of deed requires the asset to be free of liens. Our focus on this role of collateralization is new to the literature, which stresses not how it can prevent sales, but how it can enforce them through foreclosure.

B can prepay her secured debt or renegotiate with C to effect the sale. But we assume renegotiation takes place under asymmetric information, as B is privately informed of her creditworthiness, i.e. the likelihood that she repays C after producing y.

[1] See www.newyorkfed.org/microeconomics for households and Benmelech, Kumar, and Rajan (2019) for firms.

Results preview. Our first main result is that, absent renegotiation, if B finances her investment with secured debt, the asset is sold too infrequently. The reason is that selling the asset after one period requires B to pay her secured debt in full, thereby forgoing her option to default one period later. For more creditworthy types, who are unlikely to default, this option has little value. Hence they sell the asset. However, for less creditworthy types, the default option is more valuable. Hence they prefer not to sell the asset so as to keep their default option alive.

We call this result the "collateral hangover," as the inefficiency is caused by a default-option-induced distortion akin to debt overhang (Myers (1977)). Here the inefficiency is not too little investment, as in debt overhang, but too little reallocation—moving is hard for assets pledged as collateral, which might as well be bedridden with a hangover (cf. Section 4.1).

Our second main result is that B and C cannot always renegotiate their way out of the collateral hangover. Despite p and y, and therefore the gains from the sale being known, B and C might not realize those gains. With symmetric information about B's creditworthiness, type-dependent debt write-downs effect efficient sales: C offers smaller write-downs for more creditworthy types, compensating each for giving up her default option. With asymmetric information, however, we show that C cannot separate types. Hence, to get all types to agree to sell the asset, C must offer them all a write-down to induce the least-creditworthy type to sell, and thereby over-compensate all other types. This can be prohibitively costly to C, causing renegotiation to fail.

We call this result "renegotiation unraveling" to highlight the analogy with the market for lemons, in which the inability to separate types leads to too little trade (Akerlof (1970)). Here, however, the asset market is frictionless: The asset characteristics (p and y) are known. Its failure is entirely due to debt market frictions.

Next, we ask whether B, who ultimately bears the efficiency cost of the collateral hangover, would ever issue secured debt in the first place. Our third main result is that B taps her sources of funds according to a strict order: She uses internal funds first, then unsecured debt, and then secured debt only as a last resort. Although B tries to avoid pledging the asset as collateral so as to maintain the flexibility to sell it after one period, doing so can be necessary to boost her debt capacity and get her project off the ground.

We call this result the "pecking-order theory of debt structure" as B exhausts her capacity to access one source of funds before moving on to another, as in Myers and Majluf's (1984) pecking-order theory of capital structure. However, the order is reversed, in the sense that "more expensive" instruments are preferred to "cheaper" ones (in the sense of interest rates).

Our pecking-order theory resonates with practice as firms tend to exhaust their unsecured debt capacity before issuing secured debt (Benmelech, Kumar, and Rajan (2019)). It also suggests that firms rely more on secured debt in recessions, viz. when internal funds are low, and thus that efficient asset sales will be frustrated in those times. It therefore connects two stylized facts: The aggregate share of secured debt is countercyclical and aggregate reallocation procyclical (Eisfeldt and Rampini (2006) and Lanteri (2018)).

Contribution. We propose a theory of asset misallocation based on collateralization. It complements those based on regulation, imperfect competition, financing constraints, and other frictions (see, e.g., Restuccia and Rogerson (2013)). It also contrasts with the literature on collateral constraints (e.g., Kiyotaki and Moore (1997)), which emphasizes their role in preventing best users from acquiring assets. Here, instead, they prevent owners from selling their assets to best users—they are constraints on divestment, not investment.

Our result that pledging assets as collateral increases ex ante debt capacity at the expense of ex post efficiency is reminiscent of models in which borrowers relinquish control over assets to relax financing constraints (Aghion and Bolton (1992)). Here, however, the inefficiency stems from the borrower's choice to take a low-pledgeability action (no sale to keep the default option alive) rather than the creditors' choice to take a high-pledgeability action. Our result that borrowers hold off on using secured debt, but use unsecured debt to maintain flexibility, is reminiscent of models in which borrowers want to maintain flexibility to borrow in the future (Donaldson, Gromb, and Piacentino (2020a, 2020b, 2020c). Here, the borrower wants to maintain the flexibility to divest assets, not take on new debt to invest in new assets.

Extensions and robustness. In our baseline analysis, we focus on collateral as a means of preventing unilateral asset sales by the borrower. As such, we abstract from its other roles, e.g., enforcing repayment ex post and, relatedly, loosening financial constraints ex ante. In an extension, we explore how the role of collateral we focus on interacts with these other roles. We show that our "divestment constraint" mechanism strengthens the more classical "investment constraint" mechanism (Section 5.1) and that it can complement the classical "repayment enforcement" role of collateral (Section 5.3).[2]

In our baseline model, we adopt a stark notion of creditworthiness: Unworthy types never repay; worthy types always do. This may correspond to literally diverting output—e.g., engaging in "self-dealing" or "tunneling" (Mironov (2013)). But it is also a catch-all for a broader class of frictions: We show that our low-creditworthiness borrowers can be interpreted as borrowers with high-risk investments, low expected personal costs of default, or high propensity to shirk. For a given contract, these alternative micro-foundations imply

[2] Another, more model-specific extension generalizes the payoff to unsecured debt following a sale (Section 5.2).

identical payoffs to the baseline specification. One advantage of the baseline is that it makes the study of optimal contracting easier.

Layout. Section 2 presents the model, Section 3 several benchmarks, Section 4 our main results, Section 5 extensions, and Section 6 alternative microfoundations of "creditworthiness." Section 7 concludes. Proofs are in the Appendix.

2 Model

There are three dates, 0, 1, and 2, no discounting, and universal risk-neutrality. At Date 0, a borrower B seeks financing from a competitive creditor C to acquire a productive asset for a project. After one period, at Date 1, B can either sell the asset or continue the project until Date 2, when she can be either a "worthy" type who repays her debt or an "unworthy" one who does not. At Date 1, B has private information about the probability that she will be worthy.

2.1 Project

B's asset purchase requires investment cost I at Date 0. At Date 1, B can sell the asset in a frictionless, competitive market for price p or continue the project to get payoff $y > I$ at Date 2, where y and p are certain. We assume that B should always sell the asset at Date 1:

Assumption 1. *At Date 1, an asset sale is efficient, i.e., $p > y$.*

B is thus the asset's first best user in the first period, but not in the second, when it is worth p to outsiders but only y to her.[3] Such a change in first best user is essential to our analysis. However, it need not happen for sure; some positive probability would suffice. It can reflect a worker becoming the second-best user of a house after changing jobs, a retailer of a storefront after a shift in demand, or a manufacturer of a factory after a technological breakthrough.

2.2 Financing

At Date 0, B has wealth w and so must borrow $L \geq I - w$ to buy the asset. To do so, she sells C a claim, promising state-contingent repayments subject to limited enforcement and limited liability.

We assume three limits to enforcement:

[3]There is no formal distinction between the project with the asset used therein.

(i) C cannot enforce an asset sale at Date 1.

- This implies that asset sales must be voluntary. It could be due to foreclosure being prohibitively costly. (We relax this in Section 5.2.)

(ii) C cannot enforce repayment from an unworthy borrower at Date 2.

- This simply defines an unworthy borrower. It could reflect a number of frictions; e.g., if unworthy, B could have means to divert y freely, engaging in tunneling or self-dealing, or y could be non-pledgeable. (We provide other micro-foundations in Section 6.)

(iii) C cannot enforce saving of sale proceeds from Date 1 to Date 2.

- This could likewise reflect B's having a means to divert cash freely between Date 1 and Date 2 or cash being non-pledgeable. (We relax this in Section 5.2.)

Limited liability means that B's payments to C cannot exceed her cash flows. It implies, in particular, that (i) B cannot commit to a repayment at Date 1 if she does not sell the asset (she has no Date-1 cash flow otherwise) and (ii) B cannot commit to a repayment at Date 2 if she sells the asset (she has no Date-2 cash flow otherwise, as she cannot commit to save the proceeds from the sale).

We call a contract feasible if it satisfies these constraints, is deterministic,[4] and depends only on whether B sells the asset (i.e. not on her type or reports thereof). A feasible contract is thus an amount borrowed L and a pair of repayments (D_s, D_u) with $0 \leq D_s \leq p$ and $0 \leq D_s + D_u \leq y$, where D_s is B's repayment conditional on her selling the asset and $D_s + D_u$ is her repayment conditional on her not selling it and being worthy. We show below (Lemma 7) that ruling out contracts on reports is w.l.o.g.

For $D_u \geq 0$, the contract (D_s, D_u) can be seen as a combination of two debt contracts, both maturing at Date 2. A first loan with face value D_s is secured by the asset in that a sale at Date 1 requires D_s to be (pre)paid at Date 1, reflecting a secured creditor's right to block the sale of collateral.[5] A second loan with face value D_u is unsecured in that the asset

[4] See Donaldson, Gromb, and Piacentino (2020b) on why stochastic mechanisms might be hard to implement in environments like this one and Appendix B for sufficient conditions for them to be suboptimal here.

[5] What we call secured debt also resembles another common real-world contract, debt with asset sale covenants, which includes the majority of bonds (Billett, King, and Mauer (2007)). If not automatically accelerated following asset sales, the debt restricts the use of their proceeds. But it provides weaker protection than collateral, which conveys a property right in the asset, as creditors have only contractual remedies following covenant violations (Ayotte and Bolton (2011) and Donaldson, Gromb, and Piacentino (2020a)).

sale does not require prepayment. If $D_u < 0$, there is only secured debt in that, as shown later, it arises for $D_s > y$, in which case $D_u = y - D_s$, so the final repayment final repayment is $\min\{y, D_s\}$ (Proposition 3).

2.3 Creditworthiness

At Date 0, neither B nor C knows whether B will be worthy at Date 2. But at Date 1, B privately learns the probability that she will be, which we call her "creditworthiness" and denote by a random variable $\theta := \mathbb{P}[\,\text{worthy}\,]$, distributed according to a distribution F with smooth density $f > 0$ on $[0, 1]$.

Assumption 2. *F satisfies the increasing-hazard rate property, i.e. its hazard rate $\hbar := \dfrac{f}{1-F}$ is increasing.*[6]

2.4 Payoffs

The set-up above implies the following payoffs, which we express ex interim for simplicity: If the asset is sold at Date 1, C gets paid D_s for sure; otherwise, C gets paid $D_s + D_u$ if B is worthy (with probability θ) and zero if not:

$$\mathbb{E}\Big[\text{ C's payoff }\Big|\,\theta\Big] = \begin{cases} D_s & \text{if asset sale,} \\ \theta(D_s + D_u) & \text{if no asset sale.} \end{cases} \tag{1}$$

B starts with w, borrows L, and invests I. Then she gets p if the asset is sold at Date 1 in which case she repays D_s for sure; otherwise, she gets y and repays $D_s + D_u$ if she is worthy (with probability θ) and nothing if not:

$$\mathbb{E}\Big[\text{ B's payoff }\Big|\,\theta\Big] = w + L - I + \begin{cases} p - D_s & \text{if sale,} \\ y - \theta(D_s + D_u) & \text{if no sale.} \end{cases} \tag{2}$$

Our setup captures a broad set of circumstances. Indeed, a variety of frictions generate the same payoffs, such as private information about investment risk (Section 6.1), personal costs

[6] This property is satisfied by many oft-used distributions, such as the normal, power normal, exponential, and uniform. Distributions with unbounded support, such as the normal, cannot apply to θ, which, being a probability, must be supported on a subset of the unit interval. It would be natural instead to apply such a distribution to the odds ratio $\frac{\theta}{1-\theta}$. It is easy to see that if this odds ratio has an increasing hazard rate, so does θ itself.

of default (Section 6.2), or moral hazard (Section 6.3).

2.5 Solution Concept: Equilibrium and Renegotiation Proofness

We solve for a Perfect Bayesian Equilibrium: B proposes a contract (L, D_s, D_u), C accepts/rejects, and B sells/continues given θ to maximize their respective expected payoffs given their beliefs, these beliefs being consistent with the equilibrium behavior.

A contract is renegotiation proof if there are no (menus of deterministic) contracts that both B and C would prefer to it at Date 1 or Date 2.

3 Benchmarks

Here we consider four benchmarks: the outcome that obtains (i) in first best, (ii) with complete information, (iii) with enforceable asset sales, and (iv) with only unsecured debt.

3.1 First Best

The first best outcome maximizes the total surplus, i.e. the sum of B's and C's payoffs.

Lemma 1. *In the first best outcome, B invests at Date 0 and sells the asset at Date 1.*

The result follows from $I < y$ and $y < p$ (Assumption 1).

3.2 Complete Information

Now assume complete information about B's creditworthiness at Date 1.

Lemma 2. *Assume B's creditworthiness is observable at Date 1 (and contractable at Date 0). The first best is attained if and only if $I - w \leq p - (1 - \mathbb{E}[\theta])y$.*

That B sells at Date 1 follows from the Coase theorem: B and C renegotiate to implement the efficient action (an asset sale) and are both better off. That B can invest at Date 0 follows from the condition in the lemma, which is necessary and sufficient for C to lend $I - w$, as shown in the proof.

3.3 Enforceable Sale

Now assume that the asset sale is enforceable so B can commit to selling the asset.

Lemma 3. *If the asset sale is enforceable, the first best is attained.*

B will commit at Date 0 to an asset sale at Date 1 because it is always efficient (Assumption 1) and, since C is competitive, she captures the total surplus.

3.4 Unsecured Debt Only

Last, we consider a benchmark in which B can borrow only unsecured.

Lemma 4. *If B can borrow only unsecured (i.e. $D_s = 0$), she does not invest.*

If B were to borrow unsecured, she would always sell the asset at Date 1. Indeed, selling the asset allows her to avoid repaying unsecured debt entirely. Anticipating as much, C would not lend in the first place (cf. equations (1) and (2)).

The assumption that unsecured debt gets nothing following an asset sale is stark and can be relaxed (Section 5.2). As extreme as it is, however, it highlights a basic trade-off with unsecured debt. On the one hand, there is no distortion ex post: the asset is sold, which is efficient. On the other hand, there could be a distortion ex ante: B could be unable to borrow enough to finance her project, as the asset is sold at the expense of unsecured debt. Secured debt, which must be paid if the asset is sold, could mitigate this inefficiency, increasing B's debt capacity by making it more likely that she repays C.

4 Results

Here we establish our main results: the collateral hangover (Section 4.1), renegotiation unraveling (Section 4.2), and the pecking-order theory of debt structure (Section 4.3).

4.1 Collateral Hangover

Here we characterize B's decision whether or not to sell the asset at Date 1 given she has secured debt D_s and unsecured debt D_u in place, assuming no renegotiation for now.

Because an asset sale is not enforceable, it occurs only if it is incentive compatible: B's payoff (equation (2)) is greater if she sells the asset than if she does not if

$$p - D_s \geq y - \theta(D_s - D_u) \tag{3}$$

or

$$\theta \geq \hat{\theta} := \frac{D_s - (p - y)}{D_s + D_u}. \tag{4}$$

If $\hat{\theta} \leq 0$, then B sells the asset irrespective of her creditworthiness. Otherwise, she sells only if her creditworthiness is high enough.

Proposition 1. (Collateral hangover) *Absent renegotiation, if $D_s \geq p - y$, sales are inefficiently few.*

The result says that secured debt can impede an asset sale. The reason is that by selling the asset, B forgoes her option to default on secured debt. This is prohibitively costly if (i) secured debt D_s is high (so default is valuable) or (ii) her creditworthiness θ is low (so default is likely).

Default option-induced distortions are tenets of the corporate finance literature, including the debt-overhang problem (Myers (1977)) and the risk-shifting problem (Jensen and Meckling (1976)). Here the inefficiency is too few asset sales, rather than underinvestment or excessive risk-taking.

The inefficiency is also specific to the type of debt. An asset sale kills the option to default on secured debt but not on unsecured debt. Empirically, the option to default on secured debt seems more valuable on average than that for unsecured debt: The spreads on secured debt are about triple those on unsecured debt (Benmelech, Kumar, and Rajan (2020)). A back-of-the-envelope calculation suggests this option could be worth about 16% of asset value for the average firm, surely enough to outweigh asset reallocation gains.[7] And it is likely to be worth even more for highly levered firms, such as distressed and financial firms.

4.2 Renegotiation Unravels

Not selling the asset is inefficient (Proposition 1). Hence, we explore renegotiation of the initial debt contract, which solves the debt overhang and risk-shifting problems,[8] but that we have ignored so far.

First, we ask what kinds of debt renegotiation might mitigate this inefficiency.

Lemma 5. *Any renegotiation is (equivalent to) a decrease in D_s conditional on a sale.*

In particular, there is no point in either (i) renegotiating to a higher repayment conditional on no sale (B would not accept) or (ii) offering menus to screen types (B's sale payoff $p - D_s$

[7]The calculation is as follows: Secured debt trades at a spread of about 3.5%/year over the risk-free rate and has maturity of about 11 years (Benmelech, Kumar, and Rajan (2020)). Neglecting coupons, its price relative to a risk-free bond with the same face value is $e^{-3.5\% \times 11} \approx 68\%$. The discount for credit risk is thus about $(100 - 68)\% = 32\%$ per unit debt, so its discount per unit assets is about $\frac{1}{2} \times 32\% = 16\%$, as the average firm's ratio of debt to assets is about half (Graham, Leary, and Roberts (2015)).

[8]See, however, Bhattacharya and Faure-Grimaud (2001) and Giammarino (1989).

does not depend on her type). The only way to induce an asset sale is therefore to write-down secured debt.

The next result gives a necessary and sufficient condition for such secured debt write-downs to induce asset sales.

Proposition 2. (Renegotiation unravels) *The contract (D_s, D_u) is renegotiation proof if and only if for the threshold type $\hat{\theta}$ (defined in equation (4))*

$$\hbar(\hat{\theta}) \leq \frac{D_s + D_u}{p - y}. \tag{5}$$

Otherwise, secured debt is written down and more types sell.

To understand the result, consider a small write-down from D_s to $D_s - \mathrm{d}D_s$ conditional on a sale. C would accept the write-down only if the gain it induces exceeds its loss:

- C's gain is that some types just below $\hat{\theta}$, of whom there is a mass $f(\hat{\theta})\mathrm{d}\hat{\theta}$, accept the renegotiation and now sell the asset and repay $D_s - \mathrm{d}D_s$ instead of continuing and repaying $D_s + D_u$ with probability just below $\hat{\theta}$:

$$\begin{aligned} \text{C's gain } &= \big((D_s - \mathrm{d}D_s) - \hat{\theta}(D_s + D_u)\big) f(\hat{\theta})\mathrm{d}\hat{\theta} &(6)\\ &= (p - y) f(\hat{\theta}) \frac{\mathrm{d}D_s}{D_s + D_u} + o(\mathrm{d}D_s^2). &(7)\end{aligned}$$

  In effect, C captures the gains from trade $p - y$ from a mass $f(\hat{\theta})\frac{\mathrm{d}D_s}{D_s+D_u}$ of types.

- C's loss is that all inframarginal types $\theta \geq \hat{\theta}$, of whom there are $1 - F(\hat{\theta})$, who would sell the asset even without a write-down, accept the renegotiation and make a lower repayment $D_s - \mathrm{d}D_s$:

$$\text{C's loss } = \mathrm{d}D_s\big(1 - F(\hat{\theta})\big). \tag{8}$$

The contract is robust to this renegotiation if the loss in equation (8) exceeds the gain in equation (7), which corresponds to equation (5) for small $\mathrm{d}D_s$ ($\mathrm{d}D_s^2 \to 0$). We show in the proof (Appendix A.7) that, given Assumption 2, robustness to a marginal write-down implies robustness to any.

The result reflects the trade-off C faces in writing down debt: C gains from inducing asset sales from low creditworthiness types who would not sell the asset otherwise but loses from making an unnecessary concession to high creditworthiness types who would have sold it anyway.

This market breakdown, due to the inability to separate types, is reminiscent of the market for lemons (Akerlof (1970)). There, the worst types are the most eager to sell, and the buyer is therefore reluctant to buy from them.Here, in contrast, the best types are the most eager to sell, and the creditor is therefore reluctant to part with them. There, the asset market fails due to asymmetric information about the assets themselves. Here, it fails due to asymmetric information about debt. Using the asset as collateral causes debt market frictions to spill over in the asset market, preventing efficient trade.[9]

Ex ante contractual arrangements, such as warranties, help solve the lemons problems. Hence, in the next section, we turn to whether the initial choice of debt structure prevents this market breakdown. But, first, we argue that the condition in Proposition 2 is weak.

Example: $\theta \sim$ beta$(1, \beta)$. Proposition 2 is stated in terms of some things that are relatively abstract (notably, the hazard rate $\hbar$ of creditworthiness θ) and others that are determined in equilibrium (viz., the marginal type $\hat{\theta}$ and the level of secured debt D_s). Here we consider an example that allows us to state condition (5) in terms of concrete things and to write a sufficient condition for it in terms of primitives alone.

Suppose $\theta \sim$ beta$(1, \beta)$ (i.e. $F(\theta) = 1 - (1-\theta)^\beta$), in which case $\hbar(\theta) = \beta/(1-\theta)$. Substituting in for $\hat{\theta}$ from equation (4) and rearranging, condition (5) becomes

$$\frac{D_u}{p - y} \geq \beta - 1. \tag{9}$$

I.e. renegotiation unravels whenever the unsecured debt D_u is sufficiently high relative to the gains from sale $p - y$. How high? Whenever $\beta \leq 1$, the RHS above is negative, so the condition is satisfied for any other parameters whatsoever. $\beta \leq 1$ is a relatively weak condition, given our application: It holds exactly when the average type is more likely to repay than default, i.e. when $\mathbb{E}[\theta] \geq 1/2$ (as the expectation of a beta$(1, \beta)$ variable is $(1+\beta)^{-1}$). This includes the uniform on [0,1], which is beta$(1, 1)$.

4.3 A Pecking-order Theory of Debt Structure

We have shown that for given D_s and D_u, an inefficient outcome can arise (Section 4.1 and Section 4.2). But can B choose a debt structure to avoid an inefficient outcome? To address this question we start with the full-commitment debt structure, i.e. the one B and C agree to if renegotiation is not allowed.

Define the debt capacity DC of a contract as its value to C, hence the amount C is willing

[9] Papers in which assets are misallocated due to asymmetric information about the assets themselves include Eisfeldt (2004), Fuchs, Green, and Papanikolaou (2016), and Kurlat (2013).

to lend against it:

$$\mathrm{DC}(D_s, D_u) := \mathbb{E}\big[\,\text{C's payoff}\;\big|\; D_s, D_u\big] \tag{10}$$

$$= \mathbb{E}\left[\mathbb{1}_{\{\theta \geq \hat{\theta}\}} D_s + \mathbb{1}_{\{\theta < \hat{\theta}\}} \theta (D_s + D_u)\right], \tag{11}$$

where the second line follows from C's payoff in equation (1) given that B sells the asset if $\theta \geq \hat{\theta}$ per equation (4).

Hence the equilibrium contract is the solution to an optimization program.

Lemma 6. *Suppose financing is feasible, i.e.* $\max\big\{\mathrm{DC}(D_s, D_u)\,;\, D_s + D_u \leq y\big\} \geq I - w$.

The optimal full-commitment debt structure minimizes the collateral hangover subject to C's participation constraint and feasibility, i.e. solves the program

$$\begin{cases} & \min\limits_{L, D_s, D_u} \; \hat{\theta} \\ \text{s.t.} & \mathrm{DC}\left(D_s, D_u\right) = L, \\ & L \geq I - w, \\ & D_s + D_u \leq y. \end{cases} \tag{12}$$

The intuition is that C being competitive, B chooses a feasible contract maximizing total surplus. Given that asset sales are efficient but occur only for $\theta \geq \hat{\theta}$, this amounts to minimizing $\hat{\theta}$.

But to get her investment off the ground, B could have to resort to a debt structure that impedes efficient sales, i.e. one with secured debt D_s in excess of the gains from trade $p - y$ (Proposition 1). The next result says that using secured debt is indeed a last resort.

Proposition 3. (Pecking order) *The optimal full-commitment debt structure is a mix of secured debt* ($D_s \geq 0$) *and unsecured debt* ($D_u \geq 0$ *unless* $D_s > y$ *in which case* $D_s + D_u = y$) *with the smallest amount of secured debt* D_s *such that B's debt capacity* $\mathrm{DC}\left(D_s, D_u\right)$ *exceeds her financing need* $I - w$.

The full-commitment debt structure could, however, be renegotiated, and therefore ultimately need not arise in equilibrium. The next result says that it is not the case and so our focus on the full-commitment debt structure is w.l.o.g.:

Corollary 1. *The optimal full-commitment debt structure in Proposition 3 is renegotiation-proof, i.e. satisfies condition* (5).

The full-commitment debt structure not only cannot be renegotiated ex post conditional on private information, it also cannot be improved upon by contracting ex ante on (reports of) private information:

Lemma 7. *Any (deterministic) full-commitment (menu) contract is equivalent to a mix of secured debt D_s and unsecured debt D_u.*

The pecking order result (Proposition 3) captures a trade-off inherent in secured debt. On the one hand, increasing D_s inhibits asset sales ex post per the collateral hangover (Proposition 1). On the other, it relaxes financial constraints ex ante. Indeed, B needs secured debt to have positive debt capacity (Lemma 4).

The proposition helps us map our results to empirical facts. It implies that B relies more on secured debt when her initial wealth w is low. Thus, to the extent that low wealth corresponds to a recession, it implies that the secured debt share is countercyclical and, as secured debt impedes reallocation (Proposition 1), reallocation is procyclical. Thus we suggest a possible causal link between empirical facts about debt structure (Badoer, Dudley, and James (2019) and Benmelech, Kumar, and Rajan (2019)) and about reallocation (Eisfeldt and Rampini (2006) and Lanteri (2018)). Moreover, we explain why the measured gains from potential reallocation are high in recessions, exactly when reallocation itself is low (see, e.g., Eisfeldt and Shi (2018)): Absent any asset market friction, gains from sales are unrealized, entirely as a result of debt market frictions (Proposition 1 and Proposition 2).

5 Extensions

We make several stark abstractions to focus and streamline our analysis; for example, to focus on the role of collateral in preventing sales, we entirely switch off its possible role in enforcing them stressed in the literature. Here we relax some of our assumptions, including that one, to explore how our mechanism interacts with others.

5.1 Constrained Buyers

So far, we have focused on impediments to selling assets, abstracting from impediments to buying them. We have assumed that the secondary market is frictionless and competitive, so the asset price p equals its buyer's value, say v. But buyers may not be able to pay v, but only a fraction of it, say χv, due to, e.g., borrowing constraints. Such constraints are known to lead to misallocation: For trade, the price must exceed the seller's (B's) value, $p > y$, but not the buyer's ability to pay, $p < \chi v$.[10] Here, we show that in our "divestment constraints" model, such "investment constraints" can exacerbate misallocation even if $\chi v > y$. Substituting $p = \chi v$ into the expression for the cut-off type $\hat{\theta}$ yields:

$$\hat{\theta}^{\chi} = \frac{D_s - (\chi v - y)}{D_s + D_u}, \tag{13}$$

which is decreasing in χ. Hence, tighter investment constraints (lower χ) lead to less asset trade. This is because with lower asset sale prices, forgoing the default option is less attractive to B.

5.2 Reinvestment of Sale Proceeds

So far, we have assumed that unsecured debt is repaid nothing following a sale. But the assumption is starker than the reality. In practice unsecured debt is likely to be paid something sometimes after a sale. Here we show that allowing sale proceeds to be reinvested in a risky technology amplifies the collateral hangover.

To see this, suppose that following a sale, B reinvests its cash—its sale proceeds p net of its secured debt repayment D_s—in a technology that returns $1/\eta$ with probability η and zero with probability $1 - \eta$.[11] And suppose that $1/\eta$ is large enough so B repays C in full when the return is $1/\eta$. Thus, following a sale, B not only repays secured debt for sure, but also repays unsecured debt with probability η. B thus sells if the following IC is satisfied (cf. equation (3)):

$$p - D_s - \eta D_u \geq y - \theta(D_s - D_u) \tag{14}$$

or

$$\theta < \hat{\theta}^{\eta} := \frac{D_s + \eta D_u - (p - y)}{D_s + D_u}. \tag{15}$$

Observe that $\hat{\theta}^{\eta}$ is increasing in η: Increasing η decreases the likelihood of a sale and therefore there are fewer sales than in the baseline, which is the $\eta = 0$ case. Intuitively, increasing η decreases the value of the default option on unsecured debt following a sale.

[10] See, e.g., Buera, Kaboski, and Shin (2011), Lanteri and Rampini (2021), Midrigan and Xu (2014), Moll (2014), and Shleifer and Vishny (1992).

[11] That the expected return is one ($1 = \eta \times \frac{1}{\eta} + (1 - \eta) \times 0$) is just for simplicity.

5.3 Costly Foreclosure

So far, we have focused on collateral's role in preventing sales, abstracting from its role in forcing them. Secured creditors often do have the right to force sales, e.g., following default (but outside of bankruptcy in which assets are stayed). Here we show that these rights might fail to mitigate misallocation even if they are useful to enforce repayment.

Suppose that at Date 2 cash flow y is verifiable only under foreclosure which entails a cost κ for the creditor. The condition for foreclosure to be credible is $y - \kappa > 0$. The condition at Date 1 is different. If C does not foreclose, it gets paid $D_s + D_u$ later if B is worthy. Thus, ex interim, C forecloses if $p - \kappa \geq \mathbb{E}_{\text{Date 1}}[\theta(D_s + D_u)]$. The continuation value on the RHS makes it likely that this ex interim constraint is harder to be satisfied than the ex post constraint above. The standard foreclosure role cannot provide C with a credible threat when its continuation value is high and therefore might not have teeth ex interim. Perhaps the role of collateral in preventing sales matters more ex interim, per our focus, whereas its role in forcing them matters more ex post, as usually emphasized.

6 Applicability and Robustness

Our analysis applies whenever B has private information about the value of her default option. In the baseline model, B's default option is somewhat reduced form: She repays if she is creditworthy and does not if she is not. Here we give three alternative specifications that generate a valuable default option—indeed they generate the same equations as the baseline—suggesting the model could apply widely.

6.1 Investment Risk

Here we replace the assumption that B might be unworthy with the assumption that B's investment is risky and B defaults when it fails.

Suppose that B's investment pays off $\tilde{y} = y/\theta$ with probability θ and $\tilde{y} = 0$ with probability $1 - \theta$. And suppose that y is sufficiently large that B can repay in full when the investment succeeds (i.e. $y/\theta > D_s + D_u$). B's and C's payoffs given a sale are as in the baseline. Their payoffs given no sale are

$$\mathbb{E}\Big[\text{ C's payoff } \Big| \theta \text{ \& no sale}\Big] = \theta(D_s + D_u) \tag{16}$$

and

$$\mathbb{E}\Big[\text{ B's payoff } \Big| \theta \text{ \& no sale}\Big] = \mathbb{E}\Big[\tilde{y} - \text{ C's payoff } \Big| \theta \text{ \& no sale}\Big] \tag{17}$$

$$= y - \theta(D_s + D_u), \tag{18}$$

which are exactly as in the baseline (equations (1) and (2)).

6.2 Default Costs

Here we replace the mechanical assumption that the unworthy borrower never repays but the worthy always does with the assumption that the unworthy borrower can default at no cost, whereas the worthy one bears a cost ξ, representing, e.g., costs of moral guilt or social ostracism for households or costs of exclusion from future financial and goods markets for firms.

In this set-up, the unworthy does not suffer and therefore never repays; the worthy, in contrast, suffers cost ξ and therefore repays whenever

$$y - (D_s + D_u) \geq y - \xi. \tag{19}$$

Thus, if $\xi > D_s + D_u$, the worthy always repays and the equations coincide with those in the baseline.

6.3 Moral Hazard

Here we replace the assumption that the unworthy borrower avoids repaying ex post but the worthy does not with the assumption that absent a sale a borrower must "work" to produce output y, but working entails forgoing private benefits β as in, e.g., Tirole (2006). Here, unlike in the baseline, we assume that y is always pledgeable; however β never is.

Specifically, we suppose that if B works she produces y and repays her debt, making her payoff $y-(D_s+D_u)$ and C's D_s+D_u, whereas if she shirks her payoff is β and C's is zero. If β is random, equal to y with probability $1-\theta$ and 0 with probability θ, B's and C's payoffs are exactly as in the baseline (equations (1) and (2)).

7 Conclusion

What does it mean to use an asset as collateral? It means several things that have been explored in the literature, e.g., that creditors have priority over assets in bankruptcy and could be able to seize them out-of-court absent bankruptcy. But it also means something that has not been explored in the economics and finance literature, namely, that creditors can prevent their sales. We explore this role of collateral and argue that it could be a relevant source of asset misallocation.

A Proofs

A.1 Proof of Lemma 1

The (brief) argument is in the text. $\square$

A.2 Proof of Lemma 2

The main argument is in the text. It remains only to show that borrowing is feasible if and only if the condition in the lemma is satisfied.

($\Longrightarrow$) We show sufficiency by construction. Let $D_s(\theta) = p - (1-\theta)y$ and $D_u = \max\{y - D_s(\theta), 0\}$.

From equation (1), C gets $D_s(\theta)$ if B sells and $D_s(\theta) + D_u(\theta)$ if B does not. From equation (2), B sells whenever θ is such that

$$y - \theta\big(D_s(\theta) + D_u(\theta)\big) \leq p - D_s(\theta) \tag{20}$$

or

$$\max\{(1-\theta)y - (p-y), 0\} \leq (1-\theta)y \tag{21}$$

which is satisfied for all θ given $y < p$. Hence financing is feasible whenever $I - w \leq \mathbb{E}[D_s(\theta)]$, which is the condition in the lemma.

($\Longleftarrow$) We show necessity by comparing B's rent to the total surplus. A lower bound on type-θ's rent is what she can divert: $(1-\theta)y$. An upper bound on the total surplus is the first-best surplus p. Thus C gets at most $p - (1-\theta)y$. Taking the expectation gives the condition in the proposition. $\square$

A.3 Proof of Lemma 3

The argument is in the text. $\square$

A.4 Proof of Lemma 4

The argument is in the text. $\square$

A.5 Proof of Proposition 1

The argument is in the text. □

A.6 Proof of Lemma 5

For both B and C to accept, any renegotiation must increase the total surplus. Therefore, an asset sale being efficient (Assumption 1), the renegotiation must induce the sale. The renegotiation can thus be no more than a change in the repayment conditional on the sale, D_s.[12]

A type-θ B accepts to change the secured debt level from D_s to $\hat{D}_s$ conditional on a sale if doing so increases her payoff relative to rejecting the change and either selling the asset or continuing the project, i.e. if

$$p - \hat{D}_s \geq \max\big\{p - D_s, y - \theta(D_s + D_u)\big\}. \tag{22}$$

Thus if $\hat{D}_s > D_s$, no type accepts. The renegotiation can thus be only a decrease in D_s (or equivalent to one[13]). □

A.7 Proof of Proposition 2

The argument for small write-downs $\mathrm{d}D_s$ is in the text.

We need to show only that if no small write-down is possible, than no large one is either, i.e. that C's gain from the increased repayment from types who choose to sell only due to renegotiation is lower than the loss from types that would have sold absent renegotiation.

To compute the gains, we find the analog of equation (6) for $\mathrm{d}D_s > 0$:

$$\text{gain } = \int_{\hat{\theta}_{-\mathrm{d}D_s}}^{\hat{\theta}} \big(D_s - \mathrm{d}D_s - \theta(D_s + D_u)\big) f(\theta)\mathrm{d}\theta, \tag{23}$$

[12] You could try to use a menu to screen types, but conditional on a sale all types make the repayment with the same probabilities—repaying debt due at Date 1 for sure and that due at Date 2 never—so all types will choose the same contract from the menu: that with the lowest Date-1 repayment. Screening is impossible.)

[13] Such equivalent renegotiations induce the same outcome via other means. E.g., rather than "a write-down of secured-debt from D_s to $\hat{D}_s$ conditional on a sale," the renegotiation could specify "an unconditional write-down of secured debt from D_s to $\tilde{D}_s$ and an increase of unsecured debt from D_u to $\tilde{D}_u$." If $\tilde{D}_s = \hat{D}_s$ and $\tilde{D}_s + \tilde{D}_s = D_s + D_u$ than the same types accept the renegotiation and all that do sell the asset, as if the renegotiation had been conditional on the sale in the first place.

where $\theta_{-\mathrm{d}D_s}$ denotes the threshold type if secured debt is reduced conditional on a sale:

$$\hat{\theta}_{-\mathrm{d}D_s} := \frac{D_s - \mathrm{d}D_s - (p-y)}{D_s + D_u}. \tag{24}$$

To bound the gain from below, define

$$\theta^* := \arg\min \left\{ \big(D_s - \mathrm{d}D_s - \theta(D_s + D_u)\big) f(\theta) \,\middle|\, \hat{\theta}_{-\mathrm{d}D_s} \le \theta \le \hat{\theta} \right\}, \tag{25}$$

so that

$$\begin{aligned}
\text{gain} &\ge \big(\hat{\theta} - \hat{\theta}_{-\mathrm{d}D_s}\big)\big(D_s - \theta^*(D_s + D_u)\big) f(\theta^*) - \big(F(\hat{\theta}) - F(\hat{\theta}_{-\mathrm{d}D_s})\big)\mathrm{d}D_s && (26)\\
&\ge \big(\hat{\theta} - \hat{\theta}_{-\mathrm{d}D_s}\big)\big(D_s - \hat{\theta}(D_s + D_u)\big) f(\theta^*) - \big(F(\hat{\theta}) - F(\hat{\theta}_{-\mathrm{d}D_s})\big)\mathrm{d}D_s && (27)\\
&= \frac{\mathrm{d}D_s}{D_s + D_u}(p-y) f(\theta^*) - \big(F(\hat{\theta}) - F(\hat{\theta}_{-\mathrm{d}D_s})\big)\mathrm{d}D_s, && (28)
\end{aligned}$$

having used the fact that $\theta^* \le \hat{\theta}$, substituted in for $\hat{\theta}$ and $\hat{\theta}_{-\mathrm{d}D_s}$, and rearranged.

The loss is $\mathrm{d}D_s(1 - F(\hat{\theta}))$ per equation (8). Comparing it with the lower bound in equation (28) gives a sufficient condition for the gain to be lower than the loss:

$$\frac{f(\theta^*)}{1 - F(\hat{\theta}_{-\mathrm{d}D_s})} \le \frac{D_s + D_u}{p - y}, \tag{29}$$

which is satisfied whenever condition (5) is, given

$$\begin{aligned}
\frac{f(\theta^*)}{1 - F(\hat{\theta}_{-\mathrm{d}D_s})} &\le \frac{f(\theta^*)}{1 - F(\theta^*)} && (30)\\
&= \hbar(\theta^*) && (31)\\
&\le \hbar(\hat{\theta}) && (32)
\end{aligned}$$

having used that $\theta_{-\mathrm{d}D_s} \le \hat{\theta}^*$ and that F and $\hbar = f/(1-F)$ are increasing. □

A.8 Proof of Lemma 6

The result has two parts: first that C's IR binds and second that the equilibrium contract minimizes $\hat{\theta}$.

Part 1: C's IR binds: Suppose (in anticipation of a contradiction) that it is slack in equilibrium, i.e. that C gets rent. B has a profitable deviation: to increase the amount borrowed L by an amount less than C's rent (so C will still accept). B is better off and the outcome is otherwise the same, contradicting the equilibrium.

Part 2: Minimize $\hat{\theta}$: There is symmetric information at Date 0, so the equilibrium contract maximizes the total surplus. That corresponds to maximizing the sale probability, i.e. minimizing $\hat{\theta}$. □

A.9 Proof of Proposition 3

We first show that $D_u \geq 0$ whenever $D_s < y$, then that $D_u = y - D_s$ whenever $D_s > 0$, and finally that D_s is minimized.

($D_u \geq 0$ if $D_s < y$.) We show that (D_s, D_u) with $D_u > 0$ is always worse than $(D_s, 0)$, since $(D_s, 0)$ induces (i) a more efficient outcome and (ii) higher debt capacity. (The argument applies only for $D_s < y$ as otherwise $(D_s, 0)$ is infeasible.)

(i) Efficiency is higher when there are more sales or $\hat{\theta}$ (as defined in equation (4)) is lower:

$$\hat{\theta}(D_s, D_u) = \frac{D_s - (p-y)}{D_s + D_u} > \frac{D_s - (p-y)}{D_s} = \hat{\theta}(D_s, 0). \tag{33}$$

(ii) Debt capacity is given by equation (11): for $D_u < 0$,

$$\begin{aligned} \mathrm{DC}(D_s, D_u) :=& \mathbb{E}\left[\mathbb{1}_{\{\theta \geq \hat{\theta}(D_s, D_u)\}} D_s + \mathbb{1}_{\{\theta < \hat{\theta}(D_s, D_u)\}} \theta (D_s + D_u)\right] && (34)\\ \leq& \mathbb{E}\left[\mathbb{1}_{\{\theta \geq \hat{\theta}(D_s, 0)\}} D_s + \mathbb{1}_{\{\theta < \hat{\theta}(D_s, 0)\}} \theta (D_s + D_u)\right] && (35)\\ \leq& \mathbb{E}\left[\mathbb{1}_{\{\theta \geq \hat{\theta}(D_s, 0)\}} D_s + \mathbb{1}_{\{\theta < \hat{\theta}(D_s, 0)\}} \theta D_s\right] && (36)\\ =& \mathrm{DC}(D_s, 0), && (37)\end{aligned}$$

having used that $\hat{\theta}(D_s, D_u) > \hat{\theta}(D_s, 0)$ per inequality (33).

($D_u = y - D_s$ if $D_s > 0$.) The program (11) says that B minimizes $\hat{\theta}$. If $D_s > p - y$, $\hat{\theta}$ is decreasing in D_u, so D_u is maximized: $D_u = D_s - y$ is (strictly) optimal. If $D_s \leq p - y$, $\hat{\theta}$ is negative and the first best is attained for any D_u: $D_u = D_s - y$ is (weakly) optimal.

(D_s minimized.) The program (11) says that B minimizes $\hat{\theta}$. If $D_s > p - y$, $\hat{\theta}$ is increasing in D_s, so minimizing D_s is (strictly) optimal. If $D_s \leq p - y$, $\hat{\theta}$ is negative, so minimizing D_s is (weakly) optimal. □

A.10 Proof of Corollary 1

First observe that B's debt capacity DC must be locally increasing in the fraction of secured debt at the equilibrium, i.e. if (D_s, D_u) is offered in equilibrium, then

$$\left.\frac{\partial}{\partial D_s}\right|_{D_s+D_u=\text{const.}} \mathrm{DC}(D_s, D_u) \geq 0. \tag{38}$$

The reason is that otherwise, B could decrease D_s and increase D_u slightly and increase her objective (per the program (11) given $\hat{\theta}$ is increasing in D_s).

Now the result follows directly from differentiating the debt capacity with respect to D_s holding total debt constant (so $\mathrm{d}(D_s + D_u) = 0$)):

$$\begin{aligned}
\mathrm{d}\,\mathrm{DC}(D_s, D_u) &:= \mathrm{d}\Bigg(\big(1 - F(\hat{\theta})\big)D_s + F(\hat{\theta})\mathbb{E}\big[\theta \,\big|\, \theta \leq \hat{\theta}\big](D_s + D_u)\Bigg) && (39)\\
&= \mathrm{d}\Bigg(\big(1 - F(\hat{\theta})\big)D_s + \int_0^{\hat{\theta}} \theta f(\theta)\mathrm{d}\theta(D_s + D_u)\Bigg) && (40)\\
&= \big(1 - F(\hat{\theta})\big)\mathrm{d}D_s - f(\hat{\theta})\Big(\hat{\theta}(D_s + D_u) - D_s\Big)\mathrm{d}\hat{\theta} && (41)\\
&= \big(1 - F(\hat{\theta})\big)\mathrm{d}D_s + f(\hat{\theta})\left(\frac{D_s - (p-y)}{D_s + D_u}(D_s + D_u) - D_s\right)\mathrm{d}\hat{\theta} && (42)\\
&= \big(1 - F(\hat{\theta})\big)\mathrm{d}D_s - f(\hat{\theta})(p-y)\mathrm{d}\hat{\theta} && (43)\\
&= \big(1 - F(\hat{\theta})\big)\mathrm{d}D_s - f(\hat{\theta})(p-y)\frac{\mathrm{d}D_s}{D_s + D_u}. && (44)
\end{aligned}$$

So we have that $\mathrm{dDC}(D_s, D_u) \geq 0$ iff

$$1 - F(\hat{\theta}) \geq \frac{f(\hat{\theta})(p-y)}{D_s + D_u}$$

or

$$\frac{D_s + D_u}{p-y} \geq \frac{f(\hat{\theta})}{1 - F(\hat{\theta})} \equiv \hbar(\hat{\theta}),$$

which is equation (5). □

A.11 Proof of Lemma 7

Consider any set of menus of deterministic repayments, which we write as direct mechanism per the revelation principle: $\{(D_s(\theta), D_u(\theta)\}_\theta$, where $D_s(\theta)$ is type-θ's repayment conditional on a sale and $D_s(\theta) + D_u(\theta)$ is her repayment absent one if she is worthy. Type-θ B's IC for truthful reporting is

$$\max\left\{p - D_s(\theta), y - \theta\big(D_s(\theta) + D_u(\theta)\big)\right\} \geq \max\left\{p - D_s(\theta'), y - \theta\big(D_s(\theta') + D_u(\theta')\big)\right\} \quad (45)$$

for all $\theta, \theta' \in \operatorname{spt} f$.

We show that all types who sell the asset and all who do not receive equivalent contracts, so it suffices to consider contracts conditional on selling and not selling as in the baseline:

- Types who sell: For any type θ such that $p - D_s(\theta) \geq y - \theta\big(D_s(\theta) + D_u(\theta)\big)$, the IC (inequality (45)) implies that $p - D_s(\theta) \geq p - D_s(\theta')$, i.e. $D_s(\theta) \leq D_s(\theta')$, for all θ'. I.e. $D_s(\theta) = \min_{\theta'}\{D_s(\theta')\}$, which does not depend on θ. Hence all types who sell must receive equivalent contracts.
- Types who do not sell: If $p - D_s(\theta) < y - \theta\big(D_s(\theta) + D_u(\theta)\big)$, the IC (inequality (45)) implies that $y - \theta(D_s(\theta) + D_u(\theta)) \geq y - \theta(D_s(\theta') + D_u(\theta'))$, or $D_s(\theta) + D_u(\theta) \leq D_s(\theta') + D_u(\theta')$, for all θ'. I.e. $D_s(\theta) + D_u(\theta) = \min_{\theta'}\{D_s(\theta') + D_u(\theta')\}$, which does not depend on θ. Hence all types who do not sell must receive equivalent contracts.

□

B Stochastic Mechanisms

In our baseline analysis, we ruled out stochastic mechanisms. This need not be w.l.o.g. in theory (e.g., Aghion and Bolton (1992)). The next result gives sufficient conditions for them to be in our model (we conjecture that they are far from necessary).

Lemma 8. *Suppose that (i) the total surplus, $F(\hat{\theta})p + \big(1 - F(\hat{\theta})\big)y$, and (ii) the debt capacity (equation 11) are concave in D_s if $D_u = y - D_s$. Any optimal mechanism (satisfying the constraints in Section 2.2) is deterministic.*

Proof. Randomization could arise (i) conditional on B's action (sale or not) or (ii) before it (the action itself cannot be random, as B cannot commit).

We rule out each in turn:

(i) Randomization conditional on B's action cannot improve on the deterministic mechanism because everyone is risk-neutral. Any random transfers are equivalent to deterministic transfers equal to their expected values.

(ii) A random contract realized before the action cannot improve because, by Lemma 6, the contract is chosen to maximize efficiency under symmetric information subject to C being willing to lend:

- By Lemma 7 (and (i) above), it suffices to consider randomization over pairs (D_s, D_u) instead of over menus.
- By Proposition 3, any contract that occurs with positive probability must have $D_s + D_u = y$.
- By hypothesis, no two contracts can occur with positive probability as, by concavity, taking their average (i.e. the average of the secured debt levels) would increases the objective without violating the constraint (it would increase both efficiency and debt capacity).

□

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