# Term Structure of Equity and Bond Yields over Business Cycles *

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**Abstract**

Recent findings on the term structure of equity and bond yields pose serious challenges to existing equilibrium asset pricing models. This paper presents a new equilibrium model to explain the joint historical dynamics of equity and bond yields (and their yield spreads). Equity/bond yields movements are mainly driven by subjective dividend/GDP growth expectation. Yields on short-term dividend claims are more volatile because the short-term dividend growth expectation is mean-reverting to its less volatile long-run counterpart. The procyclical slopes of spot and forward equity yields are due to the counter-cyclical slope of dividend growth expectations. Returns on long-term dividend claims have higher volatilities and co-move more strongly with the market, because of stronger belief revisions over long-term dividend growth. The correlation between equity returns/yields and nominal bond returns/yields switched from positive to negative after the late 1990s, owing to (1) procyclical inflation and (2) higher correlation between expectations of real GDP and of real dividend growth post-2000. The model is also consistent with the data in generating persistent and volatile price-dividend ratios, excess return volatility, and return predictability.

**Keywords:** Term structure, equity yields, subjective expectations, bond-stock correlation, return predictability

**JEL Classification:** G00, G12, E43.

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# 1 Introduction

The fundamental question in asset pricing research is what drives the joint equity and bond price movements. To see the link between them, equity price at time $t$ can be expressed as its discounted cash flow:

$$P_t = \sum_{n\geq 0} \frac{E_t(D_{t+n})}{R_{t+n}} = \sum_{n\geq 0} \frac{E_t(D_{t+n})}{\exp\left(n\left(y_t^{(n)} + \theta_t^{(n)}\right)\right)},$$

where $E_t(D_{t+n})$ is the expected nominal dividend and $R_{t+n}$ is the required return. $R_{t+n}$ can be further decomposed into nominal bond yield $y_t^{(n)}$ and dividend risk premium $\theta_t^{(n)}$. This equation holds under both rational and subjective expectations. Motivated by the findings that prices are too volatile than dividend (holding $R_{t+n}$ constant) and future returns are predictable by the price-dividend ratio under rational expectation (Shiller, 1981; Campbell and Shiller, 1988b; Cochrane, 2008, 2011), several leading asset pricing models based on time-varying dividend risk premium $\theta_t^{(n)}$ have been proposed, for example, the disaster model (Barro, 2006; Gabaix, 2012; Gourio, 2012), the long run risk model (Bansal and Yaron, 2004), and the habit formation model (Campbell and Cochrane, 1999).

Recent empirical findings, however, pose some new challenges to existing equilibrium asset pricing models from different dimensions: (1) De la O and Myers (2020) show that short-term cash flow growth expectations obtained from survey, rather than subjective return expectations, account for most aggregate stock price movements. And Bordalo et al. (2020) show that long-term earning growth expectations over-react to macro news, which is responsible for return predictability and excess price volatility. (2) Both short- and long-term subjective beliefs from survey are also important drivers of bond yields and bond return predictability (Froot, 1989; Piazzesi et al., 2015; Cieslak, 2018). And it is difficult for existing equilibrium models to jointly explain the trend, cycle, and spread movements in bond yields $y_t^{(n)}$ (Zhao, 2020b). (3) The procyclical equity forward yield spread is mainly driven by countercyclical dividend growth expectations (van Binsbergen et al., 2013), rather than the term structure of dividend risk premium $\theta_t^{(n)}$. (4) long-term dividend strip returns co-move more strongly with the market returns and have higher volatitilies than short-term strip returns (Lettau and Wachter, 2007; Van Binsbergen and Koijen, 2017; Gonçalves, 2019). (5) The correlation between stock returns and long-term nominal bond returns has switched from positive to negative after the late 1990s (Li, 2002; Fleming et al., 2003; Campbell et al., 2017). Duffee (2018a) shows empirically that changes in the correlation between stock returns and real bond returns plays a significant role in explaining this fact, rather than the changes in inflation cyclicality as proposed in recent equilibrium models (Burkhardt and Hasseltoft, 2012; Bansal and Shaliastovich, 2013; David and Veronesi, 2013; Song, 2017; Campbell et al., 2020).

This paper contributes to the literature by proposing an equilibrium model that explains the joint dynamics of the term structure of equity and bond yields and is consistent with the above empirical findings. Variations in equity (bond) yields are due to subjective dividend growth (GDP growth) expectations, instead of dividend risk premium (bond risk premium). The ex-post realized bond and equity returns are predictable because of the expectation errors, and prices are volatile because of the volatile subjective beliefs. We build our analysis on the Treasury bond pricing framework of Zhao (2020b) by further modeling the expectation formation over aggregate dividend. Naturally, our model inherits the explanatory power for several important facts in the bond market.[1] More importantly for this paper, the model can match the historical dynamics of equity yields in data provided by Giglio et al. (2020).[2] Yields on short-term dividend claims are more volatile because the short-term dividend growth expectation is mean-reverting to the less volatile levered long-run GDP growth expectations. The negative slope of spot and forward equity yields during recessions reflects the countercyclical slope of dividend growth expectations. Returns on long-term dividend claims have higher volatilities and co-move more strongly with the market, because of stronger belief revisions

[1] Zhao (2020b) shows that learning about the long-run mean and short-run deviation of GDP and inflation can jointly explain the trend, cycle, and spread movements in bond yields. Trend in nominal bond yields is driven by long-run GDP growth and inflation expectations. Business cycle movements in short-term nominal bond yields and in the yield spreads are driven by short-run GDP growth and inflation expectations.

[2] Dividend strip yields/prices can be calculated directly from dividend strip future contracts, which is available only starting from early 2000s. Giglio et al. (2020) estimate an affine model of equity prices and derive the strip yields for the aggregate market that cover a much longer history.

over long-term dividend growth. Long-term Treasury bonds switched from risky assets to safe assets after the late 1990s, due to (1) a procyclical inflation and (2) a higher correlation between real GDP and real dividend growth expectations during the past 20 years.

We first follow Zhao (2020b) and decompose both the GDP growth (as endowment growth) and inflation into two components: one stable and one transitory/volatile. The representative agent with a constant relative risk aversion (CRRA) utility learns about the unconditional mean output growth and inflation rates from the stable component (PCE and core inflation), and learns about the stationary deviations from the mean from the transitory/volatile component. Zhao (2020b) shows that the subjective expectations resulting from these two types of learning can explain most of the variations in bond yield dynamics. Then to model aggregate dividend, we sort firms by their cash flow durations as measured by the long-term growth expectations and decompose the logarithm of aggregate dividend into two components: long-duration dividend $d_t^l$ and share of long-duration dividend in total dividend $d_t^s$. The aggregate endowment risk is embedded in the long-duration dividend $d_t^l$, which is assumed to be levered on log real GDP. The share of long-duration dividend $d_t^s$ carries no aggregate risks and follows a stationary process. The subjective expectation of dividend growth from this model matches well the median dividend growth survey expectation of the S&P 500, constructed from Thomson Reuters I/B/E/S, with a correlation of 0.79 for both 1-year and 2-year ahead dividend growth. Model-implied long-term subjective dividend growth expectation over-reacts to macro news similar to Bordalo et al. (2020). All the subjective expectations (for GDP growth, inflation, and dividend growth) are formed endogenously through constant learning as in Malmendier and Nagel (2016); Nagel and Xu (2019).

Agents in this model have limited information about the stochastic environment and hence face both risk and ambiguity. Risk refers to the situation where there is a probability law to guide choice. However, the ambiguity-averse representative agent (with recursive multiple priors, or maxmin preferences by Epstein and Schneider, 2003) lack the confidence to assign probabilities to all relevant events. The agent has in mind a benchmark or reference measure of the economy's dynamics (all the posterior beliefs about inflation, GDP growth, and dividend growth), but she is concerned that the reference measure is misspecified and acts as if she evaluates future prospects using a worst-case probability drawn from a set of multiple distributions. We follow Ilut and Schneider (2014) and measure the size of ambiguity using forecast dispersion. We find that ambiguity has negligible effects on short-term equity yields but bigger effects on long-term equity yields. The ambiguity effects are significantly higher during recessions than during expansions and most of them are driven by dividend-specific ambiguity.

Our model captures the entire time-series dynamics of equity spot and forward yields over the past three decades. Figure 1 shows some salient features in the data: (1) more volatile short-term equity yields, (2) a secular decline in equity yields since the late 1980s, followed by an upward trend post-2000, (3) sharp increase in yields during recessions, and (4) procyclical equity yield spreads. The CRRA utility implies a negligible subjective dividend risk premium in our model, and thus equity yield movements are mostly driven by subjective dividend growth expectations. The short-term subjective dividend growth expectation is more volatile and mean-reverting to the less volatile long-run growth expectation, thus the long-term equity yields are more stable. The subjective dividend growth expectations experienced an upward trend starting from the late 1980s and steadily decreased after 2000, which causes the equity yields to have the opposite trend movements. During recessions, growth expectations are exceptionally low, with short-term expectation being much lower than its long-run counterpart, therefore we observe sharp increases in equity yields and procyclical equity yield spreads.

Even if this model implies a negligible ex-ante subjective risk premium, the ex-post realized returns generated from the model still align well with the empirical counterparts. The implied 2-year (10-year) strip futures returns have a correlation of 0.62 (0.51) with the data. In particular, we match two notable features regarding the return variations: long-term claim returns are more volatile and co-move more strongly with the aggregate market returns (Lettau and Wachter, 2007; Van Binsbergen and Koijen, 2017; Gonçalves, 2019). Existing literature reconciles these two facts via discount rate variations, which seems to be inconsistent with recent findings that cast doubts on the relevance of discount rate variations in stock returns (De la O and Myers, 2020; Bordalo et al., 2020). In contrast, our channel for return variations stems from the belief changes over future dividends. The strip futures return equals a forecast error (depending on holding period, but not strip maturity $n$) plus an expectation revision (depending on strip maturity $n$). Belief shocks about fundamentals do not move strip returns evenly, with the effect increasing over the strip maturity because of bigger accumulated effects in longer expectation revisions. As a result, long-term claims are more volatile, and since aggregate market return is simply a portfolio of strip returns across maturities, the same channel also makes long-term claims co-move more with the aggregate market returns. In terms of return predictability, using the data from Giglio et al. (2020), we confirm the findings in van Binsbergen et al. (2012a) that the strip return is predictable by its own lagged equity yield. We further document empirically that the strength of such predictability declines with the strip maturity. Our model quantitatively replicates these two results. Returns are predictable because of dividend forecast errors, yet the expectation revisions whose volatility increases with strip maturity will alleviate the relative importance of forecast errors, rendering weaker predictability.

Given that our model can successfully match time-series dynamics of both equity and bond yields, we next investigate the relationship between these two markets. A well known stylized fact is that the long-term nominal bonds switched from risky assets to safe assets after the late 1990s, that is, the correlation between bond and stock returns changed from positive to negative. The same conclusion can be reached using the correlation between nominal bond yields and real equity yields: the so-called "Fed model". Changes in inflation cyclicality (from countercyclical to procyclical) can help inflation risk premium in equity returns to switch signs (from negative to positive), and hence can potentially explain these stylized facts.[3] However, Duffee (2018a) finds that changes in the *real*

[3] In equilibrium models, inflation risk premium in equity returns can be generated through, for example, "money illusion" (David and Veronesi, 2013), time varying risk aversion(Campbell et al., 2020), or long-run risk (Bansal and Shaliastovich, 2013; Song, 2017). Zhao (2017) provides an alternative approach relying on time-varying impacts of inflation on ambiguity about real growth, instead of inflation cycality. And Zhao (2020a) shows that these equilibrium models relying on changes in inflation cycality can successfully match changes in nominal bond risks, but fail to generate upward-sloping yield curves.

bond returns and stock returns correlation play a significant role in explaining this fact, rather than relying solely on changes in inflation cyclicality. We reconcile these two findings using subjective expectations of GDP growth, inflation, and dividend growth (rather than inflation risk premium). While the inflation real effect, defined as the covariance between subjective inflation and subjective *real* cash-flow growth, explains approximately 38% of the total changes in bond-stock covariance, we find that changes in the *nominal* bond-stock covariance indeed is mainly driven by changes in the *real* bond-stock covariance. Furthermore, such changes are due to stronger co-movements between *real* GDP and *real* dividend growth expectations post-2000, which accounts for around 60% of total bond-stock covariance changes. That is, a new channel driving the bond-stock correlation is that the real bonds provide a better hedge to aggregate real dividend risks after 2000.

Finally, the model can also capture several major aggregate stock market puzzles: (1) the model-implied time series of aggregate dividend yield and market return are close to the data, (2) the unconditional mean and volatility of model-implied dividend yield and market return are comparable to data, and the model-implied dividend yield is as persistent as in the data, and (3) market returns are predictable by the aggregate dividend yield. During bad times/recessions, expected cash flow growth is low (dividend yield is high), but higher than expected cash flow realizations (or positive forecast errors) likely follow, and hence higher future returns are expected.

### Related literature

This paper is motivated by some new evidences in the empirical asset pricing literature, for example, the importance of subjective expectation in equity markets (De la O and Myers, 2020; Bordalo et al., 2020) and in bond markets (Froot, 1989; Piazzesi et al., 2015; Cieslak, 2018; Duffee, 2018b), the term structure of equity yields (van Binsbergen et al., 2012a, 2013; Van Binsbergen and Koijen, 2017; van Binsbergen, 2020; Giglio et al., 2020), and the relationship between stock and bond markets (Li, 2002; Fleming et al., 2003; Campbell et al., 2017; Duffee, 2018a). To our knowledge, this is the first equilibrium pricing model that explains the jointly historical dynamics of the term structure of equity and bond yields and consistent with these recent empirical findings.

This paper is related to a large equilibrium asset pricing literature focusing on (1) rational expectation and aggregate stock market puzzles (Campbell and Cochrane, 1999; Bansal and Yaron, 2004; Barro, 2006; Gabaix, 2012; Gourio, 2012; ALBUQUERQUE et al., 2016), (2) rational expectation and bond markets (Piazzesi and Schneider, 2007; Wachter, 2006; Bansal and Shaliastovich, 2013), (3) the link between stock and bond markets (David and Veronesi, 2013; Campbell et al., 2020; Song, 2017; Zhao, 2017), (4) risk premium and the term structure of equity returns (Hasler and Marfe, 2016; Bansal et al., 2020; Breugem et al., 2020; Gonçalves, 2020b; Li and Xu, 2020), and (5) subjective beliefs in equity and bond markets (Barberis et al., 2015; Adam et al., 2016; Nagel and Xu, 2019; Zhao, 2020b). We extend the literature by providing a unified framework of bond and equity pricing under subjective expectations. Finally, this paper is related to a number of papers that study the implications of ambiguity and robustness for finance and macroeconomics.[4]

The paper continues as follows. Section 2 outlines the theoretical framework. Section 3 describes the data, calibration and estimation. Section 4 shows the empirical results. Section 5 provides some robustness analysis. Section 6 provides concluding comments.

## 2 Theoretical Framework

In this section we introduce our settings for the endowment economy, expectation formation and preference. We describe how agents form their subjective beliefs over economic fundamentals, and how equilibrium bond and equity prices are consistent with those beliefs. Note that we do not derive implications under the physical or statistical measure, as our model involves latent states governing the physical dynamics. The model and theoretical results involve a fair amount of derivations, to ease the presentation, we leave all solution details to Appendix A.

[4]Recent papers that study the multiple-priors preference and its applications include Epstein and Schneider (2007), Ilut (2012), Ilut and Schneider (2014), Ulrich (2013) , Zhao (2017), Bianchi et al. (2017), Ilut et al. (2018), and Zhao (2020a), among many others. Papers on robustness applications include Anderson, Hansen, and Sargent (2003), Cagetti, Hansen, Sargent, and Williams (2002), Hansen (2007), and Hansen and Sargent (2001, 2010). For a detailed survey, see Epstein and Schneider (2010).

## 2.1 Subjective beliefs on growth and inflation

We assume output (as endowment) growth and inflation are given exogenously, and the expectation formation of output growth and inflation are the same as in Zhao (2020b). In the data, the four components of GDP – investment spending, net exports, government spending, and consumption (PCE) – do not move in lockstep with each other. In fact, their levels of volatility greatly differ. PCE is very stable and varies less with the business cycle. In contrast, the other three components vary greatly during economic contractions and expansions. Similarly for inflation, the core inflation is much more stable than other inflation components. For this reason, it is assumed that the agent faces two types of learnings: learning about the long-run mean and learning about the short-run deviation from the mean. The agent learns about the long-run mean only from the stable components, and they learn about the short-run deviation by using only the transitory components.

Specifically, output growth and inflation can be decomposed into the following (account identity):

$$\Delta g_{t+1} = \Delta g^*_{t+1} + Gap^g_{t+1}$$
$$\pi_{t+1} = \pi^*_{t+1} + Gap^\pi_{t+1},$$

where $\Delta g_{t+1}$ is the growth rate of real output and $\pi_t$ is inflation. The stable components $\Delta g^*_{t+1}$ and $\pi^*_{t+1}$ are PCE growth and core inflation. $Gap^g_{t+1}$ and $Gap^\pi_{t+1}$ are the volatile components (GDP growth excluding the PCE and GDP deflator excluding the core inflation). Both real consumption growth and core inflation follow the i.i.d. laws of motion as follows:

$$\Delta g^*_{t+1} = \mu^*_c + \sigma_c \varepsilon^*_{c,t+1}$$
$$\pi^*_{t+1} = \mu^*_\pi + \sigma_\pi \varepsilon^*_{\pi,t+1},$$

where $\varepsilon^*_{c,t+1}$ and $\varepsilon^*_{\pi,t+1}$ are the i.i.d. normal shocks. The representative agent knows that both $\Delta g^*_{t+1}$ and $\pi^*_{t+1}$ are i.i.d., and they also know $\sigma_c$ and $\sigma_\pi$ but not the long-run mean $\mu^*_c$ and $\mu^*_\pi$. The agent forms expectations about $\mu_c^*$ and $\mu_\pi^*$, based on the constant-gain learning scheme proposed by Nagel and Xu (2019), where agent puts more weights on recent observations when updating their posteriors, instead of equal weights in Bayesian updating. And the posteriors are as follows:

$$\tilde{\mu}_{c,t}^* = \tilde{\mu}_{c,t-1}^* + \upsilon_c^* \left(\Delta g_t^* - \tilde{\mu}_{c,t-1}^*\right)$$
$$\tilde{\mu}_{\pi,t}^* = \tilde{\mu}_{\pi,t-1}^* + \upsilon_\pi^* \left(\pi_t^* - \tilde{\mu}_{\pi,t-1}^*\right),$$

where $\upsilon_c^*$ and $\upsilon_\pi^*$ are learning gains, and $(1-\upsilon_i)^j$ represents a geometric weight on each past observation.

Both $Gap_{t+1}^g$ and $Gap_{t+1}^\pi$ are assumed to contain a latent stationary component:

$$Gap_{t+1}^g = x_{c,t+1} + \sigma_c^{gap} \varepsilon_{c,t+1}^{gap}$$
$$Gap_{t+1}^\pi = x_{\pi,t+1} + \sigma_\pi^{gap} \varepsilon_{\pi,t+1}^{gap}$$
$$x_{c,t+1} = \rho_c x_{c,t} + \sigma_c^x \varepsilon_{c,t+1}^x$$
$$x_{\pi,t+1} = \rho_\pi x_{\pi,t} + \sigma_\pi^x \varepsilon_{\pi,t+1}^x,$$

where $\varepsilon_{c,t+1}^{gap}$, $\varepsilon_{\pi,t+1}^{gap}$, $\varepsilon_{c,t+1}^x$, and $\varepsilon_{\pi,t+1}^x$ are i.i.d. normal shocks. The representative agent knows all of the parameters but not $x_{c,t+1}$ and $x_{\pi,t+1}$. They form expectations about $x_{c,t+1}$ and $x_{\pi,t+1}$, based on the same learning scheme as for the long-run mean but with potentially different geometric weighting parameters, $\upsilon_c^{gap}$ and $\upsilon_\pi^{gap}$. The posteriors are given by the following:

$$\tilde{x}_{c,t} = \rho_c \tilde{x}_{c,t-1} + \upsilon_c^{gap} \left(Gap_t^g - \rho_c \tilde{x}_{c,t-1}\right)$$
$$\tilde{x}_{\pi,t} = \rho_\pi \tilde{x}_{\pi,t-1} + \upsilon_\pi^{gap} \left(Gap_t^\pi - \rho_\pi \tilde{x}_{\pi,t-1}\right).$$

The predictive distributions and more details about the learning can be found in Zhao (2020b).

## 2.2 A model of dividend expectation formation

In this subsection we describe how the representative agent forms beliefs over future aggregate dividends. We begin with the specification of a two-component dividend model under the objective measure. The logarithm of aggregate real dividend can be decomposed as

$$d_t = d_t^l + d_t^s, \tag{1}$$

where $d_t = \log D_t$ is the log total real dividend and $d_t^l = \log D_t^l$ measures the log real dividend from the sector of long-duration stocks. As a result, $d_t^s = d_t - d_t^l$ quantifies the share of the long-duration dividend in the aggregate dividend. Following Menzly et al. (2004); Lettau and Wachter (2007); Cochrane et al. (2008), we assume that such a deviation is stationary and the data generating process follows

$$d_t^s = x_{d,t} + \sigma_d \epsilon_{x,t}^*, \tag{2}$$

$$x_{d,t+1} = \rho_x x_{d,t} + \sigma_x \epsilon_{x,t+1}, \tag{3}$$

where $\epsilon_{x,t}^*$ and $\epsilon_{x,t}$ are i.i.d. shocks following the standard normal distribution. The specification captures the simple idea that dividends from long-duration stocks cannot permanently deviate from the aggregate dividend, and information from such deviation $d_t^s$ is helpful to infer future dividend growth. When the share of long-duration dividend is temporarily higher, the aggregate dividend will have to increase more.[5]

To complete the model under the objective measure and introduce the economy-wide endowment

[5]While the decomposition into sectors with different equity duration is a natural choice, our model is also motivated by the empirical findings that such a cross-section is informative on the aggregate market. For example, Kelly and Pruitt (2013) find that a factor extracted from the cross-section of book-to-market can predict the aggregate returns. Li and Wang (2018) document that the ratio of long- to short-term dividend prices also predicts market returns.

risk, we specify the following process for $d_t^l$

$$d_t^l - \lambda y_t = \mu_{d,t} + \sigma_d^* \epsilon_{d,t}^*, \tag{4}$$

$$\mu_{d,t+1} = \mu_{d,t} + \sigma_\mu \epsilon_{d,t+1}, \tag{5}$$

where $\epsilon_{d,t}^*$ and $\epsilon_{d,t}$ are i.i.d. shocks following the standard normal distribution. The log dividend level of long-duration sector is tied to the aggregate endowment through a leverage parameter $\lambda$. It is important to note that we do not take a stand on whether there exists a cointegrating relation between $d_t^l$ and the log real GDP $y_t$, because the leverage parameter may not properly cancel out the stochastic trends of two series. We use this specification solely as a parsimonious way to incorporate aggregate endowment risk, thus $\mu_{d,t}$ will be specified as a unit-root process to capture the potential stochastic trend in $d_t^l - \lambda y_t$.

We depart from the full-information rational expectation framework by assuming that $\mu_{d,t}$ and $x_{d,t}$ are latent and the agent has to infer them from the past realized dividend and output data. Specifically, the representative agent has full knowledge over the model parameters but she applies the constant-gain learning scheme to infer $\mu_{d,t}$ and $x_{d,t}$. Denote agent's subjective beliefs as $\tilde{\mu}_{d,t}$ and $\tilde{x}_{d,t}$, their dynamics thereby follow

$$\tilde{\mu}_{d,t+1} = \tilde{\mu}_{d,t} + \nu_\mu(d_{t+1}^l - \lambda y_{t+1} - \tilde{\mu}_{d,t}), \tag{6}$$

$$\tilde{x}_{d,t+1} = \rho_x \tilde{x}_{d,t} + \nu_x(d_{t+1}^s - \rho_d \tilde{x}_{d,t}). \tag{7}$$

The belief over the next-period dividend growth is then

$$\tilde{E}_t^* \Delta d_{t+1} = \underbrace{\lambda(\tilde{\mu}_{c,t}^* + \rho_c \tilde{x}_{c,t})}_{RGDP} + \underbrace{(\rho_x - 1)\tilde{x}_{d,t} + (\nu_x - 1)(d_t^s - \rho_x \tilde{x}_{d,t-1}) + (\nu_\mu - 1)(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1})}_{Div-specific}. \tag{8}$$

Several interesting features emerge from (8). First, subjective real output growth ($RGDP$) drives part of the beliefs over future dividend growth. When there is bad news regarding the economy-wide growth, the perceived dividend growth is also lower, akin to the standard implication from the literature on learning from macro (see e.g., Johannes et al., 2016; Nagel and Xu, 2019). Since we model dividend as a levered claim to aggregate endowment, the higher the leverage parameter $\lambda$, the stronger the impact from the endowment growth. Second, state learning from past dividend levels generates a dividend-specific component ($Div - specific$). When the learning gain $\nu_\mu, \nu_x$ are both unitary, the agent ignores all historical dividend data and treat current observations as the belief on the latent states. Nevertheless, once the agent relies on historical data to infer the states ($\nu_\mu, \nu_x \neq 1$), how current realizations deviate from the historical data is also informative for the future. In particular, when both $\nu_\mu$ and $\nu_x$ are smaller than 1, which is our case of interest, higher than usual realizations (e.g., $d_t^s > \rho_x \tilde{x}_{d,t-1}$) will produce downward estimate for future dividend growth.

Our model differs from the recent attempt towards modeling beliefs in stock markets. Jagannathan and Liu (2019), Nagel and Xu (2019) and De la O and Myers (2020) incorporate earnings information when modeling the beliefs over dividend growth, in addition to the standard aggregate dividend or growth data. Despite an intuitive strategy (see e.g., Campbell and Shiller, 1988a), the fundamental asset pricing theory still rests on the dividends and it is not uncontroversial on how to incorporate earnings information (see e.g., Boudoukh et al., 2007). Importantly, our methodology exploits information from dividends in different sectors so we follow a quite different path. On the other hand, Bordalo et al. (2020) and Guo and Wachter (2019) assess which kind of beliefs may be empirically consistent with leading asset pricing puzzles, yet they are silent on how those beliefs are generated from the observables. We clarify a new expectation formation that quantitatively reconciles many asset pricing puzzles in bond and stock markets, as will be clear from our subsequent empirical analysis.[6]

[6] A body of research in behavioral finance such as Barberis et al. (2015); Cassella and Gulen (2018); Greenwood and Shleifer (2014) suggest that investors may extrapolate past stock returns or fundamentals, and these irrational beliefs are consistent with several asset pricing anomalies. However, to the best of our knowledge, none of the existing papers can reconcile all the asset pricing puzzles covered in Section 4 in a unified framework.

## 2.3 Preference

Our decision framework entertains investor's fear over model misspecification. The representative agent has a recursive multiple-priors preference (see e.g., Epstein and Schneider, 2003)

$$V_t(C_t) = \min_{p_t \in \mathscr{P}_t} \mathbb{E}^{p_t}[U(C_t) + \beta V_{t+1}(C_{t+1})], \tag{9}$$

where $\mathscr{P}_t$ denotes the set of alternative models (probability measures) and we impose a CRRA utility function $U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$. $\beta$ stands for agent's subjective discount factor and $\gamma$ is the risk-aversion coefficient. Agent is ambiguous about the real endowment growth, inflation, and real dividend growth. The set of their alternative measures is generated by different mean growth rates around respective reference mean values. We assume that the reference models are the posterior distribution obtained from agent's learning over the real output, inflation and real dividend.

According to (9), the agent evaluates future prospects under the worst-case measure. More explicitly, the agent will select the lowest real GDP growth and aggregate dividend growth forecasts when forming lifetime consumption-portfolio choices (see e.g., Zhao, 2020a).[7] However, the worst-case inflation forecasts depend on the correlation between inflation expectations and real output growth, which is widely documented to be negative before 2000 and positive afterwards (see e.g, Piazzesi and Schneider, 2007; Song, 2017; Zhao, 2020a). As a result, the worst-case inflation forecast before (after) 2000 is the highest (lowest) possible forecasts from the alternative models. Following Zhao (2020a),

[7] The worst-case distortion for dividend growth belief rests on the assumption that dividend shocks are positively correlated with endowment shocks. However, when learning from past data as described in the previous subsection, we do not ask the agent to consider such correlation. This setting greatly simplifies our analysis as it avoids additional parameters that are hard to pin down facing correlated learning. Croce et al. (2015) use a similar setting where agent ignores some shock correlations when pricing assets. Given our current focus is to evaluate whether an explicit way of forming dividend expectation is consistent with leading asset pricing puzzles, we leave the extension of incorporting full correlation structure to future research.

the processes for the ambiguity over real GDP growth and inflation are specified as

$$a_{c,t} = \mu_{ac} + a_{c,t-1} + \sigma_{ac}\epsilon_{ac,t}, \tag{10}$$

$$a_{\pi,t} = \mu_{a\pi} + a_{\pi,t-1} + \sigma_{a\pi}\epsilon_{a\pi,t}. \tag{11}$$

Then the worst-case beliefs over real output growth and inflation are

$$\tilde{E}_t \Delta g_{t+1} = \tilde{\mu}^*_{c,t} + \rho_c \tilde{x}_{c,t} - a_{c,t},$$

$$\tilde{E}_t \pi_{t+1} = \tilde{\mu}^*_{\pi,t} + \rho_\pi \tilde{x}_{\pi,t} - \mathbb{I}_{t\geq 2000} \times a_{\pi,t},$$

where the time dummy $\mathbb{I}_{t\geq 2000}$ takes the value of 1 after 2000 and -1 otherwise.

On the other hand, the agent's worst-case belief over the aggregate real dividend growth is

$$\begin{aligned}\tilde{E}_t \Delta d_{t+1} = & \underbrace{\lambda(\tilde{\mu}^*_{c,t} + \rho_c \tilde{x}_{c,t})}_{RGDP} + \underbrace{(\rho_x - 1)\tilde{x}_{d,t} + (\nu_x - 1)(d^s_t - \rho_x \tilde{x}_{d,t-1}) + (\nu_\mu - 1)(d^l_t - \lambda y_t - \tilde{\mu}_{d,t-1})}_{Div-specific} \\ & \underbrace{-\lambda a_{c,t} - a_{d,t}}_{Ambiguity},\end{aligned} \tag{12}$$

where the ambiguity over total dividend growth can be decomposed to the parts related to real output growth ($\lambda a_{c,t}$) and dividend-specific growth ($a_{d,t}$). We assume that the dividend-specific ambiguity follows

$$a_{d,t+1} = \mu_{ad} + \rho_{ad} a_{d,t} + \epsilon_{ad,t+1}. \tag{13}$$

Next section will describe how we construct empirical measures for the ambiguity processes (10), (11) and (13).

## 2.4 Pricing dividend strips and Treasury bonds

The log nominal pricing kernel implied from the CRRA utility is

$$m_{t+1}^{\$} = \log\beta - \gamma\Delta g_{t+1} - \pi_{t+1}. \tag{14}$$

We derive the equilibrium price for $n-$period dividend strip, defined as the claim to the $n-$period ahead aggregate nominal dividend. Specifically, the equity *spot* yield for the $n-$period dividend strip is

$$ey_t^{(n)} = \frac{1}{n}(d_t^{\$} - p_t^{(n)}), \tag{15}$$

where $p_t^{(n)}$ is the log strip price and $d_t^{\$}$ is the log nominal aggregate dividend. Beginning from $n = 1$, the time-$t$ equilibrium price of one-period dividend strip is

$$P_t^{(1)} = \log\tilde{E}_t[M_{t+1}^{\$}D_{t+1}^{\$}], \tag{16}$$

where the conditional expectation is taken under the worst-case belief. Dividing both sides by $D_t^{\$}$ and with conditional log normality, we obtain the 1-period equity spot yield

$$\begin{aligned} ey_t^{(1)} &= A_e^{(1)} - (\lambda-\gamma)(\tilde{\mu}_{c,t}^{*} + \rho_c\tilde{x}_{c,t} - a_{c,t}) + (1-\rho_x)\tilde{x}_{d,t} \\ &\quad - (\nu_\mu - 1)(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1}) - (\nu_x - 1)(d_t^s - \rho_x\tilde{x}_{d,t-1}) + a_{d,t}, \end{aligned} \tag{17}$$

with constant $A_e^{(1)}$ given in Appendix A. Similarly, the price of $n-$period dividend strip is

$$P_t^{(n)} = \tilde{E}_t[M_{t+1}^{\$}P_{t+1}^{(n-1)}]. \tag{18}$$

Solving the iterations forward, for the $n-$period equity spot yield we obtain

$$\begin{aligned} ey_t^{(n)} = &\frac{A_e^{(n)}}{n} - (\lambda-\gamma)(\tilde{\mu}_{c,t}^* + \frac{1-\rho_c^n}{n(1-\rho_c)}\rho_c\tilde{x}_{c,t} - a_{c,t}) + \frac{1-\rho_x^n}{n}\tilde{x}_{d,t} \\ &- \frac{\nu_\mu - 1}{n}(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1}) - \frac{\nu_x - 1}{n}(d_t^s - \rho_x\tilde{x}_{d,t-1}) + \frac{1-\rho_{ad}^n}{n(1-\rho_{ad})}a_{d,t}. \end{aligned} \tag{19}$$

Then we derive equilibrium prices for nominal bonds. The time-$t$ price of $n-$period nominal discount bond satisfies the recursion

$$P_{b,t}^{(n)} = \tilde{E}_t[M_{t+1}^{\$}P_{b,t+1}^{(n-1)}]. \tag{20}$$

Conjecturing that the log nominal bond price is linear in states, we solve out the $n-$period nominal bond yield as

$$y_t^{(n)} = \frac{A_b^{(n)}}{n} + \gamma(\tilde{\mu}_{c,t}^* + \frac{1-\rho_c^n}{n(1-\rho_c)}\rho_c\tilde{x}_{c,t} - a_{c,t}) + (\tilde{\mu}_{\pi,t}^* + \frac{1-\rho_\pi^n}{n(1-\rho_\pi)}\rho_\pi\tilde{x}_{\pi,t} - a_{\pi,t}). \tag{21}$$

To better connect equity and nominal bond yields, we rewrite (19) as

$$ey_t^{(n)} = \frac{A_e^{(n)} - A_b^{(n)}}{n} + y_t^{(n)} - \frac{1}{n}\tilde{E}_t\Delta d_{t+1:t+n}^{\$}, \tag{22}$$

where $\frac{1}{n}\tilde{E}_t\Delta d_{t+1:t+n}^{\$}$ is the subjective belief over the life-time nominal dividend growth for the $n-$period dividend strip. Intuitively, higher bond yield pushes up the equity yield since bonds and stocks are competing assets for the representative investor. Higher subjective dividend growth instead lowers the equity yield because the strip price will be higher. Subtracting the nominal bond yield from both sides, we obtain the so-called *forward* equity yield

$$ef_t^{(n)} = \frac{A_e^{(n)} - A_b^{(n)}}{n} - \frac{1}{n}\tilde{E}_t\Delta d_{t+1:t+n}^{\$} = \theta_t^{(n)} - g_t^{(n)}, \tag{23}$$

where the second equality follows Equation (6) in van Binsbergen et al. (2013) by disentangling the forward equity yield into the risk premium ($\theta_t^{(n)}$) and growth components ($g_t^{(n)}$). The risk premium component in our model is constant, thus the time-variations in equity forward yields are entirely driven by the subjective beliefs over dividend growth.

# 3 Data, Calibration and Estimation

## 3.1 Data

We collect firm-level quarterly dividends and related accounting data from the CRSP/Compustat Merged Database for all firms listed on the NYSE, NASDAQ and AMEX. Following De la O and Myers (2020) and Giglio et al. (2020), we focus on cash dividends. As the benchmark measure of the equity duration, we consider firm-level long-term earnings growth median forecasts (LTG) with data available from the IBES unadjusted summary file.[8] La Porta (1996) and Gormsen and Lazarus (2020) show that such a model-free measure has a direct interpretation as the equity duration. Since equity duration is defined as the weighted sum of time with the weight given by expected cash-flow, higher long-term expected cash-flows compared with today naturally translate to higher duration.[9] We calculate the dividend from the long-duration sector as the following. At the end of each quarter, we assign all dividend-paying firms into either of the two groups based on firms' LTG forecasts in the previous quarter. If a firm's LTG is above or equal to the cross-sectional median of the LTG of all dividend paying firms, then it is assigned to the long-duration group. Otherwise it is allocated to the short-duration group. Sample split at the median ensures similar group sizes so that dividends from one sector are unlikely to dominate the other permanently, consistent with our modeling strategy in (2) and (3). Within each quarter we then sum all dividends from the long-duration and short-duration sectors respectively. We deflate the obtained two nominal dividend series using the GDP

[8]While IBES data are available at the monthly frequency, we transform them to quarterly frequency by taking the end-of-quarter readings. Results from using the within-quarter average are almost identical.

[9]Most existing measures of equity duration (see e.g., Dechow et al., 2004; Weber, 2018; Gonçalves, 2020a) require formal econometric modeling and estimation. We do not take a stand on such modeling issues and prefer to use the model-free duration. However, in Section 5.2 we run robustness checks by using these measures and results are quantitatively similar.

deflator and take a four-quarter trailing summation to remove their seasonality. By construction, the sum of two dividend series will simply be the real aggregate dividend. The equity duration data is available from 1981Q3, hence the deseasonalized dividend series ranges from 1982Q4 to 2019Q4. We use the initial 5-year training period for agent's learning and we start our empirical analysis from 1987Q4.

Now we explain how to retrieve the data of subjective dividend growth and the ambiguity. First, we extend the data of one-year subjective aggregate dividend growth constructed by De la O and Myers (2020) to 2019Q4, again using the IBES unadjusted summary file.[10] We show in the next subsection that this time-series will help us pin down the dividend learning gains. Second, the ambiguity measure over real GDP and inflation are standard and based on the survey data from the Philadelphia Fed's survey of professional forecasters (SPF) (see e.g., Ilut and Schneider, 2014; Drechsler, 2013; Zhao, 2017). For each quarter, we calculate the ambiguity over real GDP and inflation by dividing the interquartile range of forecasts by 2.

A new variable that we need to obtain from the survey data is the real dividend-specific ambiguity $a_{d,t}$ . From Equation (12), as long as the ambiguity over aggregate real cash-flows is empirically available, we can back out $a_{d,t}$ after removing the part attributed to the real GDP ambiguity. We appeal to firm-level earnings survey data to calculate such ambiguity over aggregate real cash-flows. Given that the IBES summary file does not provide the upper and lower quartiles of analyst forecasts for each firm, we retrieve them from the IBES unadjusted detail file.[11] Specifically for each firm and quarter, we first collect individual analyst forecasts of future *nominal* earning per share (EPS) for multiple forecasting horizons ranging from one fiscal year to five fiscal years. Second, for each forecasting horizon, we obtain the upper and lower quartiles of analyst forecasts. Third, we apply linear interpolations to obtain the upper and lower quartiles of forecasts at the one-year horizon,

[10] Their original data spans the period from 2003Q1 to 2015Q3. We find that the correlation between their data and our replicated series has a correlation coefficient of 0.92 over the same sample.

[11] We do not use the dividend forecast in the IBES detail file when constructing the ambiguity measure, primarily because the dividend forecast is only available after 2003 and this will shorten substantially the period for our analysis. Also, the average number of analysts providing dividend estimates in the IBES detail file is much smaller than that for the earnings, which will confound our ambiguity measurement.

similar to the method in De la O and Myers (2020). After multiplying those interpolated forecasts with the shares outstanding in each quarter and aggregate over all stocks, we obtain the 25th and 75th percentiles of predicted earnings levels for the aggregate market. Ambiguity over aggregate *real* cash-flows is then calculated as one-half of the log difference between these quartiles, minus the inflation ambiguity.[12]

In spite of using the earnings survey data when estimating ambiguity over real dividend, we show that the obtained measure is empirically sensible. Figure IA.2 in in Appendix B demonstrates the reasonable cyclicality of real dividend ambiguity within our sample, and we further find that it has a correlation of 0.62 with the ambiguity over real GDP growth constructed from the SPF. Meanwhile, Ilut and Schneider (2014) suggest that valid empirical measure of ambiguity should not exceed twice the volatility of the forecasted time-series itself (see their Section III.B). In compliance with their ambiguity bound, we find that the sample average of ambiguity over annual real aggregate dividend growth is around 4% while the volatility of realized annual real dividend growth is 7%.

Turning to the data on real output growth and GDP deflator, we collect them from the Bureau of Economic Analysis (BEA). Real personal consumption expenditure (PCE) and core inflation, i.e., the stable components of GDP and total inflation, are also obtained from the BEA. Since the learning gain on growth and inflation can be very small, as shown by Malmendier and Nagel (2016) and Nagel and Xu (2019), we need a long training sample to form reasonable beliefs. Thus we allow the agent to learn these quantities using the data back to 1959Q1, with learning gain parameters from Zhao (2020b). The end-of-quarter zero-coupon nominal bond yields are from the daily dataset of Gürkaynak et al. (2007).

Since we will draw extensive discussions on the equity term structure, we collect the full term structure of dividend strip yields from Giglio et al. (2020) with available data till 2016Q3. Using a large cross-section of US stock returns, they estimate an affine model of equity prices and derive

[12] Note that this way of deflating the ambiguity over nominal cash-flows rests on the assumption that when agent delivers a worst-case forecast for nominal cash-flows, she also simultaneously delivers the worst-case inflation forecast. That is, the agent is internally-consistent when forecasting different time-series. This is a suitable assumption in the absence of survey data on real cash-flows.

the strip yields for the aggregate market. Their method not only replicates accurately the dividend futures data used in recent studies such as van Binsbergen et al. (2013); Van Binsbergen and Koijen (2017), but also extends substantially the length of the data.[13] Longer sample creates an ideal laboratory to study the equity term structure dynamics over the business cycles.

### 3.2 Calibration and Estimation

Some of the model parameters coincide with those used in Zhao (2020b) so we adopt the same values here and they can be found in Panel A and C of Table 1.[14] The dividend model introduces additional parameters that we need to pin down. Following the literature, we set the leverage parameter $\lambda$ to 3. We estimate the AR(1) coefficient $\rho_x$ in (3) and volatility parameters from (2) to (5) from the related dividend series using the maximum likelihood via the Kalman filter, as the model involves latent states.[15] Regarding the dividend learning gains $\nu_\mu$ and $\nu_d$, we estimate them from the consensus forecast of one-year subjective dividend growth of the S&P 500, because the survey data is very informative on the subjective learning gain (see e.g., Malmendier and Nagel, 2016). Following the literature (see e.g., Ulrich, 2013), we assume that the median forecast in the data corresponds to the forecast from our reference model, i.e., the posterior distribution from agent's learning in Section 2.1 and 2.2.[16] Since the dividend survey data is only available after 2003Q1, we estimate dividend parameters based on the sample from 2003Q1 to 2019Q4.

To evaluate the performance of fitting the dividend expectation data, Figure IA.1 in Appendix B shows that the model-implied 1-year subjective dividend growth tracks well its empirical counterpart. Their unconditional correlation reaches almost 0.8, and even though we do not use 2-year

[13] Dividend futures data usually starts from 2003 and hence is not suitable for our study. van Binsbergen et al. (2012b) use option returns to assess the equity term structure with data going back to 1996. However, option-based data are only available for short maturities up to two-year, and Boguth et al. (2019) find that noises from highly levered option positions may significantly contaminate the inference from option prices.

[14] Zhao (2020a) shows that the term structure of inflation ambiguity experiences a structural change around 2000, thus Zhao (2020b) estimates related parameters governing inflation and GDP ambiguity separately for each subsample.

[15] As shown in Appendix A, even though we have two volatility parameters for each dividend series, their effect is subsumed by only one volatility parameter defined as $\sigma_{d\mu} = \sqrt{\sigma_d^{*2} + \sigma_\mu^2}$ and $\sigma_{dx} = \sqrt{\sigma_d^2 + \sigma_x^2}$, respectively.

[16] We try to match the time-series of the subjective growth data, so we attach a normally distributed measurement error to the data and run the full-sample maximum likelihood estimation. Appendix B gives more details.

survey growth in the estimation, the model-implied quantity also matches closely the data. Meanwhile, Bordalo et al. (2020) and Bordalo et al. (2019) document agent's overreaction based on the IBES survey data. In Table IA.1, we follow Coibion and Gorodnichenko (2015) and repeat their exercises. We find that our model-implied beliefs do mimick similar overreacting behavior (see more details therein). Putting together, the estimation results favors our model in capturing salient features of survey expectation data.

Finally, we obtain $\mu_{ad}, \rho_{ad}, \sigma_{ad}$ by matching the simulated moments with the mean, volatility, and AR(1) coefficient from the data of $a_{d,t}$. To be consistent with our subsequent analysis, we target the sample between 1987Q4 and 2019Q4 when matching these moments. Unlike the parameters for the ambiguity over real GDP or inflation., we use the same parameter values before or after 2000.

**Table 1: Model parameters**

The table reports model parameter values. Panel A largely follows Zhao (2020b) and displays the risk aversion coefficient $\gamma$, learning gains for real GDP and inflation and their sub-components $\nu_c^*, \nu_\pi^*, \nu_c^{gap}, \nu_\pi^{gap}$, the AR(1) coefficients for the growth and inflation cycles $\rho_c, \rho_\pi$, and the volatility parameters characterizing the trend-cycle components. Subjective discount factor $\beta$ is adjusted to help match the mean of equity yields. Panel B reports the parameters governing the dividend processes, determined following the methodology in Section 3.2. They include the leverage parameter, AR(1) coefficient of dividend share, dividend learning gains and volatility parameters. Panel C reports the ambiguity-related paramters, among those for the real GDP and inflation we follow Zhao (2020b) by specifying different values before and after 2000. Parameters for the dividend-specific ambiguity process are fixed and determined by matching moments of the empirical counterpart from 1987Q4 to 2019Q4.

| | Panel A: parameters for preference, endowment and inflation processes | | | | | | |
|---|---|---|---|---|---|---|---|
| | $\gamma$ | $\beta$ | $\nu_c^*$ | $\nu_\pi^*$ | $\nu_c^{gap}$ | $\nu_\pi^{gap}$ | $\rho_c$ |
| | 4 | 1.0215 | 0.015 | 0.05 | 0.12 | 0.2 | 0.92 |
| | $\rho_\pi$ | $\sigma_c$ | $\sigma_\pi$ | $\sigma_{cx}$ | $\sigma_{\pi x}$ | | |
| | 0.98 | 0.42 | 0.33 | 0.64 | 0.35 | | |
| | Panel B: parameters for dividend processes | | | | | | |
| | $\lambda$ | $\rho_x$ | $\nu_\mu$ | $\nu_d$ | $\sigma_{d\mu}$ | $\sigma_{dx}$ | |
| | 3 | 0.925 | 0.179 | 0.497 | 5.75 | 4.73 | |
| | Panel C: parameters for ambiguity processes | | | | | | |
| | $\mu_{ac}$ | $\mu_{a\pi}$ | $\mu_{ad}$ | $\sigma_{ac}$ | $\sigma_{a\pi}$ | $\rho_{ad}$ | $\sigma_{ad}$ |
| Before 2000 | -0.0044 | 0.0029 | 0.0089 | 0.012 | 0.010 | 0.988 | 0.063 |
| After 2000 | -0.0043 | -0.0063 | 0.0089 | 0.009 | 0.015 | 0.988 | 0.063 |

# 4 Empirical Results

After matching the time-series dynamics of dividend survey data, in this section we explore whether our general equilibrium model accounts for leading asset pricing puzzles in equity and bond markets. Since our model nests Zhao (2020b), it naturally accounts for key stylized facts in the bond markets. We thus focus more on the equity market by first studying how our model reconciles the equity term structure. Then we discuss whether the model captures the time-varying bond-stock correlation as usually observed in the data. Finally, we assess model performance in terms of matching several well-known puzzles in the aggregate stock market.

## 4.1 Term structure of equity yields

We start with the business cycle dynamics of the equity term structure. The left column of Figure 1 plots the equity spot yields defined in (19) together with the data from Giglio et al. (2020). The right column plots the corresponding forward equity yields defined in (23), after deducting either the model-implied nominal bond yields or the bond yield data of Gürkaynak et al. (2007). Roughly speaking, the model-implied yields closely track the movements in the data over the entire sample. Comparing the results for 1-year and 10-year, our model can match both the volatile 1-year equity yields and the less volatile 10-year yields. The model also generates a secular decline in equity yields since the late 1980s, followed by an upward trend post-2000, and replicates the equity yield spikes during the recession periods in 1990s and around 2008. Results are similar when looking at the forward equity yields. It is worthwhile to mention that in order to fit the forward yield data, one requires a reasonable model describing the full dynamics of both bond and equity yield curves. The last row of Figure 1 plots the slopes of equity term structure, defined as the difference between 10-year and 1-year yields. The time-series plot suggests that model-implied slopes also co-move tightly with the data. Table 2 reports their unconditional mean, volatility, and correlation. We find that the model-implied moments are close to the empirical counterparts and the correlation is also high. For example, the 1-year (10-year) model-implied spot yields have a correlation coefficient of 0.67 (0.83) with the data. Even though the model undershoots the average equity yield slopes, the correlation between slopes can reach 0.55. Overall, the evidence favors the model in terms of fitting the term structure of equity spot and forward yields.

**Figure 1: Term structure of equity spot and forward yields**

The figure compares the model-implied spot (left panel) and forward yields (right panel) of dividend strips with the data from Giglio et al. (2020). The forward yields in the data are computed by subtracting the spot yields with the maturity-matched zero-coupon nominal Treasury bond yields. The last row plots the spread between 10-year and 1-year spot or forward yields. Shaded areas correspond to NBER recessions. Model-implied quantities are from 1987Q4 to 2019Q4, while the data is only available till 2016Q3. All numbers are in annualized percentage terms.

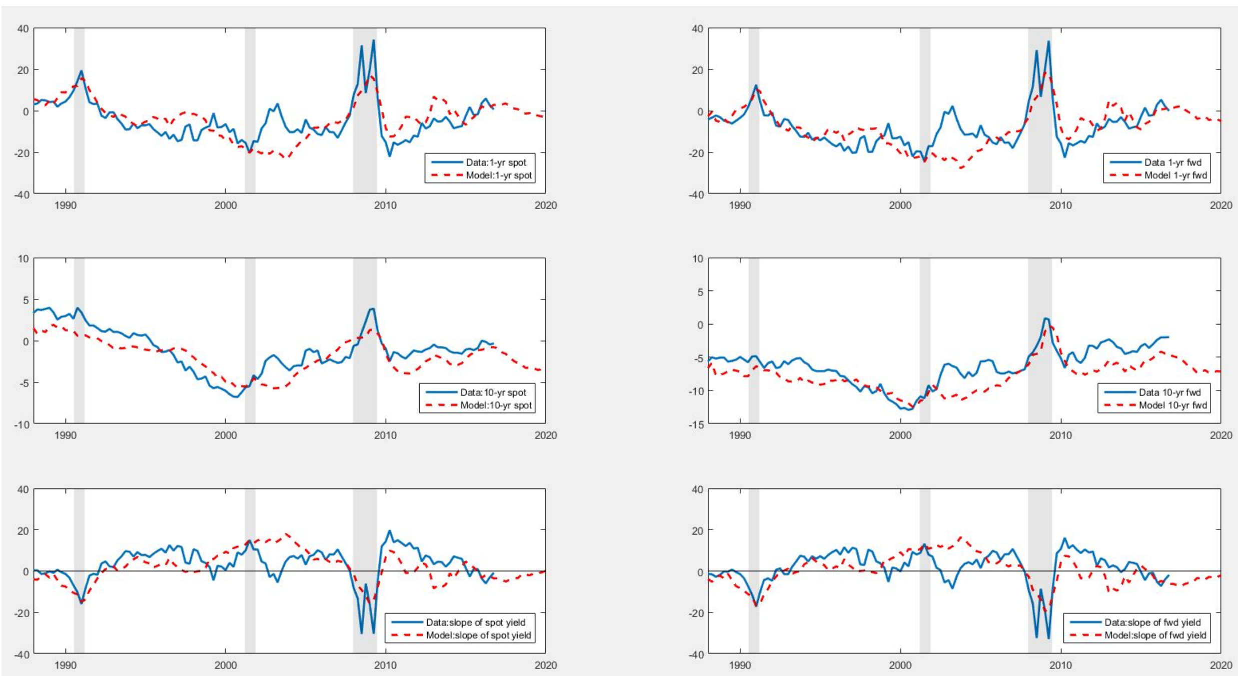


**Table 2: Summary statistics of equity yields**

The table reports the mean and standard deviation of spot and forward yields of dividend strips. These numbers are in annualized percentage terms. We report statistics from both our model and data, and also their correlation coefficients. Sample period is from 1987Q4 to 2016Q3.

| | | Spot | | | Forward | | |
|---|---|---|---|---|---|---|---|
| | | 1Y | 10Y | 10Y-1Y | 1Y | 10Y | 10Y-1Y |
| Data | Mean | -4.52 | -1.08 | 3.44 | -8.10 | -6.28 | 1.82 |
| | Volatility | 9.48 | 2.67 | 7.95 | 9.57 | 2.81 | 8.02 |
| Model | Mean | -3.96 | -1.86 | 2.10 | -8.49 | -7.90 | 0.59 |
| | Volatility | 9.31 | 2.11 | 7.54 | 9.58 | 2.39 | 7.60 |
| | Corr | 0.67 | 0.83 | 0.55 | 0.67 | 0.81 | 0.55 |

The slope of equity yield is often found to be procyclical (see e.g., van Binsbergen et al., 2013; Bansal et al., 2020), that is, during the recession the slope is deeply negative while in normal times it can be positive. We evaluate whether the conditional moments of model-implied equity yields exhibit similar pattern. However, we shall note that the forward equity yield slope subsumes the slope of nominal bond yields, which can be distorted by the zero-lower bound after the 2008 global financial crisis (Swanson and Williams, 2014). To mitigate such impact especially when estimating the conditional mean, we focus this analysis on the sample before 2009Q4. Table 3 displays the results. We find that the equity forward curve is indeed upward sloping during the expansion period, yet it becomes negative during the recession with the average forward equity slope of -10.6%. Our model successfully generates such sign reversal, with the average forward slope of 2.8% (-7.8%) during the expansion (recession). The model-implied yields also display higher volatilities during the recession, again consistent with the data.

Panel C of Table 3 further explores which factors contribute to the sign reversal of forward slopes in our model by expanding Equation (23) similar to the decomposition in (12)

$$ef_t^{(n)} = Const^{(n)} \underbrace{-\lambda(\tilde{\mu}_{c,t}^* + \frac{1-\rho_c^n}{n(1-\rho_c)}\rho_c \tilde{x}_{c,t})}_{RGDP_t^{(n)}} + \underbrace{\frac{1-\rho_x^n}{n}\tilde{x}_{d,t} - \frac{\nu_\mu - 1}{n}(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1}) - \frac{\nu_x - 1}{n}(d_t^s - \rho_x \tilde{x}_{d,t-1})}_{Div-specific_t^{(n)}}$$
$$\underbrace{-(\tilde{\mu}_{\pi,t}^* + \frac{1-\rho_\pi^n}{n(1-\rho_\pi)}\rho_\pi \tilde{x}_{\pi,t})}_{Infl_t^{(n)}} + \underbrace{\lambda a_{c,t} + a_{\pi,t} + \frac{1-\rho_{ad}^n}{n(1-\rho_{ad})}a_{d,t}}_{Ambiguity_t^{(n)}}. \quad (24)$$

In other words, the average term structure of equity forward yields can be decomposed to (negative of) the term structure of subjective real GDP growth, real dividend-specific growth, inflation, and belief distortion by ambiguous investor. Our results show that during the recession, short-maturity forward yield is higher mainly because the agent perceives lower real dividend-specific growth in the short-run compared to the long-run. This channel alone accounts for over 87% of the average forward slope changes across the business cycles. As a secondary force, the agent also perceives a lower real GDP growth in the short-run during the recession, and such a channel contributes to around 10% of the average forward slope changes.

Remarkably, our model captures the cyclicality of equity yield slopes in a way different from the previous literature. While most prior studies rely on the procyclical term structure of risk premia to reconcile this evidence (see e.g., Hasler and Marfe, 2016; Breugem et al., 2020; Gonçalves, 2020b; Li and Xu, 2020), their channels may not be coherent with the recent survey-based evidence on the importance of cash-flow variations.[17] In contrast, the CRRA utility implies a negligible subjective dividend risk premium in our model, and equity yield movements are mostly driven by subjective dividend growth expectations. During recessions, growth expectations are exceptionally lower, with short-term expectation being much lower than its long-run counterpart, therefore we observe sharp increases in equity yields and procyclical equity yield slopes.

[17] In fact, Table 5 of van Binsbergen et al. (2013) do find that the dividend growth expectation accounts for a substantial share of equity yield variations. Cassella et al. (2020) document the term structure of biased beliefs over cash-flows may be empirically consistent with the equity term structure dynamics.

**Table 3: Conditional moments of equity yields and equity slope decomposition**

The table reports mean and standard deviation of spot and forward yields during expansion and recession periods, identified via the NBER business cycle dating. We report statistics from both our model and data within each economic regime. To mitigate the impact of zero-lower bound when computing the conditional mean, we restrict our sample period from 1987Q4 to 2009Q4. Panel C reports the decomposition of average slope of forward equity yields into the components described in Equation (24), i.e., components related to the constant, real GDP growth, dividend-specific growth, inflation, and ambiguity. All numbers are in annualized percentage terms.

| | | Spot | | | | Forward | | |
|---|---|---|---|---|---|---|---|---|
| | | 1Y | 10Y | 10Y-1Y | | 1Y | 10Y | 10Y-1Y |
| | | Panel A: Data | | | | | | |
| Expansion | Mean | -5.82 | -1.32 | 4.50 | | -10.62 | -7.45 | 3.18 |
| | Volatility | 6.50 | 2.90 | 5.03 | | 6.17 | 2.24 | 5.22 |
| Recession | Mean | 8.94 | 0.59 | -8.34 | | 5.89 | -4.73 | -10.62 |
| | Volatility | 17.63 | 3.49 | 14.72 | | 17.94 | 3.88 | 14.57 |
| | | Panel B: Model | | | | | | |
| Expansion | Mean | -5.82 | -1.84 | 3.97 | | -11.66 | -8.87 | 2.80 |
| | Volatility | 8.66 | 2.26 | 6.59 | | 7.79 | 1.85 | 6.32 |
| Recession | Mean | 4.52 | -0.69 | -5.21 | | 2.27 | -5.51 | -7.78 |
| | Volatility | 14.68 | 2.56 | 12.15 | | 15.34 | 3.71 | 12.18 |
| | | Panel C: Slope decomposition | | | | | | |
| | | Const | RGDP | Div-spec | Infl | Ambiguity | Total | |
| Expansion | | 0.13 | -0.01 | 3.30 | -0.10 | -0.52 | 2.80 | |
| Recession | | 0.05 | -1.02 | -5.92 | -0.06 | -0.80 | -7.78 | |

Following van Binsbergen et al. (2013), we run variance decomposition on forward equity yields to understand determinants of their time-variations. From (24) we obtain the following decomposition

$$\begin{aligned} var(ef_t^{(n)}) = {} & cov(ef_t^{(n)}, RGDP_t^{(n)}) + cov(ef_t^{(n)}, Div-specific_t^{(n)}) \\ & + cov(ef_t^{(n)}, Infl_t^{(n)}) + cov(ef_t^{(n)}, Ambiguity_t^{(n)}). \end{aligned} \tag{25}$$

Table 4 illustrates the average proportion of total forward yield variability explained by each component. Unconditionally, the subjective real dividend-specific growth contributes over 90% to the yield volatility at 1-year horizon, consistent with the mean decomposition results in Table 3. Interestingly, the importance of subjective real GDP growth increases steadily with the horizon. For 20-year forward yield, it explains around 40% of total yield variance while the proportion of dividend-specific growth decreases to 13%. A similar pattern is observed for the ambiguity part, which explains around

28% of total variance at 20-year horizon. Zooming in different economic regimes, we find that the explanatory power of subjective real GDP growth is stronger during the expansion period, yet the ambiguity channel is more important during the recession. For instance, it explains over 40% of the 20-year forward yield variance.[18] In sum, our model quantifies the underlying forces driving the time-variations in the term structure of equity yields, and these forces display heterogeneous impact over time and over maturities.

**Table 4: Variance decomposition of forward equity yields**

The table reports the model-based variance decomposition (25), where forward yields are decomposed to the components related to the real GDP growth, dividend-specific growth, inflation, and ambiguity. The decomposition is run over the full sample from 1987Q4 to 2019Q4, or over expansion and recession periods identified via the NBER business cycle dating. The decomposition is done for the dividend strip with the maturity of 1-year, 5-year, 10-year and 20-year.

| | | 1Y | 5Y | 10Y | 20Y |
|---|---|---|---|---|---|
| Unconditional | | | | | |
| | RGDP | 0.06 | 0.16 | 0.28 | 0.41 |
| | Div-spec | 0.91 | 0.71 | 0.44 | 0.13 |
| | Infl | -0.01 | -0.00 | 0.06 | 0.18 |
| | Ambiguity | 0.04 | 0.13 | 0.22 | 0.28 |
| Expansion | | | | | |
| | RGDP | 0.06 | 0.17 | 0.33 | 0.51 |
| | Div-spec | 0.93 | 0.78 | 0.48 | 0.12 |
| | Infl | -0.00 | -0.00 | 0.08 | 0.22 |
| | Ambiguity | 0.01 | 0.05 | 0.11 | 0.15 |
| Recession | | | | | |
| | RGDP | 0.04 | 0.13 | 0.19 | 0.25 |
| | Div-spec | 0.93 | 0.68 | 0.44 | 0.19 |
| | Infl | -0.01 | 0.02 | 0.07 | 0.14 |
| | Ambiguity | 0.04 | 0.17 | 0.30 | 0.42 |

## 4.2 Time-series of dividend futures returns

Although our analysis is silent on the expected equity returns and its term structure (under the statistical measure) due to latent states, the realized price dynamics from the model still allow for inferring the properties of ex-post realized returns.[19] Following Van Binsbergen and Koijen (2017),

[18] Table IA.2 reports further decomposition on the ambiguity channel and we find that the dividend-specific ambiguity turns out to be the most important driver at the long-horizon.

[19] There is in fact an ongoing debate on the term structure of equity expected returns or risk premia under the statistical measure. While canonical asset pricing models imply a flat or upward-sloping term structure, recent evidence suggests

we study the $h-$period realized futures return of a dividend strip with $n-$period maturity, which can be written as (see Appendix A for the derivation)

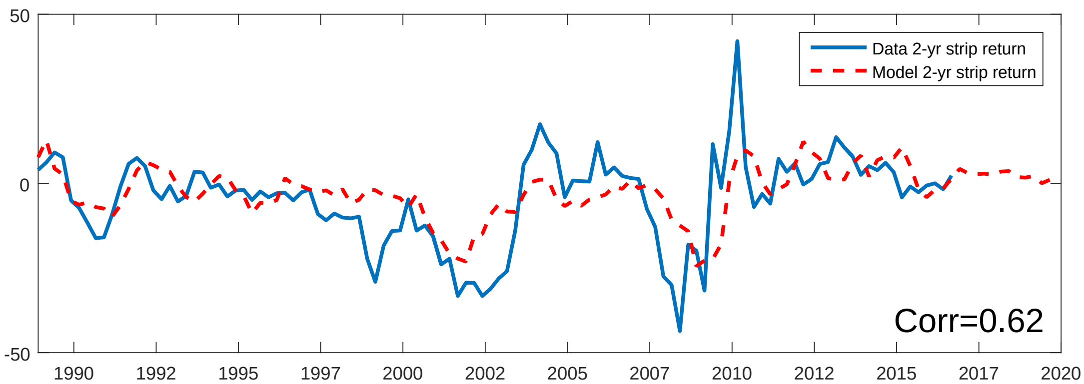
$$r_{F,t+1:t+h}^{(n)} = Cte + \underbrace{\Delta d_{t+1:t+h}^{\$} - \tilde{E}_t \Delta d_{t+1:t+h}^{\$}}_{FE_{t+h}} + \underbrace{(\tilde{E}_{t+h} - \tilde{E}_t)\Delta d_{t+h+1:t+n}^{\$}}_{FR_{t+h}^{(n)}}. \tag{26}$$

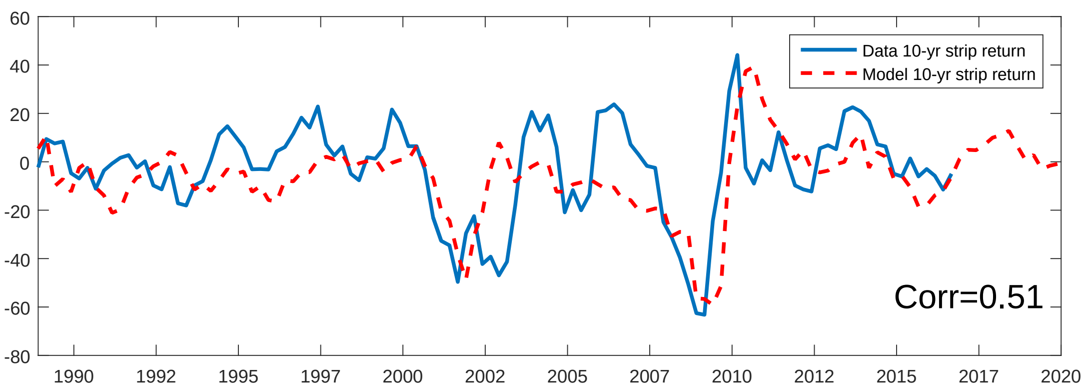
Strip futures return consists of two time-varying components. The first represents the forecast error (FE) of $h-$period dividend growth and is independent of the strip maturity, the second reflects the forecast revision (FR), or expectation changes, for the remaining dividend growth. For our subsequent analysis, we focus on the annual holding period ($h = 4$). Figure 2 suggests that our model does reasonably well in capturing ex-post returns in the data. The correlation of 2-year realized strip returns reaches 0.62, and that of 10-year strip is still at a high level of 0.51. Interestingly, the model-implied returns also mimick the significant crash and rebound during the 2008 global financial crisis. An important caveat is that the fit of equity forward yields in Figure 1 does not necessarily translate to the close fit of futures returns in Figure 2, as the latter further requires the model to capture yield *changes* in both equity and bond markets.

that it may be downward sloping unconditionally based on either options data (van Binsbergen et al., 2012a) or strip futures returns (Van Binsbergen and Koijen, 2017). However, Boguth et al. (2019) find no evidence of downward sloping term structure once accounting for noises from highly levered option positions. Using the same strip futures returns, Bansal et al. (2020) document procyclical term structure of equity risk premia and show that it can be upward sloping unconditionally after matching reasonable business cycle frequencies. Gormsen (2020) instead finds a counter-cyclical term structure when cyclicality is measured by ex-ante stock market valuation ratios.

**Figure 2: Strip futures returns: data vs. model**

The figure compares the model-implied futures returns of dividend strips with the data calculated from Giglio et al. (2020). Model-implied quantities are from 1988Q4 to 2019Q4, while the data is only available till 2016Q3. All numbers are in annualized percentage terms.




We go one step further to explore whether the model-implied returns match two stylized facts regarding return variations: (1) long-term dividend strips co-move more strongly with the market returns (Van Binsbergen and Koijen, 2017; Gonçalves, 2019); (2) return volatilities of long-term dividend strips are higher than those of short-term strips (Lettau and Wachter, 2007; Van Binsbergen and Koijen, 2017). Most previous studies attribute these two facts to the idea that long duration assets have higher exposures to discount rate variations (see e.g.,Campbell and Vuolteenaho, 2004a;

Brennan and Xia, 2006; Lettau and Wachter, 2007; Gonçalves, 2020a). Such an explanation may not be consistent with recent literature that casts doubt on the relevance of discount rate variations at both short-horizon (De la O and Myers, 2020) and long-horizon (Bordalo et al., 2020). Indeed, if discount rate variability per se does not contribute much to price volatility, we might as well expect them to explain little the above patterns for ex-post returns. Therefore, matching these two stylized facts differentiates our study from prior literature because our model specification imposes minimal discount rate variations.

Table 5 reports the comparison results, where Panel A estimates CAPM betas of strip futures returns to gauge the magnitude of comovements and Panel B calculates volatilities of strip returns.[20] Results imply that our model closely replicates the upward sloping term structure of both CAPM betas and volatilities. Despite the underestimation of return volatilities at the short end, most model-implied moments are of similar magnitude with the data. To clarify the mechanism, from Equation (26) we know that cross-maturity differences observed in Table 5 have to come from the forecast revision component, which can be expanded as

$$FR_{t+h}^{(n)} = (\tilde{E}_{t+h} - \tilde{E}_t) \sum_{j=0}^{n-1} \Delta d_{t+j+1}^{\$}, \tag{27}$$

where $\Delta d_{t+j+1}^{\$}$ denotes the nominal dividend growth realized between $t+j$ to $t+j+1$. In the model, shocks to agent's beliefs move forecast revisions of per period dividend growth in the same direction, and at all horizons. The larger the maturity $n$, the more responsive of the total forecast revisions to the belief changes, generating the upward sloping term structure of co-movements and volatilities.

[20]We discuss how to obtain model-implied aggregate market returns used to compute CAPM betas in Subsection 4.4.

**Table 5: Strip return comovements and volatilities**

Panel A reports the model-implied CAPM betas of strip futures returns, Newey-West standard errors are in parentheses. Panel B reports the volatilities of futures returns in annualized percentage terms. The sample is from 1988Q4 to 2019Q4.

| | 2Y | 5Y | 10Y | 20Y |
|---|---|---|---|---|
| | Panel A: CAPM betas | | | |
| Data | 0.46 | 0.78 | 0.90 | 0.96 |
| | (0.11) | (0.10) | (0.11) | (0.16) |
| Model | 0.24 | 0.51 | 0.81 | 1.20 |
| | (0.06) | (0.07) | (0.11) | (0.21) |
| | Panel B: return volatilities | | | |
| Data | 13.12 | 16.94 | 19.24 | 23.06 |
| Model | 7.76 | 10.27 | 15.76 | 24.87 |

## 4.3 Bond-stock comovement

Now we discuss the puzzling behavior of bond-stock correlation. A stylized finding from the literature is that the nominal long-term Treasury bonds switch from risky assets to safe assets around 2000, that is, the correlation between bond returns and stock returns change from positive to negative. Similar comovements are also identified between equity yields and long-term nominal Treasury yields, the so-called "Fed model".[21] In this subsection we address these puzzles simultaneously. To clarify the mechanism, we first study the comovements between long-term nominal bonds and dividend strips as they have analytical solutions from the model. Corresponding results between bond returns and aggregate stock returns, as typically discussed in the literature, will be presented in Section 4.4.

### 4.3.1 Return covariance

The $h-$period spot return of a dividend strip with $n-$period maturity is defined as

$$r_{S,t+1:t+h}^{(n)} = ney_t^{(n)} - (n-h)ey_{t+h}^{(n-h)} + \Delta d_{t+1:t+h}^{\$}. \tag{28}$$

[21]See more detailed discussions on bond-stock return correlations in Baele et al. (2010); David and Veronesi (2013); Campbell et al. (2017, 2020); Kozak (2020); Li et al. (2020), and on the Fed-model in Asness (2003); Campbell and Vuolteenaho (2004b); Bekaert and Engstrom (2010); Burkhardt and Hasseltoft (2012).

With Equation (22) and ignoring the constant, we can write the equity strip return as the combination of an inherent nominal bond return $r^{(n)}_{B,t+1:t+h}$ and change in subjective beliefs over the nominal dividend growth

$$r^{(n)}_{S,t+1:t+h} = r^{(n)}_{B,t+1:t+h} + (\tilde{E}_{t+h} - \tilde{E}_t)\Delta d^{\$}_{t+1:t+n}. \tag{29}$$

Then we run the decomposition of the covariance between the $n-$period strip return and the long-term nominal bond return (with $N-$period maturity)

$$Cov(r^{(n)}_{S,t+1:t+h}, r^{(N)}_{B,t+1:t+h}) = \underbrace{Cov(r^{(n)}_{B,t+1:t+h}, r^{(N)}_{B,t+1:t+h})}_{bond} + \underbrace{Cov((\tilde{E}_{t+h} - \tilde{E}_t)\Delta d^{\$}_{t+1:t+n}, r^{(N)}_{B,t+1:t+h})}_{dividend}, \tag{30}$$

where we have labeled "bond" and "dividend" parts. They represent the corresponding covariance of long-term bond return with the strip-inherent bond return and dividend expectation changes.

Table 6 presents the bond-stock return correlations and results from covariance decomposition (30) for various maturities of dividend strips, with annual holding period ($h = 4$). The analysis is implemented under two subsamples separated by 2000 and we follow the literature by setting $N =$ 10 years. From Panel A we indeed find that the bond-stock correlations turn to negative after 2000, in both the data and the model. In Panel B, despite the sign reversal on the left hand side of (30) after 2000, the bond (dividend) part on the right hand side is always positive (negative) and upward (downward) sloping in the data. Changes in their relative magnitude lead to sign change in bond-stock correlation, especially because the dividend part contributes more negatively after 2000. Therefore, while matching the overall changes in bond-stock covariance is critical, the decomposition (30) in the data introduces additional testable predictions for any equilibrium models to match. To the best of our knowledge, we are unaware of existing papers running such decomposition to justify their equilibrium models. Interestingly, results in Panel B support our model in replicating the general pattern of those underlying forces.

**Table 6: Bond-stock return correlation and covariance decomposition**

Panel A reports the bond-stock return correlation under each subsample. Panel B reports the decomposition results of bond-stock return covariance (scaled by 100) based on (30). $n$ denotes the maturity of corresponding dividend strip and we use the 10-year nominal bond throughout the analysis.

| n | 5Y | 10Y | 15Y | 20Y |
|---|---|---|---|---|
| Panel A: bond-stock return correlation | | | | |
| before 2000Q1 (data) | 0.48 | 0.42 | 0.40 | 0.39 |
| before 2000Q1 (model) | 0.23 | 0.45 | 0.49 | 0.52 |
| after 2000Q1 (data) | -0.52 | -0.51 | -0.49 | -0.47 |
| after 2000Q1 (model) | -0.64 | -0.61 | -0.63 | -0.63 |
| Panel B: covariance decomposition | | | | |
| | 1988Q4-1999Q4 | | | |
| bond (data) | 0.49 | 0.92 | 1.24 | 1.47 |
| Dividend (data) | -0.17 | -0.61 | -0.89 | -1.08 |
| bond (model) | 0.31 | 0.52 | 0.69 | 0.86 |
| Dividend (model) | -0.26 | -0.35 | -0.48 | -0.61 |
| | 2000Q1 to 2016Q3 | | | |
| bond (data) | 0.28 | 0.54 | 0.69 | 0.83 |
| Dividend (data) | -0.95 | -1.22 | -1.37 | -1.50 |
| bond (model) | 0.79 | 1.32 | 1.75 | 2.17 |
| Dividend (model) | -1.28 | -2.03 | -2.71 | -3.31 |

We go one-step ahead by decomposing the dividend part in (30) into more fundamental determinants via two identities

$$r^{(N)}_{B,t+1:t+h} = \underbrace{\gamma(\tilde{E}_t \Delta g_{t+1:t+N} - \tilde{E}_{t+h} \Delta g_{t+h+1:t+N})}_{rr^{(N)}_{B,t+1:t+h}} + \tilde{E}_t \pi_{t+1:t+N} - \tilde{E}_{t+h} \pi_{t+h+1:t+N}, \tag{31}$$

$$(\tilde{E}_{t+h} - \tilde{E}_t)\Delta d^{\$}_{t+1:t+n} = (\tilde{E}_{t+h} - \tilde{E}_t)\Delta d_{t+1:t+n} + (\tilde{E}_{t+h} - \tilde{E}_t)\pi_{t+1:t+n}, \tag{32}$$

where we denote $rr^{(N)}_{B,t+1:t+h}$ as the real return of $N-$period bond. More precisely, we follow Duffee (2018a) and Duffee (2018b) by separating nominal quantities into real and inflation-related components. The dichotomy is natural and important to distinguish our study from the related literature. As Duffee (2018b) and Gomez-Cram and Yaron (2017) point out, news about real rates explain a substantial share of nominal bond returns, inconsistent with standard habit or long-run risk models (see e.g., Wachter, 2006; Bansal and Shaliastovich, 2013). Following this line of reasoning, Duffee (2018a) documents that the bond-stock covariance can be largely explained by the changes in the covariance between short-term real bond and stock returns. Motivated by their study, we assess whether model-implied real bond components do play an important role behind the first-stage results in Table 6.

With (31) and (32), the dividend part in (30) can be decoupled to four covariance terms listed in Table 7. For the interest of space we only display the results for 5-year and 20-year dividend strips. Panel A and B show that the dividend part is predominantly driven by the negative covariance between real bond returns and real dividend growth, or the positive comovements between changes in real GDP expectation and real dividend expectation. Panel C reports the component-wise covariance changes before and after 2000. Results show that such covariance of real growth alone accounts for around 70% of the total changes in bond-stock covariance. That is, the real bonds provide a better hedge to aggregate real dividend risks after 2000. Furthermore, combing two terms in the first row for each maturity recovers the stock-*real* bond comovements as similarly studied by Duffee (2018a). We find that such covariance contributes 81% to the total bond-stock covariance. The quantitative effect coming from the real bond return is large and in line with the evidence in Duffee (2018b). Another complementary force is the inflation real effect, which we define as the correlation between subjective inflation and subjective *real* cash-flow growth.[22] Before 2000, higher inflation is bad news for both real GDP and dividend growth, whereas the situation reverses after 2000. Such an effect contributes approximately 27% to the total changes in bond-stock covariance. Thus our evidence is not inconsistent with a large body of research proposing the inflation non-neutrality as the key resolution for the bond-stock correlation (see e.g., Campbell et al. 2017; Song 2017; Campbell et al. 2020). Moreover, we simultaneously address the concern raised by Duffee (2018b) and Duffee (2018a) by showing that the *nominal* bond-stock return comovements may largely reflect *real* bond-stock comovements.

[22] Piazzesi and Schneider (2007); Bansal and Shaliastovich (2013); Song (2017) incorporate the real effect of inflation via the predictive relation under the *statistical* measure. Zhao (2017) builds a link between inflation and ambiguity over real growth. Our definition is somewhat different as it concerns the correlation between subjective beliefs.

**Table 7: Decomposing bond return-dividend comovements**

The table reports the results of decomposing comovements between 10-year nominal bond returns and dividend expectation changes, defined as the dividend part in (30). We report four covariance terms based on (31) and (32), and decomposition is run for each subsample separated by 2000. Panel C reports the changes for each covariance term across the subsample, and their contribution as the proportion of the total covariance changes. We report the results for 5-year and 20-year dividend strips.

| | 5Y | | | 20Y | | |
|---|---|---|---|---|---|---|
| | $(\tilde{E}_{t+k}-\tilde{E}_t)\Delta d_{t+1:t+n}$ | $(\tilde{E}_{t+k}-\tilde{E}_t)\pi_{t+1:t+n}$ | | $(\tilde{E}_{t+k}-\tilde{E}_t)\Delta d_{t+1:t+n}$ | $(\tilde{E}_{t+k}-\tilde{E}_t)\pi_{t+1:t+n}$ | |
| Panel A: 1987Q4 to 1999Q4 | | | | | | |
| $rr^{(n)}_{B,t+k}$ | -0.29 | 0.03 | | -0.66 | 0.09 | |
| $-(\tilde{E}_{t+k}-\tilde{E}_t)\pi_{t+k+1:t+n}$ | 0.03 | -0.03 | | 0.06 | -0.09 | |
| Total | | | -0.26 | | | -0.61 |
| Panel B: 2000Q1 to 2016Q3 | | | | | | |
| $rr^{(n)}_{B,t+k}$ | -1.02 | -0.07 | | -2.54 | -0.22 | |
| $-(\tilde{E}_{t+k}-\tilde{E}_t)\pi_{t+k+1:t+n}$ | -0.14 | -0.06 | | -0.40 | -0.15 | |
| Total | | | -1.28 | | | -3.31 |
| Panel C: Changes | | | | | | |
| $rr^{(n)}_{B,t+k}$ | -0.73 (71%) | -0.10 (10%) | | -1.88 (70%) | -0.31 (11%) | |
| $-(\tilde{E}_{t+k}-\tilde{E}_t)\pi_{t+k+1:t+n}$ | -0.17 (17%) | -0.02 (2%) | | -0.46 (17%) | -0.06 (2%) | |
| Total | | | -1.02 | | | -2.70 |

### 4.3.2 Fed model

We then explore whether similar story holds for the correlation between equity and nominal bond yield *levels*, the so-called Fed model. Based on Equation (22) and similar to (30), the yield covariance can be decomposed as

$$Cov(ey_t^{(n)}, y_t^{(N)}) = \underbrace{Cov(y_t^{(n)}, y_t^{(N)})}_{bond} + \underbrace{Cov(-\frac{1}{n}\tilde{E}_t\Delta d^{\$}_{t+1:t+n}, y_t^{(N)})}_{dividend}. \tag{33}$$

Panel A of Table 8 reports the yield correlation, and model implications are widely consistent with the data. Further in Panel B, we confirm that the correlation sign reversal after 2000 still originates from the dividend part. Table 9 runs a similar decomposition of the dividend part following Table 7. Comparing these two tables, a dominant force driving the Fed model appears to be the comovements between expected inflation and real dividend growth, which contributes to over 50% of the bond-stock yield covariance changes after 2000. Albeit with consistent sign, its impact is only modest (less than

30%) for explaining the bond-stock return covariance. The key reason is that the persistent expected inflation, though explains a substantial share of expected nominal GDP growth, cannot move enough at high-frequency to become the first order determinant of return correlation. Overall, our decomposition results uncover qualitatively similar but quantitatively different mechanisms behind the Fed model and bond-stock return correlation. The correlation between expected inflation and real dividend growth is a more prominent force at low-frequency and thus matters more to the Fed model, whereas bond-stock return correlation is primarily driven by correlated shocks to the subjective growth.

**Table 8: Decomposing bond-stock yield co-movements**

Panel A reports the bond-stock yield correlation under each subsample. Panel B reports the decomposition results of bond-stock yield covariance (scaled by 1000) based on (33). All yields are in annualized terms, $n$ denotes the maturity of corresponding dividend strip and we use the 10-year nominal bond throughout the analysis.

| n | 5Y | 10Y | 15Y | 20Y |
|---|---|---|---|---|
| Panel A: bond-stock yield correlation | | | | |
| before 2000Q1 (data) | 0.86 | 0.84 | 0.83 | 0.83 |
| before 2000Q1 (model) | 0.73 | 0.76 | 0.76 | 0.77 |
| after 2000Q1 (data) | -0.56 | -0.64 | -0.65 | -0.66 |
| after 2000Q1 (model) | -0.35 | -0.35 | -0.36 | -0.35 |
| Panel B: covariance decomposition | | | | |
| 1987Q4 to 1999Q4 | | | | |
| bond (data) | 0.14 | 0.14 | 0.13 | 0.12 |
| Dividend (data) | 0.30 | 0.17 | 0.11 | 0.07 |
| bond (model) | 0.06 | 0.05 | 0.05 | 0.05 |
| Dividend (model) | 0.09 | 0.04 | 0.02 | 0.00 |
| 2000Q1 to 2016Q3 | | | | |
| bond (data) | 0.18 | 0.16 | 0.15 | 0.14 |
| Dividend (data) | -0.44 | -0.33 | -0.27 | -0.23 |
| bond (model) | 0.28 | 0.24 | 0.23 | 0.22 |
| Dividend (model) | -0.46 | -0.35 | -0.31 | -0.28 |

**Table 9: Decomposing bond yield-dividend comovements**

The table reports the results of decomposing comovements between 10-year nominal bond yields and dividend expectation, defined as the dividend part in (33). We report four covariance terms by decomposing nominal quantities into the real quantities and inflation-related components, and decomposition is run for each subsample separated by 2000. Panel C reports the changes for each covariance term across the subsample, and their contribution as the proportion of the total covariance changes. We report the results for 5-year and 20-year dividend strips.

| | 5Y | | | 20Y | | |
|---|---|---|---|---|---|---|
| | $-\frac{1}{n}\tilde{E}_t\Delta d_{t+1:t+n}$ | $-\frac{1}{n}\tilde{E}_t\pi_{t+1:t+n}$ | | $-\frac{1}{n}\tilde{E}_t\Delta d_{t+1:t+n}$ | $-\frac{1}{n}\tilde{E}_t\pi_{t+1:t+n}$ | |
| | Panel A: 1987Q4 to 1999Q4 | | | | | |
| $r_t^{(N)}$ | -0.08 | 0.02 | | -0.04 | 0.01 | |
| $\frac{1}{N}\tilde{E}_t\pi_{t+1:t+N}$ | 0.20 | -0.05 | | 0.07 | -0.05 | |
| Total | | | 0.09 | | | 0.00 |
| | Panel B: 2000Q1 to 2016Q3 | | | | | |
| $r_t^{(N)}$ | -0.35 | -0.04 | | -0.20 | -0.03 | |
| $\frac{1}{N}\tilde{E}_t\pi_{t+1:t+N}$ | -0.05 | -0.01 | | -0.04 | -0.01 | |
| Total | | | -0.46 | | | -0.28 |
| | Panel C: Period-to-Period changes | | | | | |
| $r_t^{(N)}$ | -0.28 (50%) | -0.06 (10%) | | -0.17 (59%) | -0.04 (15%) | |
| $\frac{1}{N}\tilde{E}_t\pi_{t+1:t+N}$ | -0.26 (46%) | 0.04 (-6%) | | -0.11 (39%) | 0.04 (-13%) | |
| Total | | | -0.55 | | | -0.28 |

## 4.4 Puzzles about the aggregate stock market

In this subsection, we revisit several aggregate stock market puzzles via our equilibrium model. We model aggregate market portfolio as the portfolio of dividend strips by writing the ex-dividend aggregate stock price as

$$P_t = \sum_{n=1}^{H_t} P_t^{(n)}, \tag{34}$$

where $P_t^{(n)}$ is the price of $n-$period dividend strip, and stochastic $H_t$ may be interpreted as the life expectancy of the aggregate portfolio. Since the focus in this subsection simply asks whether previous stories explaining the equity yields also translate to the aggregate market, we do not manually set values for $H_t$ (either finite or infinite), nor clarify the mechanisms for the time-variations of $H_t$ (see e.g., Fama and French, 2004; Chen, 2011). Instead, we adopt a simple approach by modeling it as a reduced-form function of our equity duration measures for the aggregate market $LTG_t$

$$H_t = a + bLTG_t, \tag{35}$$

where $LTG_t$ is calculated as the value-weighted average of long-term growth forecasts over all firms. The specification captures the time-varying cash-flow duration of aggregate stock market in a parsimonious way. Based on (34), we pin down parameters $a$ and $b$ by asking the model-implied aggregate price-dividend ratio to match the data.[23] Although we use the data of dividend-price ratio when estimating $a$ and $b$, the time-variations of model-implied aggregate dividend-price ratio are still entirely driven by the strip yield variations and exogenous movements in aggregate equity duration $LTG_t$. Thus our empirical approach should have small, if any, mechanical effect when mimicking the whole time-series of the data.

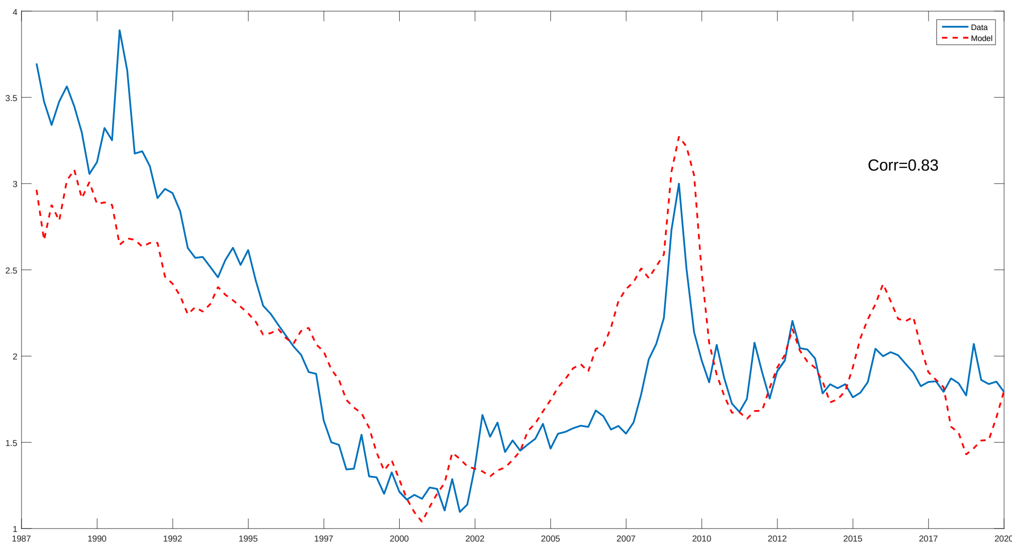
Figure 3 demonstrates that the model-implied aggregate dividend-price ratio is close to the data, with a correlation coefficient of 0.83. In Panel A of Table 10, we further report related statistics of log dividend-price ratios following the literature. The AR(1) coefficients from data and model are both 0.95, and the model-implied log dp ratio has an annualized volatility of 25%, also close to 30% in the data. We thus generate persistent and volatile aggregate dividend yield simply via strip yield variations (endogenously driven by subjective beliefs) and exogenous movements in aggregate equity duration, though the latter has a minor effect on our results.[24]

[23] Figure IA.3 in Appendix C plots the estimated time-series of $H_t$, its sample average is 37 years.

[24] Even if we fix $H_t$ at its unconditional average throughout the sample, results are only weakened marginally. These results are available upon request.

**Figure 3: Fitting aggregate dividend-price ratio**

The figure compares the model-implied aggregate dividend-price ratio with the data. The model-implied quantitiy is obtained following the method in Section 4.4. Correlation coefficient between model and data is reported in plot. Sample period is from 1987Q4 to 2019Q4 and the numbers are in annualized percentage terms.



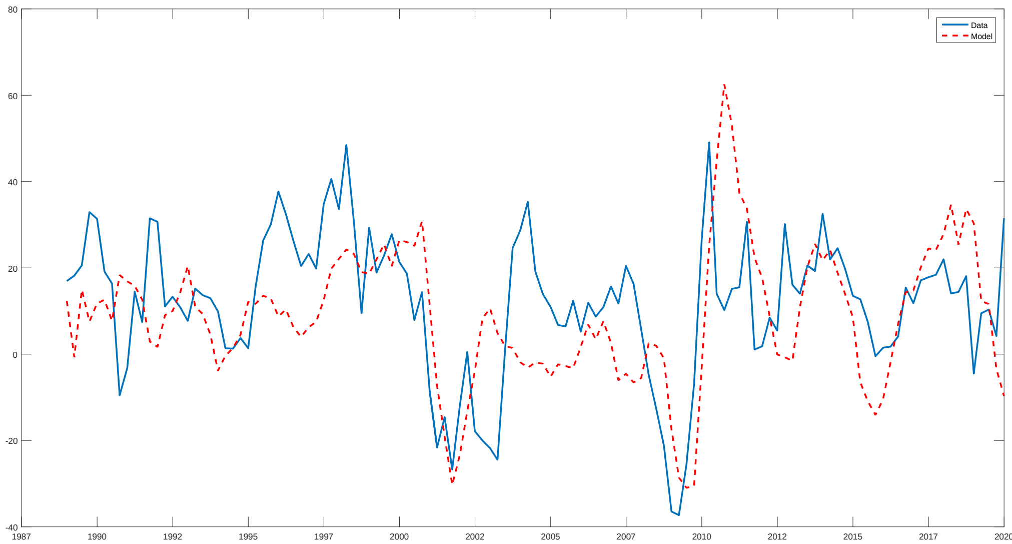

With the time-series of aggregate dividend-price ratio, we then calculate simple market return from

$$r_{M,t} = \frac{P_t + D_t}{P_{t-1}} - 1 = \frac{P_t/D_t + 1}{P_{t-1}/D_{t-1}} \times \frac{D_t}{D_{t-1}} - 1. \tag{36}$$

Figure 4 plots the model-implied market returns together with the data. They are close with each other and the correlation coefficient is 0.44. Table 10 shows that our model generates an average market return of around 9%, largely replicating the high equity return (12%) in the data. Market returns are also volatile in our model with an annualized volatility of 16%, echoing the time-series fit in Figure 4 and resolving the excess volatility puzzle. Finally, following the discussions in Section 4.3, we also document sign switches of the correlation between long-term nominal bond returns and aggregate stock returns after 2000. The correlation coefficient changes from 0.4 to -0.6, while in the data we observe similar magnitude of changes.

**Figure 4: Aggregate stock returns: data vs. model**

The figure compares the model-implied aggregate market returns with the data. The model-implied quantitiy is obtained following Equation (36). Sample period is from 1987Q4 to 2019Q4. Plotted numbers are in annualized percentage terms.




Finally, Panel B of Table 10 approaches the return predictability puzzle, i.e., stock returns can be predicted by lagged dividend yields. We run two predictive regressions, the first is on predicting the market excess returns

$$r_{M,t+1:t+h} - r_{f,t} = \alpha + \beta(d_t^{\$} - p_t) + \epsilon_{t+1:t+h}, \tag{37}$$

and the second concerns predicting the strip excess returns using its own lagged yields

$$rx_{S,t+1:t+h}^{(n)} = \alpha + \beta(d_t^{\$} - p_t^{(n)}) + \epsilon_{t+1:t+h}, \tag{38}$$

where $rx_{S,t+1:t+h}^{(n)}$ is the excess spot return. The first regression is widely used in the literature, but we run the second to uncover the source of return predictability as quantities on both sides of (38) have analytical solutions. During the sample from 1988Q4 to 2019Q4, annual market excess returns are positively predicted by lagged log dividend-price ratio, with a $t-$statistic of 2.58 and $R^2$ of 14%.

Though the $R^2$ from our model regression is comparably lower, the slope coefficient is close to that from the data and is highly significant. Similar results also hold for strip return predictability.

To understand the mechanism, we write the excess return of the $n-$period dividend strip as

$$rx_{S,t+1:t+h}^{(n)} = Cte + \underbrace{rx_{B,t+1:t+h}^{(n)}}_{Bond} + \underbrace{\Delta d_{t+1:t+h}^{\$} - \tilde{E}_t \Delta d_{t+1:t+h}^{\$}}_{FE} + \underbrace{(\tilde{E}_{t+4} - \tilde{E}_t)\Delta d_{t+5:t+n}^{\$}}_{FR}. \tag{39}$$

Realized strip excess return consists of three components, the maturity-matched realized bond excess return (Bond), the forecast error of dividend growth within the holding period (FE), and the forecast revision regarding the dividend growth after the holding period (FR). Equity yields that predict strip returns must predict some (or all) of these components. The last part of Panel B evaluates their predictability and we document somewhat different patterns for short- and long-maturity strips. For short maturities, bond predictability contributes a little whereas the forecast error and forecast revision predictability dominates. Lower subjective dividend growth (higher yields) on average strongly predicts higher subsequent forecast errors but lower forecast revisions. Intuitively, during bad time the subjective growth is low but is likely followed by higher dividend level realizations. However, Equation (8) shows that higher future realizations will likely drive down the forecast for dividend growth thereafter, leading to lower forecast revisions in the future. Furthermore, we expect such an effect decreases with the horizon as we have shown that yields of long-term claims are mainly driven by levered endowment components (see Table 4). Consistently, we find from the decomposition that most of the 10-year strip return predictability arises from the 10-year bond return predictability. Our model thus sheds light on potentially different forces operating at the term structure of return predictability.

**Table 10: Moments for aggregate market and return predictability**

Panel A reports the moments of the aggregate stock market, including the annualized mean and volatility of market returns, volatility and AR(1) coefficient of market log dividend-price ratio, and the correlation between 10-year nominal bond returns and aggregate stock returns. We also report the correlation between model and data regarding the log dividend-price ratio and market returns. Sample period is from 1987Q4 to 2019Q4. Panel B reports the results of predictive regressions (37) and (38). Due to data availability of dividend strip yields, the regressions use the sample from 1987Q4 to 2016Q3. The last part of Panel B reports the decomposition results of predictive regression (38) via (39). In brackets we report the Newey-West t-statistics with optimal lag selection following Andrews (1991).

| Panel A: Moments of aggregate market portfolio | | | | | | |
|---|---|---|---|---|---|---|
| | $E(r_M)$ | $\sigma(r_M)$ | $\sigma(d-p)$ | $\rho(d-p)$ | $Corr(r_M, r_B \mid t < 2000Q1)$ | $Corr(r_M, r_B \mid t \geq 2000Q1)$ |
| Data | 0.12 | 0.16 | 0.30 | 0.95 | 0.39 | -0.59 |
| Model | 0.09 | 0.16 | 0.25 | 0.95 | 0.46 | -0.59 |
| $Corr(dp^{data}, dp^{model})$ | 0.84 | | | | | |
| $Corr(r_M^{data}, r_M^{model})$ | 0.44 | | | | | |
| **Panel B: Return predictability** | | | | | | |
| | | 2Y | 5Y | 7Y | 10Y | MKT |
| Data | | | | | | |
| | $\beta$ | 0.53 | 0.34 | 0.25 | 0.20 | 0.20 |
| | $(t)$ | (4.33) | (3.88) | (3.15) | (2.83) | (2.58) |
| | $R^2(\%)$ | 34.82 | 22.61 | 17.43 | 15.22 | 14.00 |
| Model | | | | | | |
| | $\beta$ | 0.23 | 0.15 | 0.11 | 0.12 | 0.17 |
| | $(t)$ | (1.79) | (2.20) | (2.13) | (2.34) | (2.69) |
| | $R^2(\%)$ | 18.91 | 16.33 | 6.88 | 4.72 | 7.74 |
| Decomposition | | | | | | |
| | $Bond$ | 0.05 | 0.08 | 0.09 | 0.13 | |
| | $(t)$ | (0.99) | (1.37) | (1.28) | (1.20) | |
| | $R^2(\%)$ | 2.71 | 4.57 | 3.94 | 3.58 | |
| | $FE$ | 0.57 | 0.35 | 0.26 | 0.18 | |
| | $(t)$ | (3.30) | (2.81) | (2.32) | (1.61) | |
| | $R^2(\%)$ | 43.00 | 35.20 | 27.64 | 17.64 | |
| | $FR$ | -0.37 | -0.28 | -0.23 | -0.18 | |
| | $(t)$ | (-5.41) | (-4.37) | (-2.69) | (-1.35) | |
| | $R^2(\%)$ | 35.61 | 19.87 | 9.31 | 3.20 | |

# 5 Robustness

## 5.1 Training periods

Our benchmark analysis assumes an initial training period of 5 years so that we analyze our model results from 1987Q4 onward. In Table IA.3, we show that this choice does not affect our key results after changing the length of training period to 3 years or 10 years. As long as the training periods are of reasonable length, our model still implies close fit for the equity term structure. For example with a 10-year initial training period so that our accounting starts from 1992Q4, the model-implied 1-year and 10-year equity spot yields have a correlation coefficient of 0.54 and 0.73 with the data. The mean and volatility also match reasonable well using different training samples.

### 5.2 Duration measures and dividend components

The baseline measure of equity duration is the analyst forecast of long-term earnings growth. As a robustness check, we also experiment with alternative duration measures as proposed in recent literature, including those discussed in Dechow et al. (2004); Weber (2018); Gonçalves (2020a). In addition, we consider the book-to-market ratio as a duration measure, motivated by Lettau and Wachter (2007). Although some of these measures have data back to 1960s, we still focus on the same sample period in the benchmark setting because the availability of dividend ambiguity data prevents us from studying longer sample. Table IA.4 gives model implications after constructing the dividend series sorted over those duration measures with the method in Section 3. We find encouraging results that even if we use different measures of equity duration, the model-implied term structure of equity yields are as volatile as the data, and the time-series correlation coefficients are also high.[25]. In a related exercise, while still using forecast of long-term earnings growth as the duration measure, we change the construction of long-duration dividends by using 40th or 60th percentile of the cross-section of duration measures as the breakpoints. Results remain very similar, as found in last two rows of Table IA.4.

## 6 Conclusion

Motivated by the finding that future returns but not future cash flows are predictable by current price-dividend ratios, over the past three decades and within the rational expectations framework,

[25]Note that we always use the same parameters in Table 1 when repeating these results to avoid additional calibration. And as our focus is to capture business cycle dynamics of yields, we do not report the unconditional mean of equity yields in Table IA.4 because they may need different parameter values to match.

macro finance research is trying to come up with a force that moves prices but not expected future cash flows. This principle has guided equilibrium asset pricing literature and has given rise to model of time-varying risk attitude (habit formation) or time-varying risks (long-run risk or disaster risk).

However, new empirical findings on subjective expectations, term structure of bond and equity yields, and the stock-bond correlation pose serious challenges to existing rational models. Subjective expectations of cash flow and interest rates are found to be the most important drivers of equity and bond prices, while subjective return expectations are not as important as in the rational models. Meanwhile, dividend risk premium and bond risk premium in the rational model encounter difficulty in explaining equity and bond yield spread movements in data. Furthermore, the inflation risk premium based explanation for the change in stock-bond correlation implies too much inflation risk in equity returns.

We provide a unified framework of bond and equity pricing that is consistent with these empirical findings. Equity/bond yields movements are driven by subjective dividend/GDP growth expectation, and subjective risk premium is negligible. The model implied long- and short-yields of dividend strips and bonds and their spreads are close to the data (time-series dynamics and moments). Long-term Treasury bonds switched from risky assets to safe assets after the late 1990s, due to two changes in subjective expectations: (1) a procyclical inflation expectation and (2) a higher correlation between real GDP and real dividend growth expectations during the past 20 years. Our framework also quantitatively matches several major aggregate stock market puzzles by generating persistent and volatile price-dividend ratios, excess volatility of stock returns, and return predictability.

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# Internet Appendix for "Term Structure of Equity and Bond Yields over Business Cycles"

## A Model Solution Details

From the posterior (6) and (7), define the error terms

$$\tilde{\epsilon}^*_{d,t} = \frac{d^l_t - \lambda y_t - \tilde{\mu}_{d,t-1}}{\sqrt{(1+\nu_\mu)(\sigma^{*2}_d + \sigma^2_\mu)}}, \tag{IA.1}$$

$$\tilde{\epsilon}^x_{d,t} = \frac{d^s_t - \rho_x \tilde{x}_{d,t-1}}{\sqrt{(1+\nu_x)(\sigma^2_d + \sigma^2_x)}}. \tag{IA.2}$$

Under subjective measure, they follow i.i.d. standard normal distribution, similar to discussions in Nagel and Xu (2019); Zhao (2020b). Therefore, under the subjective measure the total log dividend follows the process

$$d_{t+1} - \lambda y_{t+1} = \tilde{\mu}_{d,t} + \rho_d \tilde{x}_{d,t} + \sqrt{(1+\nu_\mu)(\sigma^{*2}_d + \sigma^2_\mu)}\tilde{\epsilon}^*_{d,t+1} + \sqrt{(1+\nu_x)(\sigma^2_d + \sigma^2_x)}\tilde{\epsilon}^x_{d,t+1}, \tag{IA.3}$$

which implies the dividend growth under the worst-case measure

$$\begin{aligned}\Delta d_{t+1} = {} & \tilde{E}_t \Delta d_{t+1} + \lambda\sqrt{\sigma^2_c(1+\nu_c)}\tilde{\epsilon}^*_{t+1} + \lambda\sqrt{(\sigma^{x2}_c + \sigma^{gap2}_c)(1+\nu^{gap}_c)}\tilde{\epsilon}^x_{t+1} \\ & + \sqrt{(1+\nu_\mu)(\sigma^{*2}_d + \sigma^2_\mu)}\tilde{\epsilon}^*_{d,t+1} + \sqrt{(1+\nu_x)(\sigma^2_d + \sigma^2_x)}\tilde{\epsilon}^x_{d,t+1},\end{aligned} \tag{IA.4}$$

with $\tilde{E}_t \Delta d_{t+1}$ given in Equation (12).

From the CRRA utility and asset pricing equation (16), we can solve out the expression for one-period strip price

$$
\begin{aligned}
p_t^{(1)} - d_t^{\$} = \log\beta + (\lambda-\gamma)(\widetilde{\mu}_{c,t}^* + \rho_c\widetilde{x}_{c,t} - a_{c,t}) + (\rho_x - 1)\widetilde{x}_{d,t} + (\nu_\mu - 1)(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1}) \\
+ (\nu_x - 1)(d_t^s - \rho_x\tilde{x}_{d,t-1}) - a_{d,t} + \frac{1}{2}[A_1^e\sigma_c^2(1+\nu_c) + B_1^e(\sigma_c^{x2} + \sigma_c^{gap2})(1+\nu_c^{gap}) \\
+ C_1^e\sigma_a^2 + D_1^e(\sigma_d^{*2} + \sigma_\mu^2)(1+\nu_\mu) + E_1^e(\sigma_d^2 + \sigma_x^2)(1+\nu_x) + F_1^e\sigma_{ad}^2],
\end{aligned}
\tag{IA.5}
$$

where we define $A_e^{(1)} = -\log\beta - \frac{1}{2}[A_1^e\sigma_c^2(1+\nu_c) + B_1^e(\sigma_c^{x2} + \sigma_c^{gap2})(1+\nu_c^{gap}) + C_1^e\sigma_a^2 + D_1^e(\sigma_d^{*2} + \sigma_\mu^2)(1+\nu_\mu) + E_1^e(\sigma_d^2 + \sigma_x^2)(1+\nu_x) + F_1^e\sigma_{ad}^2]$ as in Equation (17). Coefficients inside are

$$
A_1^e = (\lambda-\gamma)^2, B_1^e = (\lambda-\gamma)^2, C_1^e = 0, D_1^e = 1, E_1^e = 1, F_1^e = 0.
$$

Similarly for $n-$period strip price

$$
\begin{aligned}
p_t^{(n)} - d_t = n\log\beta + \frac{n(n-1)}{2}(\gamma-\lambda)\mu_a - [\frac{n-1}{1-\rho_{ad}} - \frac{\rho_{ad} - \rho_{ad}^n}{(1-\rho_{ad})^2}]\mu_{ad} + n(\lambda-\gamma)(\widetilde{\mu}_{c,t}^* - a_{c,t}) + (\lambda-\gamma)\frac{1-\rho_c^n}{1-\rho_c}\rho_c\widetilde{x}_{c,t} \\
+ (\rho_x^n - 1)\widetilde{x}_{d,t} - \frac{1-\rho_{ad}^n}{1-\rho_{ad}}a_{d,t} + (\nu_\mu - 1)(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1}) + (\nu_x - 1)(d_t^s - \rho_x\tilde{x}_{d,t-1}) + \frac{1}{2}[A_n^e\sigma_c^2(1+\nu_c) \\
+ B_n^e(\sigma_c^{x2} + \sigma_c^{gap2})(1+\nu_c^{gap}) + C_n^e\sigma_a^2 + D_n^e(\sigma_d^{*2} + \sigma_\mu^2)(1+\nu_\mu) + E_n^e(\sigma_d^2 + \sigma_x^2)(1+\nu_x) + F_n^e\sigma_{ad}^2],
\end{aligned}
\tag{IA.6}
$$

where the loadings follow the iteration

$$
\begin{aligned}
A_n^e &= A_{n-1}^e + \Big[(\lambda-\gamma)(1+(n-1)\nu_c)\Big]^2,\\
B_n^e &= B_{n-1}^e + ((\lambda-\gamma)(1+\frac{\rho_c-\rho_c^n}{1-\rho_c}\nu_c^{gap}))^2,\\
C_n^e &= C_{n-1}^e + (\lambda-\gamma)^2(n-1)^2,\\
D_n^e &= D_{n-1}^e + \nu_\mu^2,\\
E_n^e &= E_{n-1}^e + (\rho_x^{n-1}\nu_x)^2,\\
F_n^e &= F_{n-1}^e + (\frac{1-\rho_{ad}^{n-1}}{1-\rho_{ad}})^2.
\end{aligned}
$$

We thus define $A_e^{(n)}$ after collecting all constant terms as in (19):

$$
\begin{aligned}
A_e^{(n)} = & -n\log\beta - \frac{n(n-1)}{2}(\gamma-\lambda)\mu_a + [\frac{n-1}{1-\rho_{ad}} - \frac{\rho_{ad}-\rho_{ad}^n}{(1-\rho_{ad})^2}]\mu_{ad}\\
& -\frac{1}{2}[A_n^e\sigma_c^2(1+\nu_c)+B_n^e(\sigma_c^{x2}+\sigma_c^{gap2})(1+\nu_c^{gap})+C_n^e\sigma_a^2+D_n^e(\sigma_d^{*2}+\sigma_\mu^2)(1+\nu_\mu)+E_n^e(\sigma_d^2+\sigma_x^2)(1+\nu_x)+F_n^e\sigma_{ad}^2].
\end{aligned}
$$

We then derive nominal bond yield shown in (21) from the log price of $n-$period bond in Zhao (2020b)

$$
p_t^{(n)} = -A_b^{(n)} - n\gamma(\tilde{\mu}_{c,t}^* + \frac{1-\rho_c^n}{n(1-\rho_c)}\rho_c\tilde{x}_{c,t} - a_{c,t}) - n(\tilde{\mu}_{\pi,t}^* + \frac{1-\rho_\pi^n}{n(1-\rho_\pi)}\rho_\pi\tilde{x}_{\pi,t} - a_{\pi,t}), \tag{IA.7}
$$

with the constant $A_b^{(n)}$ follows

$$
\begin{aligned}
A_b^{(1)} &= -\log\beta - \frac{1}{2}Var_t m_{t+1},\\
A_b^{(n)} &= A_b^{(n-1)} - \log\beta - (n-1)(\gamma\mu_a + \mathbb{1}_{t\geq 2000}\mu_\pi) - \frac{1}{2}Var_t[m_{t+1}+p_{t+1}^{(n-1)}].
\end{aligned}
$$

To derive (26), we note the definition of futures return is

$$
r_{F,t+1:t+h}^{(n)} = p_{F,t+h}^{(n-h)} - p_{F,t}^{(n)} - [ny_t^{(n)} - (n-h)y_{t+h}^{(n-h)}], \tag{IA.8}
$$

where $p_{F,t}^{(n)}$ is the log price of $n-$period dividend futures. It can be written in terms of the forward equity yield as

$$r_{F,t+1:t+h}^{(n)} = \Delta d_{t+1:t+h}^{\$} + nef_t^{(n)} - (n-h)ef_{t+h}^{(n-h)}.$$

Plugging in (23) implies (26). Expression (27) can be obtained by further plugging (24).

# B Estimation Details

We will run three estimation steps to determine our parameters, in addition to the calibrated ones and those borrowed from Zhao (2020b). We first estimate dividend parameters from two state-space system given in (2)-(3) and (4)-(5) using the standard MLE via Kalman filter. To estimate parameters for the dividend learning and ambiguity, we write the one-year subjective dividend growth implied from our model

$$\begin{aligned}\tilde{E}_t \Delta d_{t+1:t+4} = & \underbrace{4\lambda\tilde{\mu}_{c,t}^* + \lambda\rho_c \frac{1-\rho_c^4}{1-\rho_c}\tilde{x}_{c,t} + (\rho_x^4 - 1)\tilde{x}_{d,t} + (\nu_x - 1)(d_t^s - \rho_x \tilde{x}_{d,t-1}) + (\nu_\mu - 1)(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1})}_{\tilde{E}_t^* \Delta d_{t+1:t+4}} \\ & - 4\lambda a_{c,t} - \frac{1-\rho_{ad}^4}{1-\rho_{ad}} a_{d,t}, \end{aligned} \tag{IA.9}$$

where $\tilde{E}_t^* \Delta d_{t+1:t+4}$ is the model-implied one-year subjective aggregate dividend growth from the *reference* model. As the data of survey dividend expectation and ambiguity both correspond to a forecast horizon of one-year, we will match different components of (IA.9) to the data to obtain the parameters.

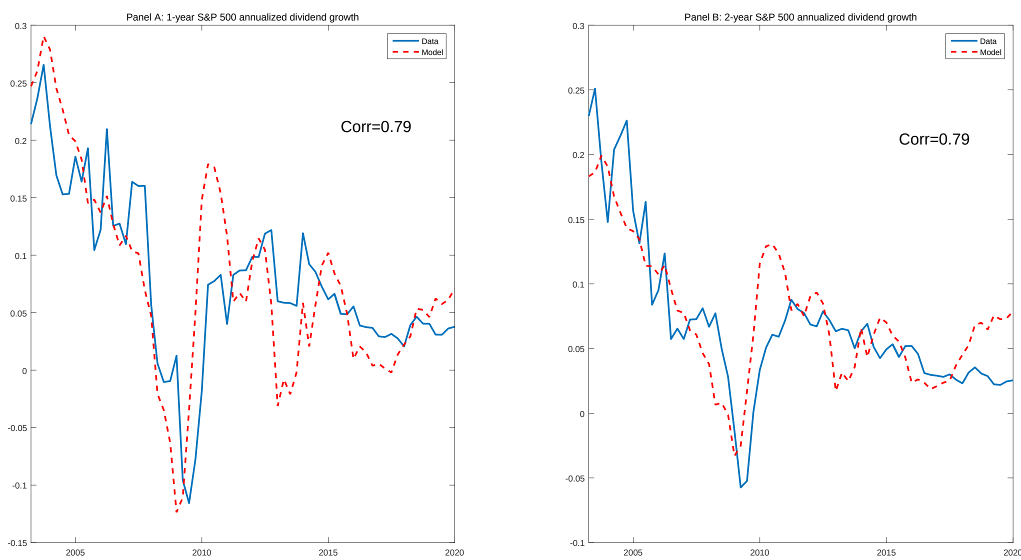
Specifically, we attach an estimation error to one-year expectation of S&P 500 dividend growth

$$DIV_{SP500,t} = E_t^* \Delta d_{t+1:t+4} + \sigma_\eta \eta_t, \tag{IA.10}$$

where $DIV_{SP500,t}$ is the quarter-$t$ expectation for the next year dividend growth of S&P 500 and $\eta_t$ is the measurement error following the standard normal distribution. Given other parameters, we estimate subjective learning gains $\nu_d, \nu_x$ and standard deviation $\sigma_\eta$ by applying MLE on (IA.10). The left panel of Figure IA.1 plots the model fit. For comparison, we also use the estimated parameters to calculate 2-year subjective dividend growth and plot it together with the survey data on the right panel.

**Figure IA.1: Fit of S&P 500 dividend growth expectations**

The figure plots the model-implied 1-year and 2-year aggregate dividend growth expectations together with the data. Sample from 2003Q1 to 2019Q4.



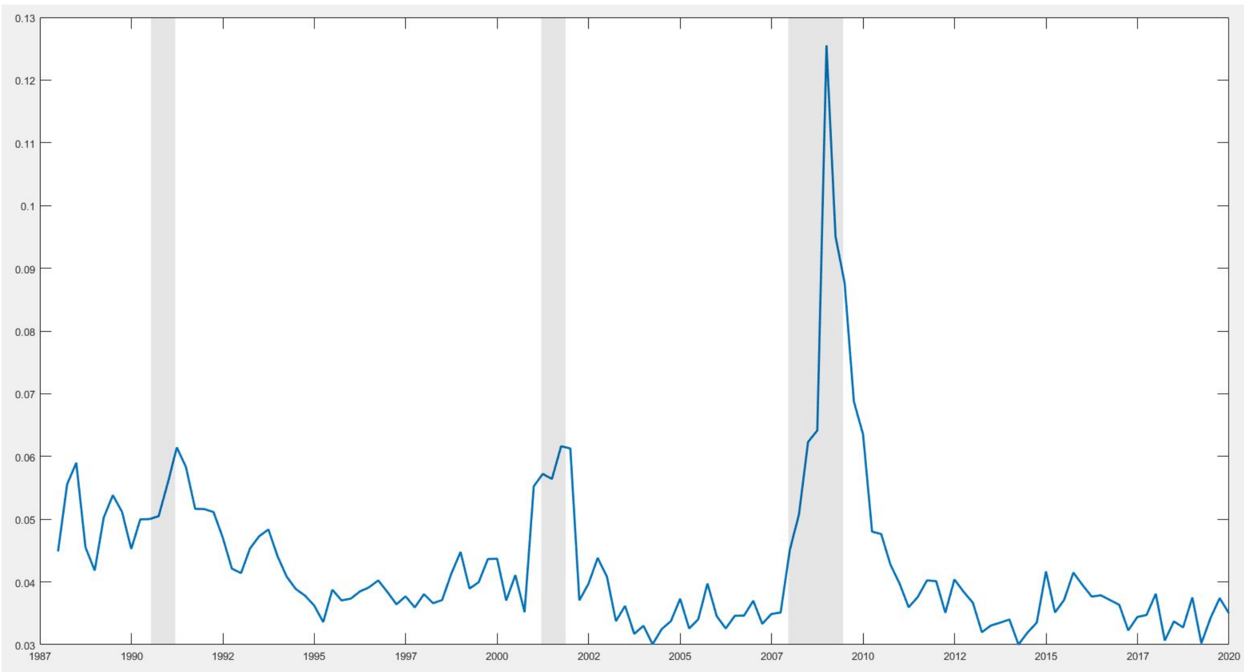

Meanwhile, we note that the ambiguity over 1-year aggregate cash-flow growth in our model is written as

$$DIVAmb_t = 4\lambda a_{c,t} + \frac{1-\rho_{ad}^4}{1-\rho_{ad}} a_{d,t}, \tag{IA.11}$$

which corresponds to the data in Figure IA.2. We determine the ambiguity parameters for $a_{d,t}$ as the following. For possible values of $\mu_{ad}, \rho_{ad}, \sigma_{ad}$, we simulate 100,000 paths of $a_{d,t}$, each with the same length as the sample from 1987Q4 to 2019Q4. We then plug simulated $a_{d,t}$ into (IA.11), together with $\lambda$ and $a_{c,t}$, to obtain simulated paths of $DIVAmb_t$. For each path we calculate the mean, volatility and AR(1) coefficient. Finally, we take average of these simulated moments across all paths and we adjust parameter values so that the simulated moments match the empirical moments.

**Figure IA.2: Ambiguity over aggregate real cash-flows**

The figure plots the annualized ambiguity over 1-year ahead aggregate real cash-flows, constructed following the method in Section 3.1. Shaded areas correspond to NBER recessions. Sample from 1987Q4 to 2019Q4.



## C Additional results

**Table IA.1:** Overreaction of subjective beliefs

The table tests the overreaction of subjective beliefs by regressing the forecast errors on lagged forecast revisions, following Coibion and Gorodnichenko (2015) and Bordalo et al. (2020)

$$x_{t+n} - x_{t+n-1} - \hat{E}_t(x_{t+n} - x_{t+n-1}) = \alpha + \beta[\hat{E}_t(x_{t+n} - x_{t+n-1}) - \hat{E}_{t-1}(x_{t+n} - x_{t+n-1})] + \epsilon_{t+n}.$$

Negative slope coefficients indicate over-reaction. We choose $n$ to be 1-year or 5-year, $x$ denotes the log earnings (E) or log dividends (D), and $\hat{E}_t(\cdot)$ denotes the subjective conditional expectation either based on the data or the model. We use two methods to extract subjective beliefs from the data. The first is simply to extract the forecasted earnings or dividends from the IBES. Due to data availability, we study the 1-year and 5-year earnings forecast data from 1987Q4 to 2019Q4, yet we can only study 1-year dividend forecasts from 2003Q1 to 2019Q4. Note that we follow Bordalo et al. (2020) by using long-term growth earnings forecast (LTG) from IBES as the proxy for 5-year forecast. Results from this method are listed in the first part. The second method is motivated by Equation (23) and results are in the second part. As long as the risk premium is negligeable, we can back out dividend expectations from the data of equity forward yields. The third part reports the results by using subjective beliefs implied from our reference model (that is, posterior distribution from learning without ambiguity).

| | Survey data | | | Yield data | | Model | |
|---|---|---|---|---|---|---|---|
| | E (1Y) | E (5Y) | D (1Y) | 1Y | 5Y | 1Y | 5Y |
| $\beta$ | -0.03 | -10.38 | -0.02 | -0.75 | -1.23 | -1.63 | -1.91 |
| $(t)$ | (-0.69) | (-2.28) | (-1.72) | (-5.87) | (-3.10) | (-1.62) | (-1.85) |
| $R^2$ | -0.66 | 8.06 | -0.64 | 16.07 | 8.38 | 11.72 | 8.28 |

**Table IA.2: Variance decomposition of ambiguity part in forward equity yields**

The table follows the model-based variance decomposition (25), where we further decompose the ambiguity part to that over real GDP growth, dividend-specific growth, and inflation. The decomposition is run over the full sample from 1987Q4 to 2019Q4, or over expansion and recession periods identified via the NBER business cycle dating.

| | | 1Y | 5Y | 10Y | 20Y |
|---|---|---|---|---|---|
| Unconditional | | | | | |
| | Ambiguity (total) | 0.04 | 0.13 | 0.22 | 0.28 |
| | RGDP | 0.01 | 0.02 | 0.01 | -0.00 |
| | Div-spec | 0.04 | 0.11 | 0.17 | 0.19 |
| | Infl | -0.00 | 0.00 | 0.03 | 0.10 |
| Expansion | | | | | |
| | Ambiguity (total) | 0.01 | 0.05 | 0.11 | 0.15 |
| | RGDP | -0.00 | -0.01 | -0.03 | -0.07 |
| | Div-spec | 0.02 | 0.07 | 0.11 | 0.11 |
| | Infl | -0.01 | -0.01 | 0.03 | 0.11 |
| Recession | | | | | |
| | Ambiguity (total) | 0.04 | 0.17 | 0.30 | 0.42 |
| | RGDP | 0.01 | 0.04 | 0.05 | 0.05 |
| | Div-spec | 0.03 | 0.12 | 0.21 | 0.27 |
| | Infl | -0.00 | 0.01 | 0.04 | 0.09 |

**Table IA.3: Robustness: alternative training periods for dividend learning**

The table reports the mean and standard deviation of spot and forward yields of dividend strips. These numbers are in annualized percentage terms. We report statistics from both our model and data, and also their correlation coefficients. For the model-implied quantities, we change the baseline training period of 5-year to either 3-year or 10-year. As a result, the sample period is from 1985Q4 (1992Q4) to 2016Q3 for 3-year (10-year) training.

| | | Spot | | | Forward | | |
|---|---|---|---|---|---|---|---|
| | | 1Y | 10Y | 10Y-1Y | 1Y | 10Y | 10Y-1Y |
| Panel A: 3-year training | | | | | | | |
| Data | | | | | | | |
| | Mean | -4.11 | -0.85 | 3.26 | -7.89 | -6.24 | 1.65 |
| | Volatility | 9.31 | 2.74 | 7.74 | 9.30 | 2.73 | 7.80 |
| Model | | | | | | | |
| | Mean | -3.30 | -1.56 | 1.74 | -8.02 | -7.75 | 0.27 |
| | Volatility | 9.41 | 2.36 | 7.47 | 9.51 | 2.38 | 7.52 |
| | Corr | 0.67 | 0.84 | 0.55 | 0.67 | 0.80 | 0.56 |
| Panel B: 10-year training | | | | | | | |
| Data | | | | | | | |
| | Mean | -6.50 | -1.88 | 4.62 | -9.38 | -6.43 | 2.95 |
| | Volatility | 8.96 | 2.18 | 7.98 | 9.82 | 3.06 | 8.13 |
| Model | | | | | | | |
| | Mean | -6.15 | -2.43 | 3.73 | -10.15 | -8.05 | 2.10 |
| | Volatility | 8.49 | 1.85 | 6.99 | 9.47 | 2.59 | 7.19 |
| | Corr | 0.54 | 0.73 | 0.44 | 0.62 | 0.81 | 0.46 |

### Table IA.4: Robustness: alternative decomposition of aggregate dividends

The table reports the standard deviation of spot and forward yields of dividend strips. These numbers are in annualized percentage terms. We report statistics from both our model and data, and also their correlation coefficients. For the model-implied quantities, we change our way of decomposing aggregate dividend in (1) based on different measures of equity durations. These include the measures proposed by Weber (2018), Gonçalves (2020a) and the book-to-market ratio in Lettau and Wachter (2007). Alternatively, we also change the breakpoint from the median to 40th or 60th percentile at the cross-section. The sample period is from 1987Q4 to 2016Q3.

| | | Spot | | | Forward | | |
|---|---|---|---|---|---|---|---|
| | | 1Y | 10Y | 10Y-1Y | 1Y | 10Y | 10Y-1Y |
| Data | Volatility | 9.48 | 2.67 | 7.95 | 9.57 | 2.81 | 8.02 |
| Weber | | | | | | | |
| | Corr | 0.58 | 0.88 | 0.41 | 0.60 | 0.79 | 0.45 |
| | Volatility | 10.59 | 2.35 | 8.48 | 10.97 | 2.75 | 8.58 |
| Gonçalves | | | | | | | |
| | Corr | 0.54 | 0.75 | 0.40 | 0.60 | 0.80 | 0.43 |
| | Volatility | 11.76 | 2.07 | 9.81 | 12.66 | 3.16 | 10.01 |
| Book-to-market | | | | | | | |
| | Corr | 0.60 | 0.90 | 0.43 | 0.60 | 0.82 | 0.44 |
| | Volatility | 9.70 | 2.05 | 7.92 | 9.58 | 2.24 | 7.85 |
| 40th | | | | | | | |
| | Corr | 0.55 | 0.77 | 0.40 | 0.58 | 0.76 | 0.41 |
| | Volatility | 8.28 | 1.65 | 6.92 | 8.72 | 2.55 | 6.96 |
| 60th | | | | | | | |
| | Corr | 0.62 | 0.76 | 0.52 | 0.64 | 0.78 | 0.53 |
| | Volatility | 11.15 | 2.26 | 9.29 | 11.64 | 2.97 | 9.35 |

**Figure IA.3: Estimated $H_t$ (in years)**