An Equilibrium Model of Career Concerns, Investment Horizons, and Mutual Fund Value Added

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Abstract

We study a dynamic equilibrium model of mutual fund investing under career concerns that features investment opportunities at different horizons. Equilibrium returns are endogenously determined by competition. Short-term investment strategies can benefit fund managers by accelerating skill revelation, while the downside risk is managed by manager exit. In the steady state, a large number of new and unskilled managers exploit the value of this call option, driving down short-term excess returns. A small number of experienced and skilled managers exploit scalable long-term investment opportunities, adding substantial value. We empirically confirm our theoretical predictions using US mutual fund data.

Keywords: career concern, investment turnover, fund manager skill, fund size, optionality

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1 Introduction

Whether active mutual fund managers are skilled has been debated in the economics and finance literature for decades. Recent advances in both theoretical and empirical research show that there is a substantial amount of skill in the fund management industry.[1] Because fund returns are highly volatile, however, it may take years or even decades for fund managers to reveal their skills to investors. Therefore, it still remains as an open question how fund managers attract enough asset under management (AUM) for their investments. This pending question relates to several puzzles about the mutual fund industry in term of the sources of value added and the distribution of fund size. For example, why does the majority of value added come from funds' long-term holdings as opposed to their short-term trades? Why does a small number of large funds with low-turnover strategies manage the majority of assets, while the industry continuously features a large number of small funds with high-turnover strategies? Why do older funds have lower fund turnover than newer funds on average? In this paper we hypothesize that the reason for these empirical patterns is that a large amount of new and small fund managers compete for short-term (high-turnover) trading opportunities to speed up revealing their investment skills to investors.

We study the questions above in a dynamic equilibrium model of mutual fund investing under career concerns. Our model builds on that of Berk and Green (2004), but deviates from it by featuring investment opportunities with different horizons whose returns are endogenously determined by competition. In our model, fund managers with a finite lifespan choose their investment strategies by trading off investment profitability and career potential. The key insight is that short-term (high-turnover) strategies maximize the expected life-time stream of management fees of new and unskilled fund mangers by offering high potential growth of their funds and also protecting against downside risk through exit options. Therefore, new and unskilled fund managers tend to invest more in short-term investment

[1] Berk and Green (2004) study a rational expectations equilibrium model of mutual fund investing where fund managers add substantial value even though their alphas are not persistent due to alpha-chasing fund flows. Berk and van Binsbergen (2015) propose value added (the product of gross alpha and size) as the measure of mutual fund manager skill, and find that value added is positive and persistent while net alphas are not. They show that the value added of top 10% skilled funds (with large AUM) persists as long as ten years.

opportunities, while experienced and skilled managers invest in long-term (low-turnover) investments. In equilibrium, a large amount of new and small fund managers compete for short-term trading opportunities, driving down their excess returns lower than those of long-term trading opportunities. We find supporting empirical evidence for our theoretical predictions using US mutual fund data.

To formalize the aforementioned ideas, we consider an infinite-horizon, discrete-time model with a continuum of mutual funds, which have access to investment opportunities that may deliver excess returns (alphas) over the passive benchmark. There are two types of investment opportunities: short-term and long-term opportunities. The investment opportunities can be interpreted as investment strategies that exploit mispricing in the market. For example, investment in those opportunities will deliver alphas over the passive benchmark when prices converge to their fundamental value. A short-term investment opportunity is more likely to converge to the fundamental value more quickly, whereas a long-term investment opportunity converges more slowly.[2]

There is a continuum of fund managers in the economy, who have finite lifespan because they randomly exit the economy in each period. They may also voluntarily exit if the value of continuing operation falls below the outside option. New managers enter in the economy so that the mass of fund managers in the economy is kept as a constant at any point of time. Following Berk and Green (2004), we assume that fund managers' ability is initially unknown to everyone in the economy. Given the history of performance, investors update their beliefs about fund managers' ability. Under updated beliefs, investors' money flows to and from each fund until its expected net alpha becomes zero. Therefore, fund sizes are tied to perceived skills of managers under the assumption of rational expectations.

In each period, fund managers can choose to either exit or continue fund operation. In case they continue, they can choose to invest in either a short-term or a long-term investment opportunity. Fund managers maximize their expected utility of consuming the stream of fund management fees after fixed and variable costs. By investing in short-term

[2]Because funds can immediately deploy their capital from realized existing investment to a new opportunity, short-term investment is equivalent to high-turnover strategy in our model. Likewise, long-term investment is equivalent to low-turnover strategy.

opportunities, they can accelerate the revelation of their talents, which are equally good or bad conditioning on the current information. The benefit of short-term investment arises from the optionality of fund operation, and the finiteness of their lifespan. Fund managers can explore the possibility of higher fund growth in case of good performance before their career is over, but can still limit the adverse impact of bad performance by choosing to exit from the industry. Such option value is more sensitive to investment turnovers if managers are new (because talents are less known) or if their funds are small (because their skills are perceived to be low). This implies that new and small funds are willing to accept lower excess returns of short-term opportunities relative to those of long-term opportunities due to the extra option value.

We show that fund managers choose to exit when their perceived skills are sufficiently low, and older fund managers with the same perceived skill are more likely to exit than new managers because they have a smaller growth potential. As a consequence, the stationary distribution of surviving fund managers' talents becomes on average higher than the initial distribution of talents. As fund managers become older or perceived by investors as skilled, they switch to long-term investment opportunities and the revelation of their talents becomes slower. The stationary distribution of perceived skills determines the distribution of fund sizes, leading to a large number of small (high-turnover) funds and a small number of large (low-turnover) funds in the economy.

Another important feature of our model is that the gross alpha of a fund's investment is affected by its skill as well as the capacity constraint at the aggregate level; the excess return of a fund's investment over the passive benchmark increases in the level of its skill, but decreases in the amount of aggregate investment by other funds in the same type of opportunities. The capacity constraint at the aggregate level is equivalent to strategic substitutability in investment. It is well known in the literature that informed arbitrage can be strategic substitute as more participation in informed trading eliminates mispricing (Grossman and Stiglitz [1980]).

Consequently, the amount of capital invested in (and the resulting excess returns of) investment opportunities are determined by the distribution of funds' perceived skills. Because we can pin down all the choices of fund managers in terms of their state variables, which is a pair of perceived skills and confidence of beliefs, we can construct a Markov transition function of fund manager states. Then, we can obtain the stationary distribution of state variables as the steady state outcome of those transitions. Under the stationary distribution, there are many new and relatively unskilled funds in the economy. They invest in short-term opportunities for growth options, which drive the excess returns of short-term opportunities down to a level lower than long-term opportunities. As a result, old and skilled fund managers optimally choose to invest in long-term opportunities to exploit higher profit margins. The aggregate value added of investing in short-term opportunities is smaller because of both the competition for growth and the low average skill of new managers, whereas the aggregate value added of investing in long-term opportunities is larger because of both the lack of competition and the high average skill of experience managers. Finally, our results also shed light on a potential source of investor short-termism. The misalignment of incentives between funds and investors makes it difficult for investors to access to long-term investment opportunities, thereby creating long-term efficiency in financial markets.

Empirically, we use 59 years of US mutual fund data to confirm our model predictions. Consistent with our prediction that high-turnover strategies reveal fund managers' talents faster, we find that the flow-performance sensitivity is higher for high- than low- turnover funds, and the results are stronger for the returns in the past quarter or year than three years. Moreover, the joint distribution of fund size, age, and turnover in the data confirms the predictions in our model. New and small funds are more likely to choose high-turnover strategies, while old and large funds are more likely to choose low-turnover strategies. Since the fund managers perceived by investors as skilled attract more capital and are likely to switch to low-turnover strategies, low-turnover funds manage substantially more assets than high-turnover funds do. For high-turnover funds, the number of new managers is substantially more than the number of old managers, whereas for low-turnover funds, the total amount of assets managed by old managers is substantially more than the amount managed by new managers. Lastly, as our model predicts, because high-turnover strategies offer higher future growth potentials, new and small fund managers are willing to accept lower current value added for high-turnover strategies. The value added of high-turnover funds (close to zero) is substantially smaller than the value added of low-turnover funds under both the CAPM and the Vanguard benchmarks. Using a unique dataset of transaction and daily holding data as in Van Binsbergen, Han, Ruan, and Xing (2020), we further document that the trade of mutual funds on average do not add value in the first seven months. These results are consistent with our conjecture that a main function/value added of the large number of small and high-turnover funds is to select skilled managers for large low-turnover funds, which add the majority of value for the mutual fund industry.

The paper is organized as follows. In Section 2, we review related literature. In Section 3, we describe our theoretical model. In Section 4, we solve for equilibrium of our model. In Section 5, we provide main theoretical findings and test them empirically. In Section 6, we conclude.

2 Literature

The academic literature has been relatively successful at solving mutual fund puzzles at the fund level in the past two decades. For example, with an innovative use of decreasing returns to scale, Berk and Green (2004) reconciled the lack of persistence in fund performance with the fact that money flows into (out of) good (bad) performing funds. By introducing portfolio liquidity and fund turnover into Berk and Green's model, Pastor, Stambaugh and Taylor (2020) explained several tradeoffs among active mutual funds' characteristics. However, the majority of puzzles at the mutual fund industry level are left unanswered, largely because of the dynamic nature of these puzzles and the complexity of the interactions between funds. Our model solves these problems and emphasizes the importance of the life cycle of mutual funds to the value added of the mutual fund industry. In particular, we show that new fund managers use high-turnover strategies to speed up investors' learning of their skills, which affects the joint distributions of mutual funds' skill and age in a dynamic equilibrium.

We focus on the relation between fund turnover and the perceived skill of a fund manager, measured by value added as proposed in Berk and Green (2004) and Berk and van

Binsbergen (2015). In contrast, previous studies investigate the relation between fund turnover and the abnormal return received by investors, measured by net alphas, or gross alphas, and the empirical evidence on this relation is mixed. For example, Pastor, Stambaugh, and Taylor (2017) documents a positive relation in both the time series and the cross section, Elton, Gruber, Das, and Hlavka (1993) and Carhart (1997) find a negative relation, and Wermers (2000), Kacperczyk, Sialm, and Zheng (2005), and Edelen, Evans, and Kadlec (2007) find no significant relation. Cremers and Pareek (2016) and Lan, Moneta, and Wermers (2019) construct direct measures for the average investment horizon of a fund and find that long-horizon funds outperform short-horizon funds in the cross-section. Our model predicts that low-turnover funds do have more skilled managers than high-turnover funds in equilibrium, since managers that are new or perceived as unskilled prefer high-turnover strategies, which reveal their skills faster. However, the larger amount capital managed by low-turnover funds have brought their net alphas to zero because of the decreasing returns to skill (as in Berk and Green [2004]).

This paper also relates to a growing literature distinguishing fund managers from funds. For example, Choi, Kahraman, and Mukherjee (2016) studies capital allocations to managers with two mutual funds and shows that investors learn about managers from their performance records in both funds. Kaniel and Orlov (2020) models the asymmetric information between fund managing firms and investors about their fund managers' skills and find that fund managing firm churns unskilled managers frequently to help retained mangers build reputation fast. Meanwhile, fund managing firms expropriate mangers' ability by threatening to undervaluate their skills. Consistently, we show that new fund managers choose high-turnover strategies to speed up the learning of their skills at the costs of lower value added, which increases their bargaining power against fund managing firms. Different from Kaniel and Orlov (2020), we focus on the relationship between managers' investment choices and investors' learning under symmetric information, instead of the agency problem between fund managing firms and managers under asymmetric information.

Finally, our paper is related to both theoretical and empirical literature on investment and performance in different horizons. Theoretically, Shleifer and Vishny (1990) and Dow and Gorton (1994) show that long-term assets should have larger mispricing wedge than short-term assets because investors can redeploy their capital faster. Dow, Han, and Sangiorgi (2021) microfound equilibrium capital distributions in a dynamic model, and show how mispricing wedge should be determined in equilibrium. Building on this intuition, our model shows that fund performance difference across horizons arise from equilibrium distribution of fund skills. Our model further suggests larger mispricing wedges can arise from an alternative channel of career concern unlike above papers in the literature; fund managers are willing to accept smaller trading profits in short-term investments for growth options. Empirically, Cella, Ellul, and Giannetti (2013) shows that institutional investors with short investment horizons sell more during market turmoil, and this creates price pressure for stocks held mostly by short-horizon investors. Giannetti and Kahraman (2018) provides evidence that open-end organizational structures undermine incentives for asset managers to attack long-term mispricing.

3 Model

We consider an infinite-horizon model in discrete time under fund managers' career concern. Our model builds on the model of Berk and Green (2004), but unlike theirs, our model features investment opportunities with different investment horizons whose returns are endogenously determined by competition under strategic substitutability.

3.1 Mutual Funds and Investment Opportunities

There is a continuum of risk-neutral mutual funds in the economy indexed by j, and we denote the set of all active funds operating in period t by $\mathcal{J}_t$. Mutual funds have access to investment opportunities that may deliver excess returns (alphas) over the passive benchmark. In our model, we focus on the misalignment in incentives between fund managers and their investors rather than the one between funds and fund managers. Therefore, we will use funds and fund managers interchangeably, and shut down any potential misalignment in incentives between funds and fund managers.

There are two types of investment opportunities: short-term and long-term opportunity.

We index each type of investment opportunities by $i \in \{S, L\}$ where S denotes short-term opportunity and L denotes long-term opportunity. The investment opportunities can be interpreted as investment strategies that exploit mispricing in the market. For example, investment in those opportunities will deliver alphas over the passive benchmark when prices converge to fundamental. For simplicity, we assume that an investment opportunity yields a zero excess return over the passive benchmark until its payoff realizes. More formally, each fund j's investment in a type i opportunity yields a random excess return over the benchmark before costs and fees (or, gross alpha) between period t and $t+1$ as follows:

$$R^j_{t+1,i} \equiv e^j_{t+1,i} \alpha^j_{t+1,i}, \tag{1}$$

where $\alpha^j_{t+1,i}$ is the fund's excess return conditioning on the realization of payoff, and $e^j_{t+1,i}$ is an identically and independently distributed (i.i.d.) random variable that is equal to one with probability d_i, and zero with probability $1 - d_i$. A short-term investment opportunity is more likely to mature early whereas a long-term investment opportunity is less likely to do so, i.e., $0 < d_L < d_S \leq 1$. The inverse of d_i is interpreted as investment duration (i.e., the payoff takes on average $1/d_i$ periods to realize). We also assume that the realization of payoff is public information.

We denote by $q^j_{t,i}$ the amount of fund j's investment in a type i opportunity. Similarly as in Berk and Green (2004), we assume that the cost of actively managing investment in a type i opportunity increases convexly in its size, and is independent of its skill; investing $q^j_{t,i}$ creates a cost of $C_i(q^j_{t,i})$ for fund j where $C'_i(\cdot) > 0, C''_i(\cdot) > 0$, and $C_i(0) = 0$. The convexity is assumed to ensure a unique interior optimum for technical simplicity. The assumption of increasing cost in the fund's size of active management can be motivated by costs related to the price impact costs when acquiring and rebalancing its positions. In addition, we assume that, given the same size, the marginal cost of short-term investment is greater than or equal to that of long-term investment, i.e., $C'_S(q) \geq C'_L(q)$ for all $q \geq 0$. This captures the idea that short-term investment incurs higher costs from price impact.

For simplicity, we further assume that a fund can hold a position on only one type of investment opportunities at a time. This is a technical assumption that facilitates analysis on funds' choice of investment horizons. In reality, funds may diversify among different types of investment opportunities, but may also tilt toward a certain type of investment. The assumption of a fund having only one type of positions at a time captures the idea of having major positions in its portfolio. Finally, we assume that $q^j_{t,i}$ is observable to investors for each fund j (i.e., both the type and the size of investment are observable). Each fund should also cover a fixed cost of operation F which reflects overhead, back-office expenses in each period. To focus on the parameter values that are economically interesting, we assume that F is positive but cannot be too big to make all the funds in the economy choose to stop their operation.

In our model, the gross alpha of a fund's investment is affected by its skill as well as the capacity constraint at the aggregate level; the excess return of a fund's investment over the passive benchmark increases in the level of its skill, but decreases in the amount of aggregate investment by other funds in the same type of opportunities. The capacity constraint at the aggregate level is equivalent to strategic substitutability in investment. It is well known in the literature that informed arbitrage can be strategic substitute as more participation in informed trading eliminates mispricing (e.g., Grossman and Stiglitz [1980]). See, for example, Dow, Han, and Sangiorgi (Forthcoming) for a microfoundation of strategic substitutability with investment opportunities under different horizons.

To formalize the aforementioned intuition, we denote by $Q_{t,i}$ the aggregate amount invested in all i type opportunities in period t by all the funds in the economy:

$$Q_{t,i} \equiv \int_{j \in \mathcal{J}_t} q^j_{t,i} dj.$$

Fund j's excess return on a type i investment opportunity increases in the talent parameter ϕ^j, which captures the fund manager's true ability of generating alpha, but decreases in the aggregate investment $Q_{t,i}$:

$$\alpha^j_{t+1,i}(Q_{t,i}, \phi^j) \equiv g_i(Q_{t,i})(\phi^j + \epsilon^j_{t+1})$$

where $g_i(Q_{t,i})$ is a decreasing function of $Q_{t,i}$ which captures the profitability of a type i investment opportunity under capacity constraint, and ϵ^j_{t+1} is an idiosyncratic noise component specific to fund j. For technical simplicity, we assume that the marginal return on an investment opportunity is infinity if no one invests in the opportunity, i.e., $g_i(0) = \infty$ for all $i \in \{S, L\}$. This assumption allows us to focus on economically meaningful outcomes by ruling out cases where no investment is made on any of the available investment opportunities. The idiosyncratic noise term ϵ^j_{t+1} follows an i.i.d. normal distribution with mean zero and variance σ^2. We denote the precision of this uncertainty by $\omega \equiv 1/\sigma^2$.

Each fund is terminated randomly with a probability $1 - \kappa$ every period, but may also voluntarily shut down its operation. All funds that exit the economy are replaced by the same mass of new funds that enter the economy. This simplifying assumption gives us more tractability by preventing the mass of funds from becoming another state variable in the economy. At the birth of a new fund (indexed by j), the prior of the fund's talent parameter ϕ^j follows an i.i.d. normal distribution with mean ϕ_0 and variance η^2. We denote the precision of the prior by $\gamma \equiv 1/\eta^2$. We assume that the true talent parameter ϕ^j of the fund manager in fund j is not known to anyone in the economy.

Finally, we assume that all the random variables in the model are jointly independent for technical tractability.

3.2 Fund Performance and Belief Updates on Skills

The fund manager in fund j is paid a management fee f^j_t in each period t, which is a fraction of its asset under management $q^j_{t,i}$. The fund's excess total payout to investors over the passive benchmark in the subsequent period is

$$TP^j_{t+1} \equiv q^j_{t,i} R^j_{t+1,i} - C_i(q^j_{t,i}) - q^j_{t,i} f^j_t.$$

Then, the net alpha of fund j is given by

$$\hat{\alpha}^j_{t+1} \equiv \frac{TP^j_{t+1}}{q^j_{t,i}} = R^j_{t+1,i} - \frac{C_i(q^j_{t,i})}{q^j_{t,i}} - f^j_t = e_{t+1,i}\alpha^j_{t+1,i} - c_i(q^j_{t,i}), \tag{2}$$

where $c_i(q^j_{t,i})$ is the unit cost associated with investing in fund j that actively manages the size of investment $q^j_{t,i}$ in opportunity i:

$$c_i(q^j_{t,i}) \equiv \frac{C_i(q^j_{t,i})}{q^j_{t,i}} + f^j_t.$$

Therefore, the expected value of the fund's net alpha with the choice of investment in opportunity i is

$$\mathrm{E}_t(\hat{\alpha}^j_{t+1}) = d_i \phi^j g_i(Q_{t,i}) - c_i(q^j_{t,i}). \tag{3}$$

Because the size of fund as well as the type of investment opportunities, which is summarized by $q^j_{t,i}$, are observable for all funds, the aggregate amount of investment, $Q_{t,i}$, is common knowledge for each type of investment opportunity $i \in \{S, L\}$. This also implies other quantities like $c_i(q^j_{t,i}), g_i(Q_{t,i})$ are also common knowledge for each opportunity i and fund j in equilibrium. Therefore, we can define a new variable ξ^j_{t+1} which is made of only observable variables:

$$\xi^j_{t+1} \equiv \frac{R^j_{t+1} + c_i(q^j_{t,i})}{g_i(Q_{t,i})}.$$

By observing the history of $\{R^j_{s+1,i}, q^j_{s,i}, Q_{s,i}\}^t_{s=1}$, investors can infer the history of $\{\xi^j_{s+1}\}^t_{s=1}$. Then, Eqs. (1) and (2) imply that ξ^j_{t+1} is the sufficient statistic for information about the manager's true talent whenever the payoff realizes, and is zero (thus, uninformative) otherwise:

$$\xi^j_{t+1} = \begin{cases} \phi^j + \epsilon^j_{t+1} & \text{if } e_i = 1 \\ 0 & \text{if } e_i = 0. \end{cases}$$

All agents update their posterior belief on each fund's skill on the basis of the history of sufficient statistic in a Bayesian manner. Let the posterior mean of fund j's talent in period t be denoted as

$$\hat{\phi}^j_t \equiv \mathrm{E}(\phi^j | \xi^j_1, ..., \xi^j_t),$$

and let τ^j_t denote the number of payoff realizations of fund j by period t (then, there is τ^j_t informative signals in the sequence $\xi^j_1, ..., \xi^j_t$.) Because only realized payoffs deliver the signals in the sufficient statistic, investors have more precise information about fund j with higher τ_t^j.

The following lemma derives the law of motion for the posterior belief on the fund manager's talent.

Lemma 1. *The posterior belief $\hat{\phi}_t^j$ on fund j's talent parameter ϕ^j is given as a function of the prior belief $\hat{\phi}_{t-1}^j$, the number of realized performance τ_t^j, and the sufficient statistic for performance in the previous period ξ_t^j as follows:*

$$\hat{\phi}_t^j = \hat{\phi}_{t-1}^j + e_{t,i}^j \left(\frac{\omega}{\gamma + \tau_t^j \omega} \right) (\xi_t^j - \hat{\phi}_{t-1}^j). \tag{4}$$

Proof. Given the realization of $e_{t,i}^j$, the Bayes' rule implies

$$\hat{\phi}_t^j = (1 - e_{t,i}^j)\hat{\phi}_{t-1}^j + e_{t,i}^j \left[\frac{\gamma + (\tau_t^j - 1)\omega}{\gamma + \tau_t^j \omega} \hat{\phi}_{t-1}^j + \frac{\omega}{\gamma + \tau_t^j \omega} \xi_t^j \right]$$

□

3.3 Fund Manager's Optimization Problem

In period t, each fund j can choose its investment type $i \in \{S, L\}$, its size $q_{t,i}^j$ and its fee f_t^j, to maximize the present value of its expected utility of receiving the stream of fees such that

$$\mathrm{E}_t \left[\sum_{s=t}^{T^j} u\left(q_{s,i} f_s^j - F \right) \right], \tag{5}$$

where T^j is the last period in which the fund operates before exiting the economy, and $u(\cdot)$ is a infinitely-differentiable, bounded utility function with $u' > 0, u'' < 0$ and $u(0) = 0$.[3]

Following the rational expectations assumption in the literature, as described by Berk and Green (2004), we similarly assume that there is a continuum of risk-neutral investors who can invest either in funds by paying fees or in the passive benchmark without any cost. We assume that investors are unconstrained (or equivalently, their supply of capital is infinitely elastic for any investment opportunity with positive excess returns). Therefore,

[3]The assumption that the utility function is bounded and concave does not affect our result qualitatively. By bounding rewards to finite values, we can ensure that the value function is bounded. Then, we can obtain existence of value functions using the standard Banach fixed point theorem (see the proof of Theorem 1 in Appendix A). The assumption that $u(0) = 0$ normalizes the utility of zero income to be zero, simplifying our notations by eliminating the extra notation for reservation utility in case of shutting down the operation.

investors' fund flows to and from funds until each fund j has a zero net expected excess-return over the passive benchmark (net alpha):

$$\mathrm{E}_t(\hat{\alpha}_{t+1}^j) = 0 \tag{6}$$

where $\hat{\alpha}_{t+1}^j$ is the net alpha of fund j's investment in time t. Substituting Eq. (3) into Eq. (6) yields

$$d_i \hat{\phi}_t^j g_i(Q_{t,i}) = c_i(q_{t,i}^j) = \frac{C_i(q_{t,i}^j)}{q_{t,i}^j} + f_t^j. \tag{7}$$

That is, the fund flow equates the average excess return with the average cost in equilibrium. Therefore, Eq. (7) implies the revenue of the fund such that

$$q_{s,i} f_t^j = d_i \hat{\phi}_t^j g_i(Q_{t,i}) q_{s,i} - C_i(q_{t,i}^j). \tag{8}$$

We focus on stationary equilibrium where only state variables, which are perceived skills $\hat{\phi}$ and the number of belief updates τ, matter for the fund manager's optimal choice. As every variable is time-invariant, we will drop the time subscript t and fund index j from now on for notational convenience. Then, we can represent the maximization problem in Eq. (5) in a recursive form; the value of continuing the operation of an individual fund given the state variables $\hat{\phi}, \tau$ can be written as

$$V(\hat{\phi}, \tau) \equiv \max\left\{V_S(\hat{\phi}, \tau), V_L(\hat{\phi}, \tau)\right\}, \tag{9}$$

where $V_i(\hat{\phi}, \tau)$ is the value of choosing a type i investment opportunity such that

$$\begin{aligned} V_i(\hat{\phi}, \tau) \equiv \sup_{q_i \in [0,\infty)} & u\left(d_i \hat{\phi} g_i(Q_i) q_i - C_i(q_i) - F\right) \\ & + \kappa\left[(1-d_i)\max\left\{V(\hat{\phi}, \tau), 0\right\} + d_i \,\mathrm{E}\left[\max\left\{V(\hat{\phi}', \tau+1), 0\right\} \middle| \hat{\phi}, \tau\right]\right], \end{aligned} \tag{10}$$

and $\hat{\phi}'$ denotes the posterior of the perceived skill in the event of a payoff realization:

$$\hat{\phi}' = \hat{\phi} + \left(\frac{\omega}{\gamma + \tau\omega}\right)(\xi - \hat{\phi}). \tag{11}$$

The fund exits the economy when its continuation value of operation becomes less than or equal to the reservation utility of zero for the first time as is shown in Eq. (9). If the fund decides to operate by paying the fixed cost, it chooses between long-term and short-term opportunities, and then decides the size of investment. The possibility of exit gives an option value to fund's growth potentials, which is reflected in the continuation value in Eq. (10). This optionality is the key to fund manager behavior as is shown in the next section.

4 Equilibrium

4.1 Optimal Choice

Fixing the choice of the types of investment opportunities to type i, the optimal level of q_i will set the expected excess return on the marginal dollar equal to the marginal cost of expansion. From the first order condition with respect to q_i in Eq. (10), we have[4]

$$d_i \hat{\phi} g_i(Q_i) = C_i'(q_i). \tag{12}$$

Let $q_i^*(\hat{\phi})$ be the solution of Eq. (12) given the values of $\hat{\phi}$. The optimal size $q_i^*(\hat{\phi})$ increases in the perceived skill $\hat{\phi}$ because $C_i' > 0$. Furthermore, $q_i^*(0) = 0$ because the marginal benefit is less than the marginal cost, i.e., $0 = d_i \hat{\phi} g_i(Q_i) \leq C_i'(q_i)$ with $\hat{\phi} = 0$ (the fund flow becomes zero at the point.)

The following lemma shows that the optimal size increases in the level of perceived skill.

Lemma 2. *Fixing the type of investment opportunity, q_i^* increases in $\hat{\phi}$, i.e., $dq_i^*/d\hat{\phi} > 0$.*

Proof. Let

$$\psi(\hat{\phi}, q_i^*) \equiv d_i \hat{\phi} g_i(Q_i) - C_i'(q_i^*).$$

Then, $\hat{\phi}$ and q_i^* satisfy $\psi(\hat{\phi}, q_i^*) = 0$ under the optimal choice. By the implicity function theorem, we have

$$\frac{dq_i^*}{d\hat{\phi}} = -\frac{\frac{\partial \psi}{\partial \hat{\phi}}}{\frac{\partial \psi}{\partial q_i^*}} = \frac{d_i g_i(Q_i)}{C_i''(q_i)} > 0, \tag{13}$$

[4]Note that the total revenue in Eq. (8) is a deterministic function of the chosen fee by the fund, so the monotonic transformation $u(\cdot)$ of a deterministic function does not alter the optimization problem in any other way. This is why the concavity of the utility function does not affect the fund's choice of size. But the concavity and the boundedness of $u(\cdot)$ help achieving existence of the value function by preventing it from exploding with high values of $\hat{\phi}$.

because $C_i'' > 0$. $\square$

Using the optimal size derived under the given investment opportunity, we can represent the indirect value of choosing each type $i \in \{S, L\}$ of investment opportunity in Eq. (10) as follows:

$$V_i(\hat{\phi}, \tau) = \Pi_i(\hat{\phi}) + \kappa(1 - d_i)V(\hat{\phi}, \tau) + \kappa d_i \operatorname{E}\left[\max\left\{V(\hat{\phi}', \tau + 1), 0\right\}\right]. \tag{14}$$

where $\Pi_i(\hat{\phi})$ is the expected utility of the fund's payoff in the current period:

$$\Pi_i(\hat{\phi}) \equiv u\left(d_i \hat{\phi} g_i(Q_i) q_i^* - C_i(q_i^*) - F\right). \tag{15}$$

Using the results so far, we can establish existence and uniqueness of the value function, and also characterize it.

Theorem 1. *There exists a unique value function V that solves Eqs. (9)-(11). Furthermore, V strictly increases in $\hat{\phi}$, and strictly decreases in τ.*

Proof. See Appendix A. $\square$

Using the properties of the value function found in Theorem 1, we can obtain the optimal choice of the fund manager on exit or investment opportunities. In the following two theorems, we show that the fund manager uses threshold strategies in both choices.

The fund manager strictly prefers exiting to continuing operation whenever the maximum value of continuing investment in either of short-term or long-term opportunity is less than zero, i.e., $V(\hat{\phi}, \tau) \equiv \max\left\{V_S(\hat{\phi}, \tau), V_L(\hat{\phi}, \tau)\right\} < 0$. We can show that, given the precision of skill perception, the fund manager chooses to exit if the perception of skill is sufficiently low, and continue operation otherwise. Furthermore, the fund manager chooses to exit at a higher level of perceived skill as the confidence of the perceived belief increases.

Theorem 2. *(Exit choice) If at least some funds find it optimal to invest at given τ, there exists a unique positive threshold $\hat{\phi}_E(\tau)$ such that $V(\hat{\phi}, \tau) < 0$ if and only if $\hat{\phi} < \hat{\phi}_E(\tau)$. Furthermore, the threshold $\hat{\phi}_E(\tau)$ increases in τ.*

Proof. See Appendix A. $\square$

The fund manager's incentive to continue operation with a low perception of skill becomes weaker if the perception is more precise. Intuitively, the option value to growth potentials becomes smaller as the volatility of outcomes becomes smaller. That is, a new fund perceived as unskilled may continue operation even with loss for a while, hoping for better performance that will upgrade their skill perception. But, an old, unskilled fund will not continue operation when loss becomes large.

Similarly, the option value to growth potentials also drives the fund manager's choice in investment opportunities. Conditioning on continuing operation (i.e., $\hat{\phi} \geq \hat{\phi}_E(\tau)$), the fund manager strictly prefers short-term investment if and only if $V_S(\hat{\phi}, \tau) > V_L(\hat{\phi}, \tau)$, or equivalently:

$$(d_S - d_L)\kappa \left(\mathrm{E}\left[\max\left\{ V(\hat{\phi}', \tau+1), 0 \right\} \middle| \hat{\phi}, \tau \right] - V(\hat{\phi}, \tau) \right) > \Pi_L(\hat{\phi}) - \Pi_S(\hat{\phi}). \tag{16}$$

The L.H.S represents the difference between long-term and short-term investment opportunities in expected increases of value by updating its perception of skill, whereas the R.H.S represents the expected utility differentials in the current period. Note that the benefit of revealing talent arises from the protection against downward risk due to the optionality of fund operation. The fund manager enjoys higher expected payoff in the future in case of good performance, but can minimize the impact of bad performance by simply exiting from the industry. By choosing short-term investment, a fund can exploit the option value, but the benefit becomes smaller as the confidence of skill perception gets higher.

Eq. (16) implies an important aspect of equilibrium value added. It has to be the case that long-term investment opportunity is more profitable than short-term investment by offering higher value added fixing the type of fund manager. Otherwise, no one will invest in long-term investment because it is dominated on both dimensions of growth option and trading profits. Therefore, from now on, we conjecture that the R.H.S is strictly positive (i.e., long-term investment is more profitable than short-term investment), and verify this later.

Using Eq. (16), we can show that the fund manager strictly prefers short-term investment whenever its perceived skill is low enough at the given level of belief precision. The threshold is unique if the value function is sufficiently concave, and the threshold decreases with lower precision of belief.

Theorem 3. *(Investment choice) There exists a threshold $\hat{\phi}_S(\tau)$ such that a fund with τ chooses short-term investment if $\hat{\phi} < \hat{\phi}_S(\tau)$. Furthermore, the threshold is unique if $-V_{11}/V_1$ is sufficiently large for $\hat{\phi} \geq \hat{\phi}_E(\tau)$ given any τ where $V_1 \equiv \partial V/\partial\hat{\phi}, V_{11} \equiv \partial^2 V/(\partial\hat{\phi})^2$, and the threshold $\hat{\phi}_S(\tau)$ decreases in τ in that case.*

Proof. See Appendix A. □

The R.H.S of Eq. (16) increases in $\hat{\phi}$ because long-term investment opportunity offers higher value added per unit of investment. Therefore, funds benefit more from higher perception of skill by investing long-term. On the other hand, the L.H.S of Eq. (16) decreases in $\hat{\phi}$ because the option value of exit disappears as the exit threshold becomes increasingly less relevant as $\hat{\phi}$ increases. The intersection between the R.H.S and the L.H.S gives the unique threshold of choosing different investment opportunities.

Fund managers choose to invest in short-term if perceived skill is low. This is to exploit the growth option by speeding up the revelation of skills. Furthermore, funds tend to choose short-term investment more if the perception of skills is less certain. The interpretation is that new funds take advantage of exit option by continuing their operations, and also by investing in short term opportunities. On the other hand, old funds find more stable profits from long term opportunities more attractive.

4.2 Markov Transition Function and Stationary Distribution

So far, we have shown that the optimal decisions of fund managers are completely specified by the state variable $(\hat{\phi}, \tau)$, which is a pair of perceived skill and confidence of belief. Now, we construct the transition function of the state variable. For expositional convenience, we introduce the following new notations. We denote by $I(\hat{\phi}, \tau)$ an indicator function which equals one if a fund with $(\hat{\phi}, \tau)$ continues its operation and zero otherwise. We denote by $d(\hat{\phi}, \tau)$ the probability of payoff realization as a result of optimal choice given $(\hat{\phi}, \tau)$, i.e., $d(\hat{\phi}, \tau) = d_S$ if short-term strategy is optimally chosen and $d(\hat{\phi}, \tau) = d_L$ otherwise. Finally, we denote by $\tau = E$ the state of exit.

The state process of each individual fund manager follows a Markov process. Using the results in Section 4.1, we can represent the Markov transition function as follows:

Lemma 3. *The transition function from the current state $(\hat{\phi}, \tau)$ to the future state $(\hat{\phi}', \tau')$ is given by*

$$Z\left(\hat{\phi}', \tau' \middle| \hat{\phi}, \tau\right) = \begin{cases} \kappa I(\hat{\phi}, \tau) d(\hat{\phi}, \tau) n\left((\gamma + \tau\omega)\left(\hat{\phi}' - \hat{\phi}\right)\right) & \text{if } \tau' = \tau + 1; \\ \kappa I(\hat{\phi}, \tau)(1 - d(\hat{\phi}, \tau)) & \text{if } \tau' = \tau; \\ 1 - \kappa I(\hat{\phi}, \tau) & \text{if } \hat{\phi}' = \hat{\phi}, \tau' = E; \\ 0 & \text{otherwise}, \end{cases} \tag{17}$$

where $n(\cdot)$ is the probability density function of the standard normal distribution.

Proof. See Appendix A. □

We denote by $\nu(\hat{\phi}, \tau)$ the joint density of the state $(\hat{\phi}, \tau)$ in the current period. Because the perceived skill is continuous and the realization of payoffs is discrete, $\nu(\cdot, \cdot)$ is a mixed joint density function. Given $\nu(\hat{\phi}, \tau)$ in the current period, we can represent $\nu(\hat{\phi}', \tau')$ in the subsequent period using the transition function in Lemma 3:

$$T\nu(\hat{\phi}', \tau') = \begin{cases} \sum_{\tau=0}^{\infty} \int_{\phi_E(\tau)}^{\infty} Z\left(\hat{\phi}', \tau' \middle| \hat{\phi}, \tau\right) d\nu(\hat{\phi}, \tau) & \text{for } \tau' \geq 1 \text{ and } \tau' \neq E; \\ \begin{pmatrix} \kappa(1 - d(\hat{\phi}, \tau))\nu(\phi_0, 0) \\ +1 - \kappa \sum_{\tau=0}^{\infty} \int_{\hat{\phi}_E(\tau)}^{\infty} I(\hat{\phi}, \tau) d\nu(\hat{\phi}, \tau) \end{pmatrix} & \text{for } (\hat{\phi}, \tau') = (\phi_0, 0); \\ 0 & \text{otherwise.} \end{cases} \tag{18}$$

In Eq. (18), the density of any state $\hat{\phi}', \tau'$ in the subsequent future can be generally calculated by counting all flows to the state. In the second line, we treat one exception of the first line, which is the initial entry point $\hat{\phi} = \phi_0, \tau = 0$. The first component in the second line captures the remaining mass of managers after the transition and the second component captures the mass of new managers who enter the economy to replace exiting managers.

In a stationary equilibrium, the distribution of types is time-invariant. That is, the density of state variables in the subsequent period should be equal to the one in the current period. That is, the stationary distribution $\nu(\cdot,\cdot)$ solves the following functional equation:

$$\nu(\hat{\phi}', \tau') = T[\nu(\hat{\phi}', \tau')]. \tag{19}$$

The stationary distribution is a long-run outcome which is the result of convergence of distribution for any given initial distribution. We can show that such a stationary distribution exists and is actually unique.

Theorem 4. *There exists a unique stationary distribution ν that solves Eq. (19).*

Proof. See Appendix A. □

4.3 Stationary Equilibrium

We focus on stationary equilibrium where fund managers make decisions only based on state variables and the distribution of state variables are invariant over time. That is, optimal decision making is a function of a pair of perceived skills and the number of belief updates $(\hat{\phi}, \tau)$, and their distribution is stationary as in Eq. (19).

The steady state equilibrium is pinned down by the amount of total investment in each investment opportunity (Q_S, Q_L) where

$$Q_i = \sum_{\tau=0}^{\infty} \int_{\phi_E(\tau)}^{\infty} q_i(\phi, \tau)\nu(\phi, \nu) \quad \text{for all } i \in \{S, L\}.$$

For this, let us define $q_i(\hat{\phi}, \tau; Q_S, Q_L)$ as the optimal size of investment in investment opportunity i given state variables $\hat{\phi}, \tau$ and aggregate investment (Q_S, Q_L). Likewise, let $I(\hat{\phi}, \tau; Q_S, Q_L)$ be the continuation choice that equals one if a fund optimal chooses to stay and zero otherwise given $\hat{\phi}, \tau$ and (Q_S, Q_L). In a stationary equilibrium, the equilibrium mapping is given by the vector of the aggregate investment in each investment opportunity:

$$H(Q_S, Q_L) \equiv \big(h_S(Q_S, Q_L), h_L(Q_S, Q_L)\big), \tag{20}$$

where

$$h_S(Q_S, Q_L) \equiv \int_{j\in\mathcal{J}} q_S^j dj = \sum_{\tau=0}^{\infty} \int_{\phi_E(\tau)}^{\infty} I(\hat{\phi}, \tau; Q_S, Q_L) q_S(\hat{\phi}, \tau; Q_S, Q_L) d\nu(\hat{\phi}, \tau);$$
$$h_L(Q_S, Q_L) \equiv \int_{j\in\mathcal{J}} q_L^j dj = \sum_{\tau=0}^{\infty} \int_{\phi_E(\tau)}^{\infty} I(\hat{\phi}, \tau; Q_S, Q_L) q_L(\hat{\phi}, \tau; Q_S, Q_L) d\nu(\hat{\phi}, \tau).$$

The fixed point of the mapping in Eq. (20), which solves the following equation, yields the stationary equilibrium:

$$H(Q_S, Q_L) = (Q_S, Q_L). \tag{21}$$

We denote by $\sigma(\hat{\phi}, \tau)$ the optimal operational decision of a fund with $\hat{\phi}, \tau$ such that $\sigma \in \{E, S, L\}$ where E, S, L stand for exit, short-term investment, and long-term investment, respectively. Now, we define stationary equilibrium as follows:

Definition 1. *A stationary equilibrium consists of optimal operational decision $\sigma(\hat{\phi}, \tau)$, optimal size $q_i(\hat{\phi}, \tau)$, value function $V(\hat{\phi}, \tau)$, transition probabilities $Z(\hat{\phi}', \tau' | \hat{\phi}, \tau)$, stationary distribution $\nu(\hat{\phi}, \tau)$, aggregate investment Q_S, Q_L such that*

1. *Value function $V(\hat{\phi}, \tau)$ solves the recursive problem in Eqs. (9)-(11);*
2. *Transition probabilities $Z(\hat{\phi}', \tau' | \hat{\phi}, \tau)$ are given by Eq. (18);*
3. *The stationary distribution solves the functional equation in Eq. (19);*
4. *Aggregate investment solves Eq. (21).*

To prove that there exists equilibrium, we need to verify that there exist a fixed point for Eq. (21) while satisfying all the requirements in Definition 1.

Theorem 5. *There exists a stationary equilibrium.*

Proof. See Appendix A. □

Given that we have established existence of equilibrium, the following result is a direct consequence from an indifference condition of individual fund managers between short-term and long-term investment in Eq. (16).

Corollary 1. *In equilibrium, the expected profit of long-term investment is strictly greater than that of short-term investment fixing the skill level, i.e.,* $\Pi_L(\hat{\phi}) > \Pi_S(\hat{\phi})$.

Although Corollary 1 is a consequence of the indifference condition, it is an equilibrium result in that endogenous returns with capacity constraints driven by strategic substitutability play a key role in achieving it. Because of the high option value attached to it, short-term investment will attract fund managers until its alpha becomes sufficiently smaller than that of long-term investment. Our finding also provides an alternative source of investor short-termism based on moral hazard.

5 Main Findings and Empirical Tests

In this section, we demonstrate our main theoretical findings using numerical analysis, and test those predictions empirically.

5.1 Parametric Model

We first numerically solve our stationary equilibrium model using some parametric assumptions. For simplicity, we assume a cost function which is quadratic for each opportunity $i \in \{S, L\}$:

$$C_i(q_{t,i}) = \frac{a_i}{2} {q_{t,i}}^2$$

We also assume a decaying function for returns to scale:

$$g_i(Q_{t,i}) = \frac{b_i}{Q_{t,i}}$$

Table 1 shows the parametric values of the model employed in our numerical analysis. We choose an adjustment cost for short-term opportunity of $a_S = 1.2$ which is larger than that for long-term opportunity $a_L = 1$ and a return scale for short-term opportunity $b_S = 1$ which is smaller than the return scale for long-term opportunities $b_L = 1.2$ since that is mostly the case in reality. For example, the trading costs of momentum strategy is higher than that of value strategy and the scalability of the former is smaller than the later. Given the parameter values provided in the table, we can numerically solve for the fixed point in

Eq. (21). We use policy function iteration method as a numerical algorithm.

[Insert Table 1 about here]

5.2 Data

For empirical tests, we obtain mutual fund data from the Center for Research in Security Prices (CRSP) survivor-bias-free database and the fund manager tenure data from the Morningstar Direct, which is until the end of 2019. Following the data cleaning process of Kacperczyk, Sialm, and Zheng (2008), we remove bond, money market, balanced, index, ETFs/ENFs, international, and sector funds. We merge funds with multiple share classes into a single fund. We end up with a sample of 3,390 actively managed US equity mutual funds from 1961 to 2019 that only invest in US domestic equities. A more detailed description of the data cleaning process is in the Online Appendix.

Table 2 reports the summary statistics for our sample of mutual funds. Fund size and value added are adjusted by inflation into January 1, 2020 dollars. Our sample has an average fund size of 1,374 million dollars and an average turnover of 81% per year. The average age of a fund is 12.9 years and the average manager tenure is 5.7 years. The value added of a fund under both the CAPM model and the Vanguard benchmarks are positives, which are consistent with the numbers in Berk and van Binsbergen (2015). Since the asset pricing literature is still debating on whether pricing factors such as value and momentum are risk factors or anomalies, we use the CAPM model and the four Vanguard US index funds including S&P 500 Index (VFINX), Extended Market Index (VEXMX), Small-Cap Index (NAESX), and Mid-Cap Index (VIMSX) in our benchmark analyses. Our main findings still hold after including value and momentum factors or the corresponding factor related Vanguard index funds.

[Insert Table 2 about here]

5.3 Main Finding 1: Optimal Investment

Theorem 2 shows that fund managers choose to exit when their perceived skills are sufficiently low, and the threshold becomes higher as fund managers get older because their growth potential become smaller. Also, Theorem 3 shows that, conditioning on continuing their operations, fund managers choose to invest short-term when their perceived skills are sufficiently low, the threshold becomes lower as fund managers get older because their growth potential become smaller. These findings are related to the value of exit option attached to short-term investment strategies. That is, the call-option-like value becomes more sensitive to choices as the current value of state is nearer to the exercise boundary (low perceived skill) or the volatility is higher (new).

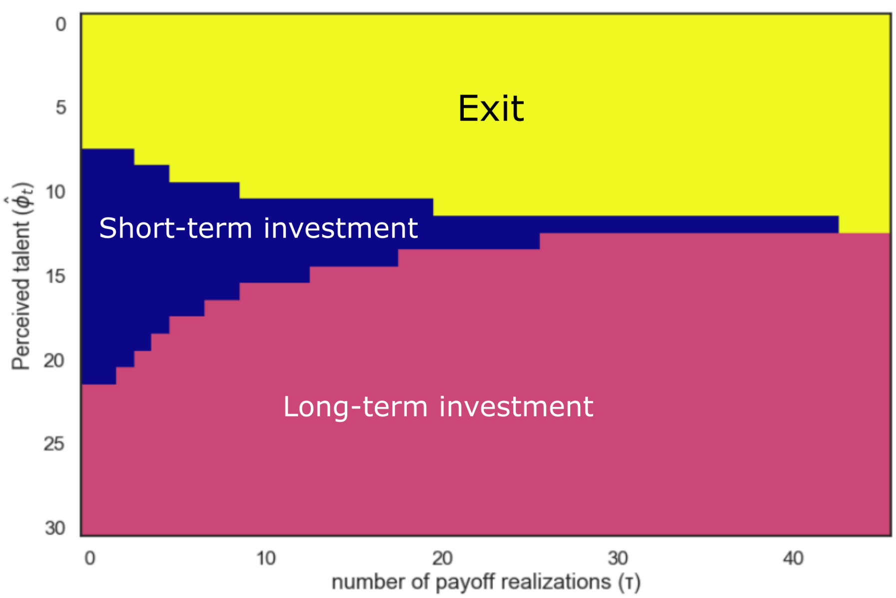


Figure 1: Optimal Choice by Perceived Skill and the Number of Belief Updates under the Parametric Model

Figure 1 shows the area of optimal choice in terms of state variables. As the tradeoff demonstrated in Eq. (16), funds with new managers and small size are more likely to invest in short-term opportunities to speed up investors' learning of their talents and, thus, increase the values of their growth options, but it sacrifices their current-period profits. On the other hand, because the information about old fund managers' talents is more precise and the growth potential of large funds are smaller, growth option is less important for them. As a result, funds with old managers and large size are more likely to choose long-term opportunities which prioritize current-period value added to growth option.

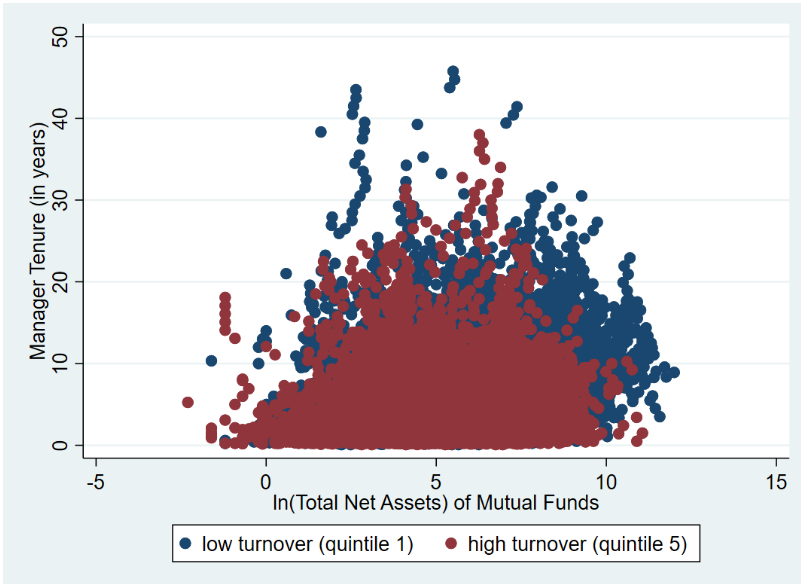


Figure 2: Scatter Plot of Manager Tenure and Total Net Assets by Fund Turnover
This figure shows the scatter plot of manager tenure versus the ln value of funds' total net assets for low-turnover funds (quintile 1) and high-turnover funds (quintile 5) separately.

Figure 2 shows that the distribution of fund size and manager tenure of high- and low-turnover funds resembles our model's prediction in Figure 1. Funds with new managers and small size are more likely to choose high-turnover strategies, while funds with old managers and large size are more likely to choose low-turnover strategies.

We further plot the average fund turnover by manager tenure for each fund size quintile separately in Figure 3 to look into the correlation between manager tenure and fund turnover controlling for fund size. It shows that fund turnover almost decreases monotonically with the increase of manager tenure for fund size quintile 2 to 5. We formally test this correlation using the regression below:

$$Turnover_{j,y} = cons + \beta * Tenure_{j,y} + \gamma * ln(TNA)_{j,y-1} + \upsilon_y + \varepsilon_{j,y}, \tag{22}$$

where $Turnover_{j,y}$ is turnover of fund j in year y reported in CRSP mutual fund database, and $Tenure_{j,y}$ is the manager tenure from Morningstar. We control for the year fixed effect in our benchmark setting since there might be some common variations in funds' turnovers over time, and we also control for fund fixed effect as a robustness check. Since the fund turnovers are highly persistent over time (as shown in van Binsbergen, Han, Ruan, and Xing [2020]), we cluster the robust standard errors at the fund level. We find that, on average, the annual fund turnover is 1.9% lower for a manager with one more year of experience (as reported in Panel A of Table 3). Since the standard deviation of manager tenure is 5.1 years as reported in Table 2, one standard deviation increase in manager tenure on average leads to a 9.7% (1.9%*5.1) decrease in annual fund turnover. This negative correlation is statistically significant for all fund size quintiles, except the quintile 1. It is because small funds always have the incentive to choose short-term opportunities which update investors' believes of their skills faster (as shown in Figure 1). Besides, the correlation between fund size and turnover is also significantly negative which is consistent with the prediction of our model.

Brown, Harlow, and Starks (1996) and Chevalie and Ellison (1997) argue that younger fund managers have a higher propensity to take risks. To distinguish our story that new fund manager use high-turnover strategies to speed up the learning from this risk-taking story, we measure the fund's portfolio risk by the standard deviation of the fund's monthly excess return per year. We regress this measure of portfolio risk on manager tenure controlling for fund turnover in the same year and report the result in setting (1) of Panel B. The result shows that the correlation between manager tenure and portfolio risk is indifferent from zero after controlling for fund turnover, suggesting that fund managers are not simply using high-turnover strategies to increase their risk taking. Consistently, we find that the relation between fund turnover and manager tenure remains the same after including the return volatility as a control variable (as reported in setting [4] of Panel B). Our result is also robust to including the fund fixed effects (setting [2] of Panel B).

Since fund age and manager tenure are positive correlated (with a correlation of 0.347), it is unclear whether it is the new fund or the new manager that tend to choose high-turnover strategies. To distinguish whether this effect is at the fund level or manager level, we include both fund age and manager tenure in the same regression in setting (3) of Panel B and find that the effect of manager tenure remains the same while the effect of fund age is close to zero, indicating that it is the new managers, not the new funds, that incline to use high-turnover strategies.

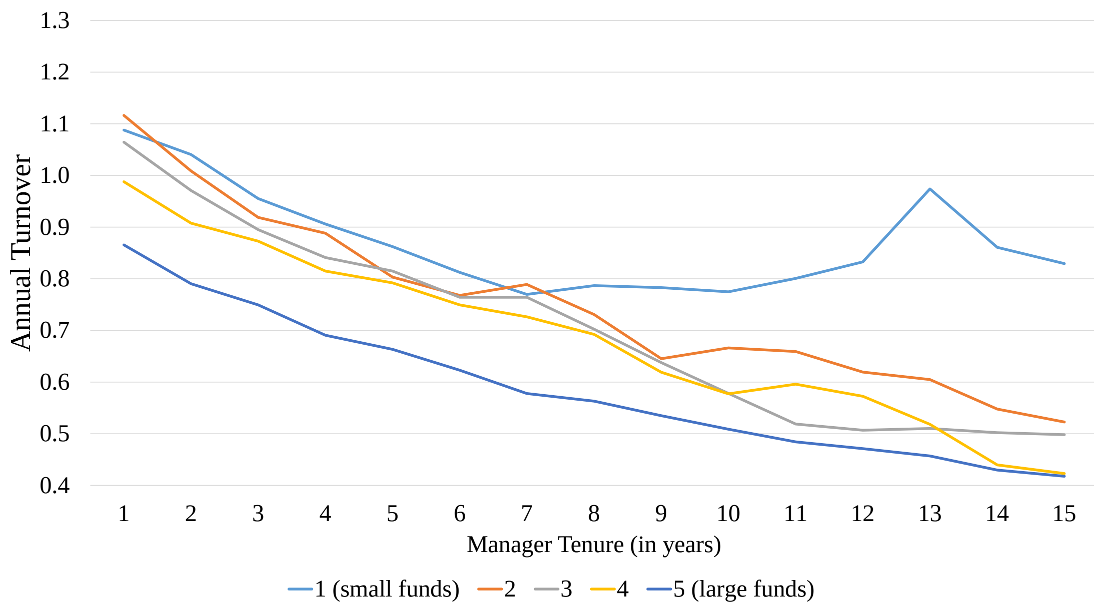


Figure 3: Fund Turnover by Manager Tenure and Fund Size Quintiles

This figure plots the average fund turnover by manager tenure for each fund size quintile separately. Manager tenure is the number of years a manager has worked in a given fund. Every quarter, we sort funds into fund size quintiles based on their total net assets at the end of last quarter.

[Insert Table 3 about here]

5.3.1 Flow-Performance Sensitivity of High- and Low- Turnover Funds

If investing in short-term opportunities speeds up investors' learning of funds' skill, we expect the flow-performance sensitivity is higher for high-turnover funds than low-turnover funds, especially for the performance in the recent past.

We estimate the fund flows of mutual fund j in quarter t as

$$Flow_{j,t} = \frac{TNA_{j,t} - TNA_{j,t-1} * (1 + R_{j,t})}{TNA_{j,t-1} * (1 + R_{j,t})} \tag{23}$$

where $TNA_{j,t}$ is the CRSP TNA value for fund j at the end of quarter t, and $R_{j,t}$ is the quarterly return of fund i during quarter t. When all the money of a fund is withdrawn by its investor ($TNA_{j,t} = 0$), this measure of fund flow is -100%. Following Huang, Wei, and Yan (2007), we winsorize the fund flows at 2.5 percent level at both tails to avoid the errors with mutual fund mergers and splits in the CRSP mutual fund database.

Because our main interest is in the flow-performance sensitivity of high-turnover funds versus low-turnover funds, we regress quarterly fund flows on funds' return ranks in the past quarter, year, and three years and their interactions with fund turnovers, as the following:

$$\begin{aligned} Flow_{j,t} =& cons + \beta_1 * Ret\ Rank_{j,t-1} \times Turnover_{j,t-1} + \gamma_1 * Turnover_{j,t-1} \\ &+ \beta_2 * Ret\ Rank_{j,t-1} \times Age_{j,t-1} + \gamma_2 * Age_{j,t-1} \\ &+ \beta_3 * Ret\ Rank \times ln(TNA)_{j,t-1} + \gamma_3 * ln(TNA)_{j,t-1} \\ &+ \eta * Ret\ Rank_{j,t-1} + \upsilon_t + \varepsilon_{j,t}. \end{aligned} \tag{24}$$

Each quarter we rank all funds based on their past quarter (year or 3-year) returns and assign them a continuous rank ranging from zero (worst) to one (best). *RetRank* is the return rank, *Age* is the number of years since the fund's starting date, and $ln(TNA)$ is the ln value of the fund's total net asset at the end of last quarter.

As our model predicts, Table 4 shows that the flow-performance sensitivity is higher for high-turnover than low-turnover funds, and this effect is stronger for the sensitivities to last quarter's returns and last year's returns than to last three-year returns. The coefficient of *Ret Rank* in column (1) reports a quarterly flow-performance sensitivity of 10%, that is, the quarterly fund flow is 9% of fund TNA higher for the fund with the highest last-quarter return (*Ret Rank* = 1) compared to the fund with the lowest (*Ret Rank* = 0), and this flow-performance sensitivity is 14.9% and 12.3% for last-year return and last-3-year return ranks respectively as in column (2) and (3). The coefficient of $Ret\ Rank * Turnover$ in column (1) reports that this flow-performance sensitivity is 0.9% higher for a fund with an annual turnover 100% higher. Since the standard deviation of funds' annual turnover is 82% as reported in Table 2, that is, the flow-performance sensitivity is 0.74% (0.9%*0.82) higher for a fund with an annual turnover one standard deviation higher. This effect is 0.57% (0.7%*0.82) and 0.41% (0.5%*0.82) for last-year return and last-3-year return ranks respectively as in column (2) and (3). Therefore turnover has a substantial effect on the flow-performance sensitivity. Consistent with our model prediction and Huang, Wei, and Yan (2007), we find that fund age has a negative effect on the flow-performance sensitivity. We control for quarterly fixed effects and cluster the standard errors per quarter since fund flows are positively correlated in the same quarter and we are interested in the sensitivity of fund flows to the relative performance ranks of funds. The results are similar withour controlling for quarterly fixed effects.

[Insert Table 4 about here]

5.4 Main Finding 2: Stationary Distribution

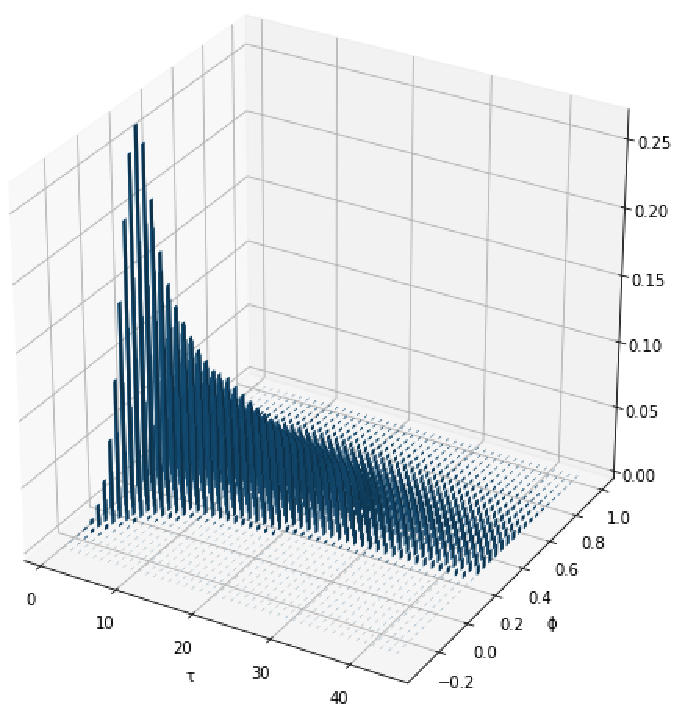


Figure 4: Stationary Distribution of Perceived Skill and the Number of Belief Updates under the Parametric Model

Figures 4 and 5 show the stationary distribution of state variables. Theorem 4 guarantees existence and uniqueness of such a distribution. One can also observe gradual attrition

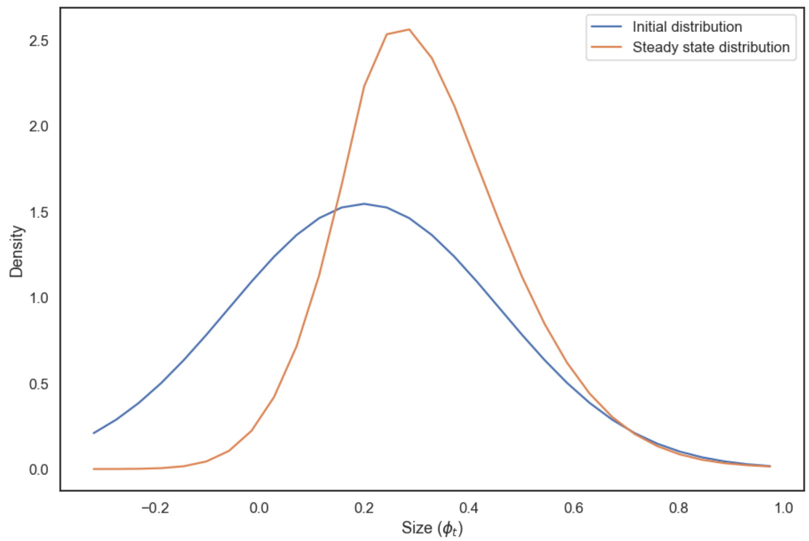
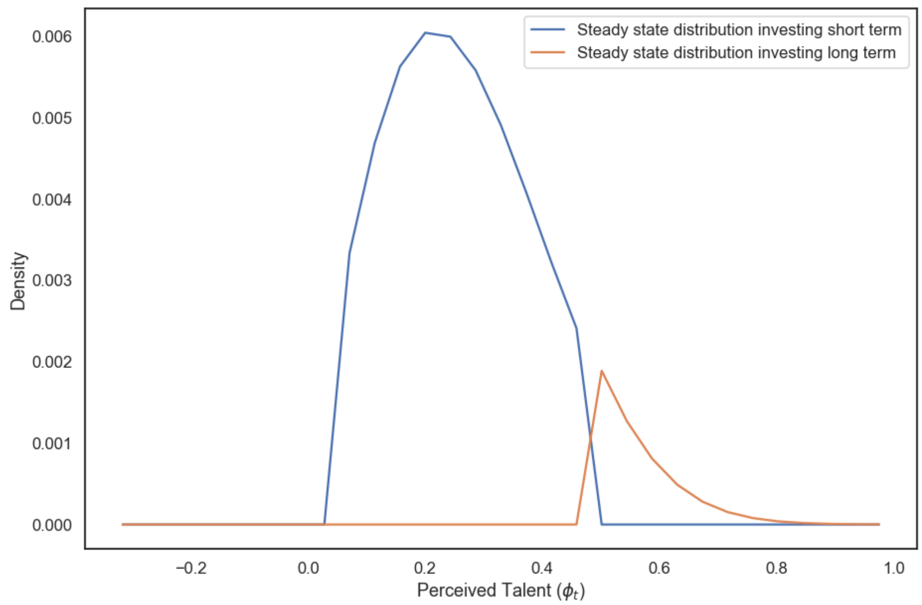


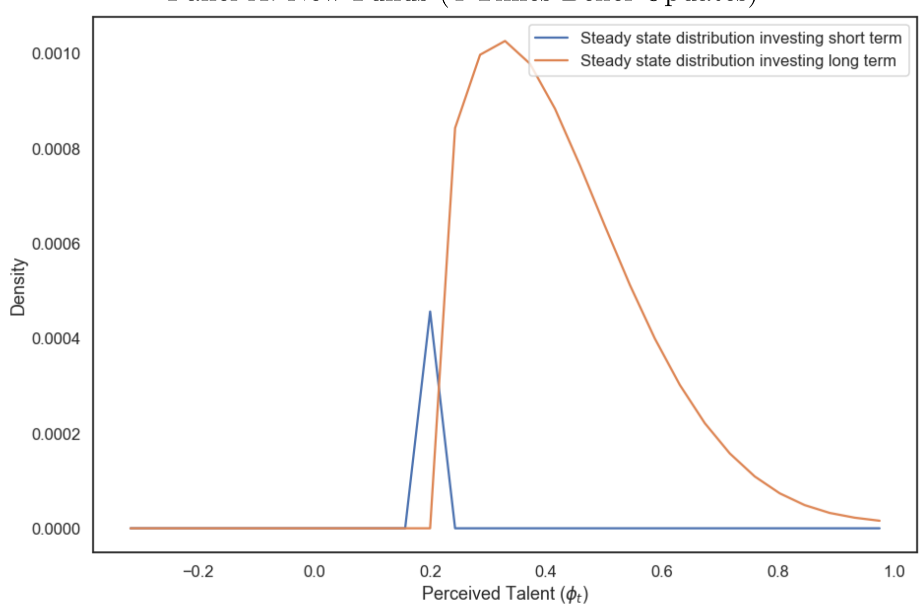
Figure 5: Stationary Distribution of Perceived Skill under the Parametric Model

of fund managers for both voluntary and random exits. As a consequence of voluntary exits, steady state distribution of perceived skills, which is the distribution of surviving funds, naturally contains better skilled fund managers compared to the initial distribution. Another important feature is the kurtosis of the distribution. Compared to the initial distribution, it is heavily concentrated around the mean partially because it takes a long time for fund managers to prove that they are exceptionally talented and deserve to manage a large amount of capital. Fund managers switch to long-term investment once their perceived skills are high enough, further slowing down information revelation. Therefore, there are a large number of small funds, yet very small number of large funds in the economy. It is worth noting that fund sizes and perceived skills are in one-to-one relationship in our model as in Berk and Green (2004).

Figure 6 plots the steady state distributions of short-term and long-term funds based on our numerical results. Panel A is for new fund managers (with 4 times belief updates), and Panel B is for old fund managers (with 28 times belief updates). As shown in Panel A, most new fund managers choose short-term opportunities to increase their growth option. Only a small group of funds who have proved their skills through extraordinary past performance (because of either luck or skill) choose long-term opportunities. For old fund managers,



Panel A. New Funds (4 Times Belief Updates)



Panel B Old Funds (28 Times Belief Updates)

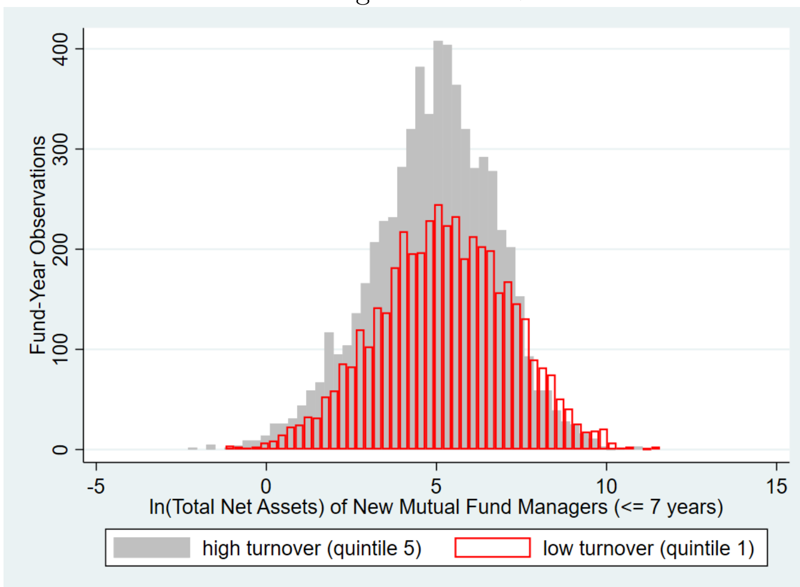
Figure 6: Distribution of Fund Size by Investment Horizon

in Panel B, most of them choose long-term opportunities for higher current-period value added. Only a small fraction of funds with low perceived skills choose high-turnover funds.

Figure 7 plots the distribution of fund size from the data for high-turnover funds (quintile 5) and low-turnover funds (quintile 1) separately and for new and old managers separately. The distributions in Panel A for new managers (with tenure <= 7 years) resemble the distributions in Panel A of Figure 6, where the density of high-turnover funds is higher than the density of low-turnover funds. The distributions in Panel B for old managers (with tenure > 7 years) also resemble the distributions in Panel B of Figure 6, where the density of low-turnover funds is higher than the density of high-turnover funds. The average fund size of high-turnover funds is smaller than low-turnover funds in both panels.

Next we look into the compositions of high-turnover and low-turnover funds and the total amount of asset managed by each category. Figure 8 plots the number of funds and the total net assets for each fund turnover quintile and by manager tenure. As shown in Panel A, for high-turnover funds, the number of new managers (with tenure <= 7 years) is about three times the number of old managers (with tenure > 7 years); while for low-turnover funds, new managers are only slightly more than old managers. However, Panel B shows that, for low-turnover funds, the total amount of assets managed by old managers is more than twice the amount managed by new managers, and low-turnover funds manage substantially more assets than high-turnover funds do. These results are consistent with our conjecture that a main function/value added of the large number of small and high-turnover funds is to select skilled managers for large low-turnover funds, which add the majority of value for the mutual fund industry. There are two ways that a manager of small and high-turnover fund can become a manager of large and low-turnover funds. (1) investors reward the fund with more capital and the fund becomes large and switches to a low-turnover strategy, and (2) this manager is hired (or reassigned by the fund family) to manage a large and low-turnover fund.

Panel A: Manager Tenure <= 7 Years



Panel B: Manager Tenure > 7 Years

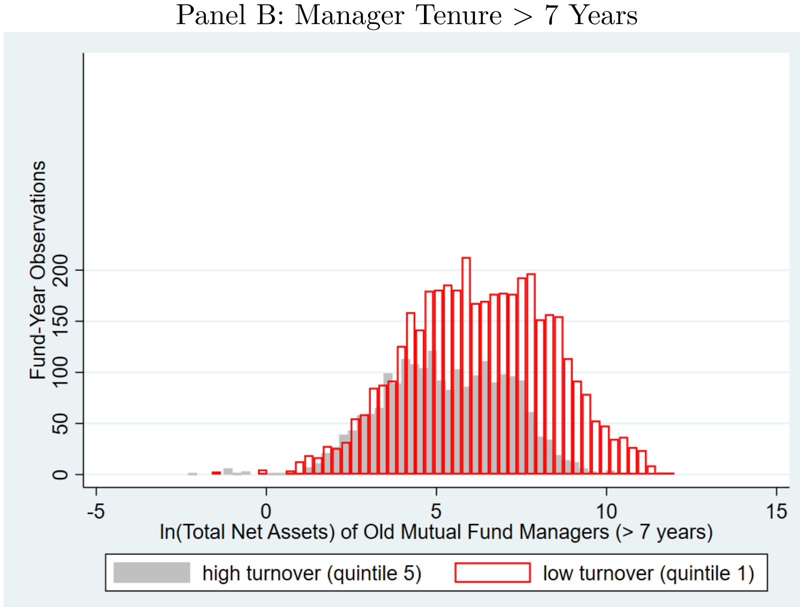
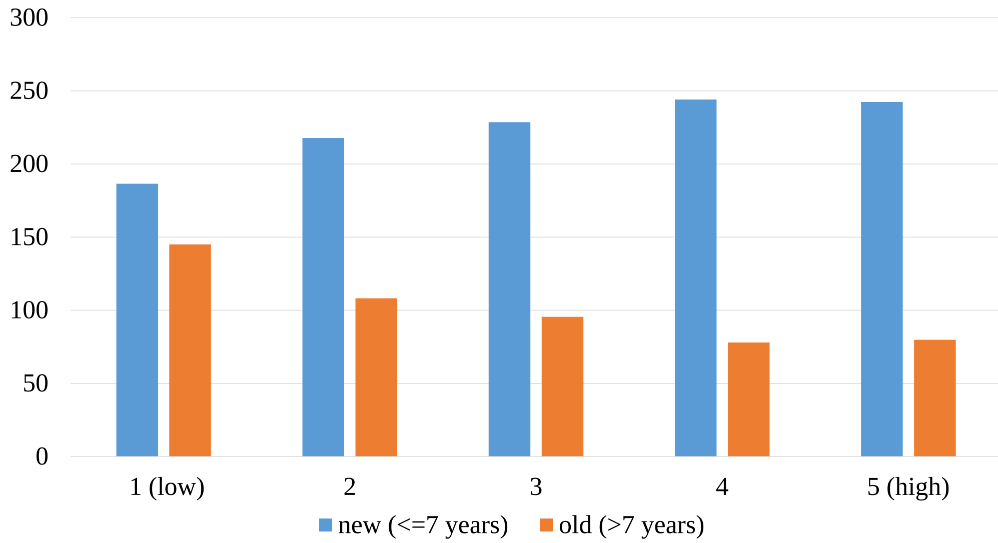


Figure 7: Distribution of Fund Size by Turnover and Manager Tenure: New (<= 7 Years) vs Old (> 7 Years)

This figure plots the distribution of fund size for high-turnover funds (quintile 5) and low-turnover funds (quintile 1) separately and for new and old managers separately. Panel A is for new managers with tenure <= 7 years, and Panel B is for old managers with tenure > 7 years. We sort funds into turnover quintiles every year based on their turnover ratios in CRSP mutual fund database. The vertical axis is the number of fund-year observations for all the funds in our sample from 1961 to 2019, and the horizontal axis is the ln value of total net assets inflation adjusted to the dollar amounts on 2020 January 1.

Panel A: Average Number of Funds per Year



Panel B: Total Net Assets (in billion $s)

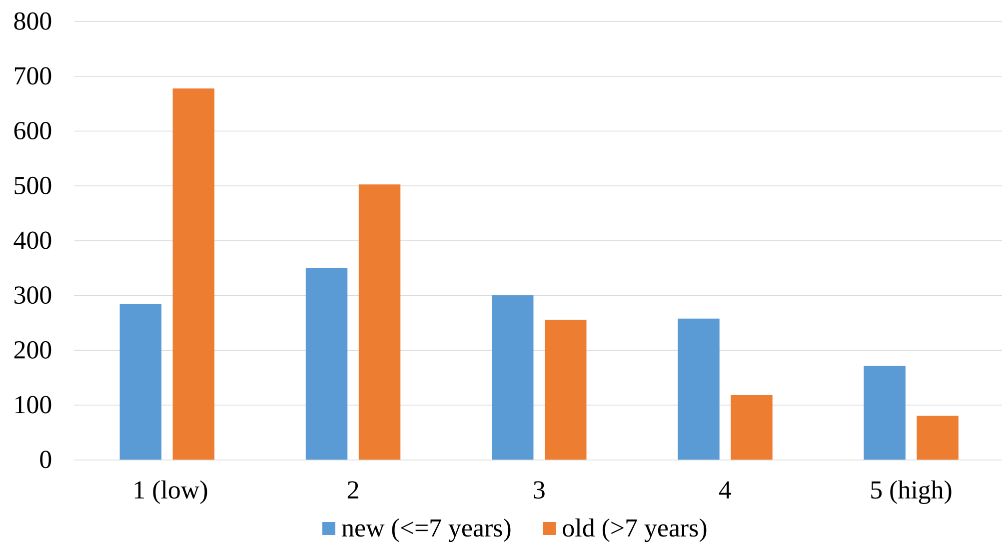


Figure 8: Number of Funds and Total Net Assets by Manager Tenure and Fund Turnover Quintiles

This figure plots the number of funds and the total net assets for each fund turnover quintile and by manager tenure. Panel A is for the average number of fund per year in each category, and Panel B is for the total net assets of all the funds in each category. We sort funds into turnover quintiles every year based on their turnover ratios in CRSP mutual fund database. Blue bars are for new managers with tenure $<=7$ years, and orange bars are for old managers with tenure >7 years. All dollar amounts are inflation adjusted to 2020 January 1, and all numbers are averaged across years from 1961 to 2019.

5.5 Main Finding 3: Equilibrium Value Added

Figure 9 shows the total value added and average fund gross alphas in the steady state for investments in short-term and long-term opportunities separately. Under the stationary distribution in our model, there are many new and relatively unskilled funds in the economy. They invest in short-term opportunities for growth options, which drive the excess returns of short-term opportunities down to a level lower than long-term opportunities. As a result, old and skilled fund managers optimally choose to invest in long-term opportunities. The aggregate value added of investing in short-term opportunities is small because of both the competition for grow options and the lack of skill for new managers, whereas the aggregate value added of investing in long-term opportunities is large because of both higher skills of old managers and smaller growth options. Therefore, long-term opportunities add more value than short term opportunities do in equilibrium. Because short term opportunities offer higher future growth options, fund managers are willing to accept lower current value added for short term opportunities. Competition makes short term opportunities less profitable (i.e., prices are more efficient).

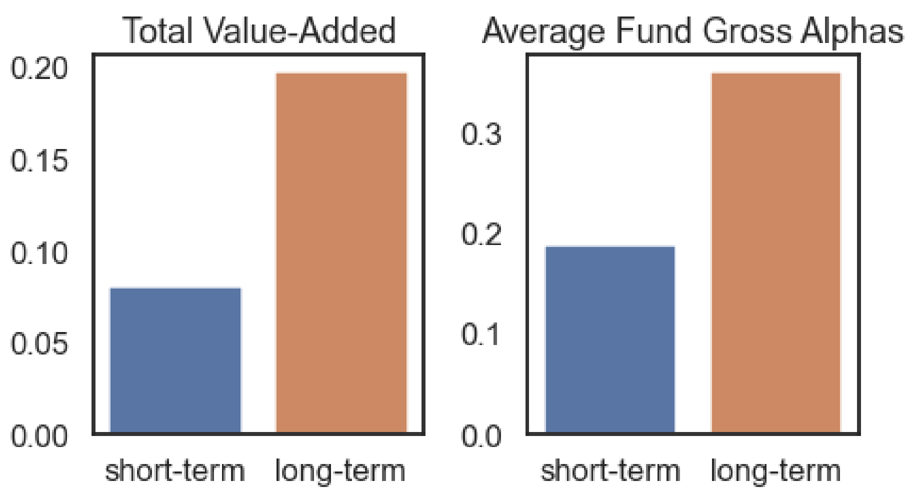
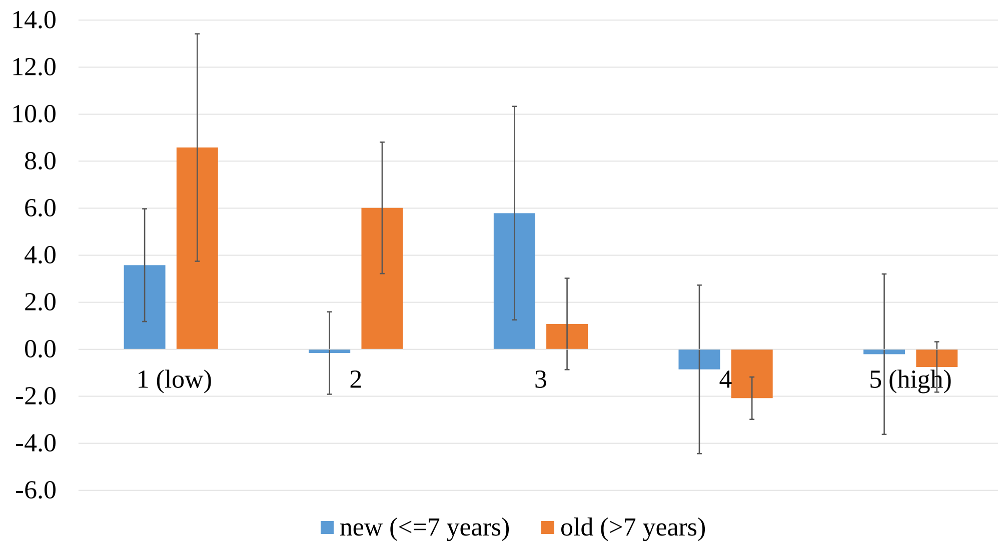


Figure 9: Value Added and Gross Alphas in Equilibrium under the Parametric Model

Figure 10 plots the total value added for each fund turnover quintiles by manager tenure. We sort funds into turnover quintiles every year based on their turnover ratios in the CRSP mutual fund database. Panel A reports the value added calculated based on the CAPM, Panel A: Total Value Added Based on the CAPM (in billion $s)



Panel B: Total Value Added Based on the Vanguard Benchmarks (in billion $s)

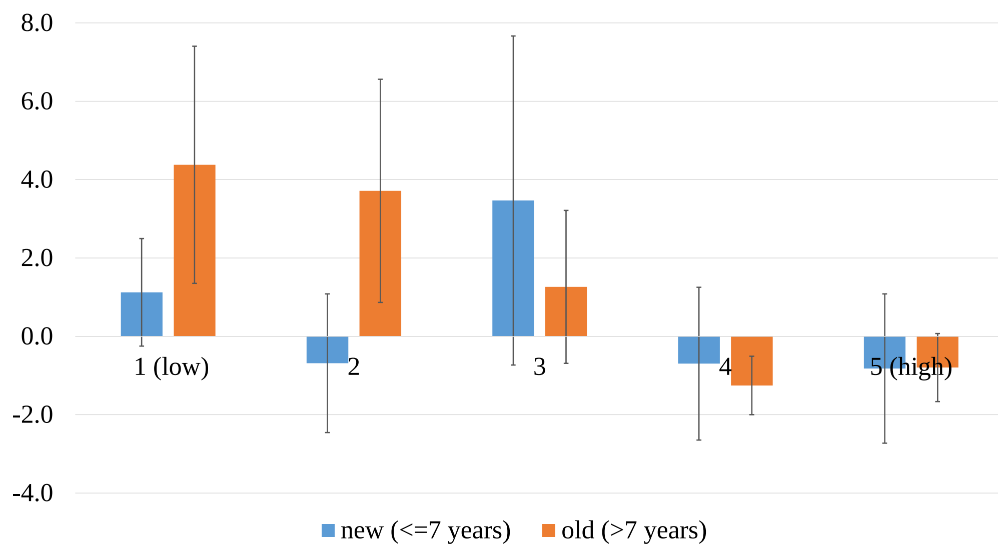
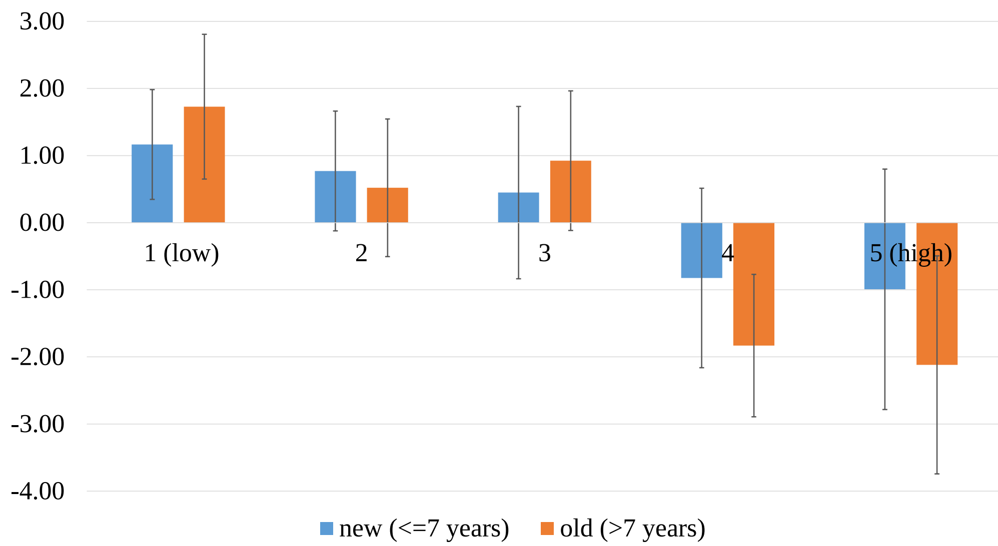


Figure 10: The Value Added by Fund Turnover Quintiles and Manager Tenure

This figure plots the total value added for each fund turnover quintiles by manager tenure. We sort funds into turnover quintiles every year based on their turnover ratios in the CRSP mutual fund database. Panel A reports the value added calculated based on the CAPM, and Panel B based on four US Vanguard Index funds as benchmarks (including S&P 500 Index (VFINX), Extended Market Index (VEXMX), Small-Cap Index (NAESX), and Mid-Cap Index (VIMSX)). Blue bars are for new managers with tenure $<= 7$ years, and orange bars are for old managers with tenure > 7 years. All dollar amounts are inflation adjusted to 2020 January 1. All numbers are averaged across months from 1961 January to 2019 December and annualized. The 90% confidence intervals are calculated across months.

Panel A: Fund Gross Alphas Based on the CAPM (in %)



Panel B: Fund Gross Alphas Based on the Vanguard Benchmarks (in %)

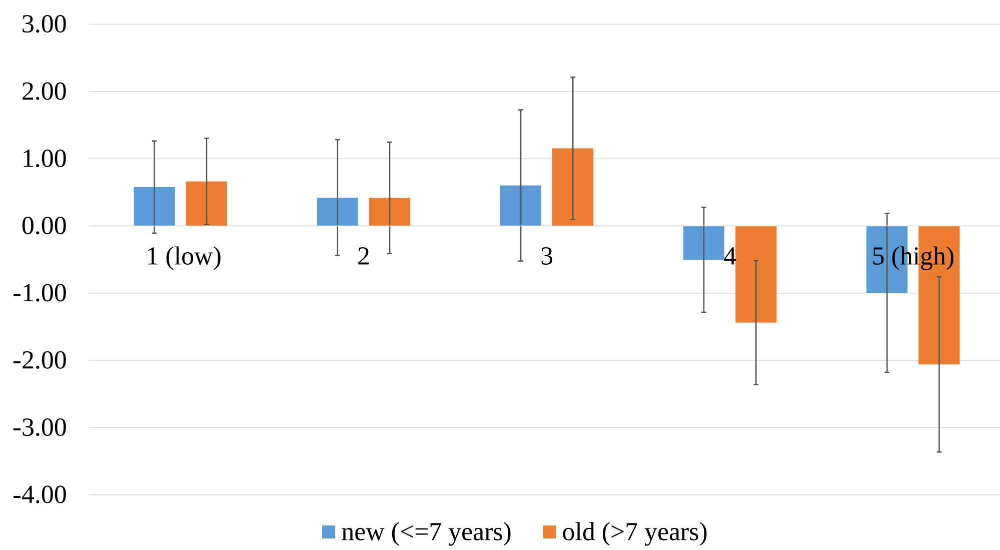


Figure 11: Fund Gross Alphas by Fund Turnover Quintiles and Manager Tenure

This figure plots the average fund gross alphas for each fund turnover quintiles by manager tenure. We sort funds into turnover quintiles every year based on their turnover ratios in the CRSP mutual fund database. Panel A reports the gross alphas calculated based on the CAPM, and Panel B based on four US Vanguard Index funds as benchmarks (including S&P 500 Index (VFINX), Extended Market Index (VEXMX), Small-Cap Index (NAESX), and Mid-Cap Index (VIMSX)). Blue bars are for new managers with tenure $<= 7$ years, and orange bars are for old managers with tenure > 7 years. We first calculate the value-weighted average gross alphas across funds in each month. Then the gross alphas are averaged across months from 1961 January to 2019 December and annualized. The 90% confidence intervals are calculated across months.

and Panel B based on the Vanguard benchmark composed by four US Vanguard Index funds (including S&P 500 Index (VFINX), Extended Market Index (VEXMX), Small-Cap Index (NAESX), and Mid-Cap Index (VIMSX)). As our model predicts, the value added of high-turnover funds (close to zero) is substantially smaller than the value added of low-turnover funds. According to our model, it is because short-term opportunities offer higher future growth options, new and small fund managers are willing to accept lower current value added for short-term opportunities. As a consequence, the value added of low-turnover funds are mostly from old and skilled managers as Figure 10 shows. The value added of relatively low-turnover funds (quintile 1 and 2) managed by old managers are significantly postive under both CAPM and Vanguard benchmarks under the 10% significant level. The value added relatively high-turnover funds (quintile 4 and 5) managed by new and old managers are all negative, though only significantly negative for funds in quintile 4 managed by old managers. In addition, we find that new managers that choose medium-turnover strategies (in quintile 3) add a substantial amount of value at least under the CAPM benchmark.

Figure 11 plots the average fund gross alphas for each fund turnover quintiles by manager tenure. Our model predicts that for old fund managers, the unskilled ones are more likely to choose high-turnover strategies. For new fund managers, some skilled ones choose high-turnover strategies as well to speed up investors' learning. Consistently, the gross alphas of old high-turnover funds in quintile 4 and 5 are between -1.44% and -2.12% per year and significantly negative, whereas the gross alphas of new high-turnover funds in quintile 4 and 5 are relatively higher and statistically indifferent from zero.

5.6 Further Empirical Analysis: Value Added of Trades by Investment Horizons

Although our model equates the value added of high- (low-) turnover funds to that of short- (long-) term investment opportunities for simplicity, both high- and low- turnover funds have relatively short- and long- term holdings in their portfolios. In this section we calculate the value added and alphas of funds' trades at different investment horizons directly using their transactions and holdings data. We use the method developed in Van

Binsbergen, Han, Ruan, and Xing (2020) for this calculation based on the past length of funds' actual holdings. Consistent with our model prediction, we show that the majority of mutual funds' value added and gross alphas are from their long-term holdings instead of short-term trades.

5.6.1 332 Mutual Funds with Daily Holdings Data

We merge the transaction data provided by Abel Noser Solutions with the quarterly holdings data in the Thomson Reuters database from 1999 to 2010 (using the method in Busse, Chordia, Jiang, and Tang [2020]) and the mutual fund data (including fund characteristics) from the Center for Research in Security Prices (CRSP). Using these data, we identify 332 US mutual funds and construct a unique dataset of their daily holdings.[5]

5.6.2 Measures of Price Impact Costs and Other Trading Costs

We measure the contributions of both explicit and implicit trading costs to fund value added. Explicit trading costs include commissions, taxes, and fees. Implicit trading costs include the intra-day implicit costs related to the price impact of trades, and the multi-day implicit costs related to the liquidity consumption/provision across days.

Trades' commissions, taxes, and fees are reported directly by Abel Noser Solutions in dollars. We calculate their (negative) contribution to daily fund value added as the average dollar amount of those costs per day. We measure the intra-day price impact costs using the execution shortfalls of trades. The execution shortfall is the difference between the actual execution price of a stock and the price at the time of order placement (measured by the last executed price of the same stock) as a percentage of the price at the time of order placement. The expression is

$$ES_{i,t} = D_{i,t} \frac{P_{i,t}^e - P_{i,t}^0}{P_{i,t}^0}, \tag{25}$$

where $D_{i,t}$ is 1 for buys and -1 for sells. $P_{i,t}^0$ is the stock price at order placement, and $P_{i,t}^e$ is the order's actual execution price. If you buy (sell) at a price $P_{i,t}^e$ higher (lower) than $P_{i,t}^0$,

[5]See Van Binsbergen, Han, Ruan, and Xing (2020) for a detailed description of this data and the matching quality.

the price impact costs of this trade measured by $ES_{i,t}$ is positive. The execution shortfall can be positive or negative depending on market conditions, and the extent to which an order demands or supplies liquidity. Funds split large trades into smaller trades to reduce price impact. The intra-day price impact costs measured by the execution shortfalls are paid during trade executions. The total contribution of intra-day price impact costs to a fund's daily value added for all trades on day t is

$$ES_t = \sum_{i=1}^{I} (D_{i,t} V_{i,t} ES_{i,t}), \tag{26}$$

with the intra-day price impact costs of each trade in dollars calculated as the product of the absolute trading amount, $D_{i,t}V_{i,t}$, and the execution shortfall, $ES_{i,t}$.

5.6.3 Method of Decomposition by Investment Horizon

We decompose the daily value added of mutual funds using their current holdings together with their past trades to estimate the contribution of trades in the past 1 to 240 business days (one year) to funds' daily value added.

A fund's value added on day t can be expressed as the sum of the value added from its holdings at the beginning of day t and the value added from its trades on day t:

$$VA_t = \sum_{i=1}^{N} H_{i,t-1} R_{i,t} + \sum_{i=1}^{N} V_{i,t} R^e_{i,t}. \tag{27}$$

VA_t is the fund's value added on day t. $H_{i,t-1}$ is the fund's holding of stock i at the end of day $t-1$ in dollars. For $R_{i,t}$ we use the abnormal return based on the CAPM.[6] [7] $V_{i,t}$ is the fund's trading amount of stock i in day t, which is positive for purchases and negative for sales, and $R^e_{i,t}$ is defined as

$$R^e_{i,t} = (P^c_{i,t} - P^e_{i,t})/P^e_{i,t}, \tag{28}$$

where $P^e_{i,t}$ is the execution price and $P^c_{i,t}$ is the closing price for stock i on day t. Since the exact time of trade execution within a day is not reported in the Abel Noser dataset, we

[6]The CAPM alphas of stocks is calculated using the one-year rolling regression, as on French's website.
[7]We also use the raw return and the Fama-French-Carhart four factor alpha as a robustness check.

use raw returns instead of risk adjusted returns for intra-day trading profits/losses. Given that the daily market risk premium is negligible, this does not materially affect our results. In particular, if we assume trades on average occur at mid day, the contribution of trades to fund value added through market exposure within the same day is approximately 9000 dollars per year, which is only 0.1 bps of the average funds' TNAs and 2 bps of average value added of funds in our sample.

In summary, the first term on the right-hand side of equation (27) is the contribution of the holdings at the end of day $t-1$ to fund value added on day t, and the second term is the contribution of the trades on day t to the same-day fund value added.

We distinguish the change in holdings caused by trades within the past n days (10 to 240 days), $H^{s(n)}_{i,t-1}$, from the holdings $n+1$ days ago, $H^{p(n)}_{i,t-1}$. The expression of this decomposition is

$$H_{i,t-1} = H^{s(n)}_{i,t-1} + H^{p(n)}_{i,t-1}, \tag{29}$$

where $H^{s(n)}_{i,t-1}$ can be of either sign while $H^{p(n)}_{i,t-1}$ is non-negative.

Using this holdings decomposition, we decompose the fund's value added on day t into the value added from trades on the same day, the changes in holdings in the past n days (10 to 240 days), and the holdings n days ago,

$$VA_t = \left(\sum_{i=1}^{N} V_{i,t} R^e_{i,t} + \sum_{i=1}^{N} H^{s(n)}_{i,t-1} R_{i,t} \right) + \sum_{i=1}^{N} H^{p(n)}_{i,t-1} R_{i,t}. \tag{30}$$

The first and second terms on the right-hand side of equation (30) together (in the parenthesis) measure the contribution of the trades within n days to the fund's daily value added. The last term is the value added from the holdings $n+1$ days ago.

5.6.4 Value Added of Trades by Investment Horizons

Table 5 reports the average value added of trades by investment horizon. On the left side of the table, we provide the value added of trades for different horizons, such as: 10, 20,...240 days, on the basis of equation (30). We also compute the value added for holdings beyond 240 days. We use three performance measures to compute the value added: CAPM model, raw returns, and Fama-French-Carhart 4 factor model. The value added is equally weighted across funds and annualized. The right side of the table reports the corresponding contribution to fund annual return, which is the value added divided by the fund TNAs (total net assets). Figure 12 plots the result using CAPM model. We find that trades of these funds neither add nor destroy value in the first 7 months (150 days). The value added of trades within 150 days is close to zero based on CAPM model, and the corresponding contribution to fund alpha is constantly below 6 bps per year, which is small compared to a gross CAPM alpha of 2.55 percent per year based on daily holdings. The value added of trades increases gradually since 160 days up to 4.2 million dollars in one year, corresponding to 38 bps contribution to fund alpha, but it is still insignificantly different from zero. Results are similar when raw returns or Fama-French-Carhart 4 factor model are used.

[Insert Table 5 about here]

The last row of Table 5 reports the value added from holdings one year (240 days) ago, which is calculated as the value added of all holdings minus the value added of trades within 240 days. We find that out of the 27.5 million dollars of value added (CAPM model) from their daily holdings, 24.0 million dollars are from their holding one year ago, and it is statistically significant at 5% significance level. It means that if funds keep their holdings one year ago, they can still add 24.0 million dollars value per year. Since a fund's holdings one year ago are a result of all their trades more than one year ago, we can also say that these 24.0 million dollars of value are added by their trades beyond a year. When calculated based on raw returns and Fama-French-Carhart 4-factor model, the value added from trades beyond a year are 39.7 and 12.3 million dollars correspondingly. Both numbers represent a majority of funds' total value added and are significant under 5% significance level. This result shows that mutual funds mainly profit from their trades in the long-term. The second last row of Table 5 reports that the average value added of all daily holdings are significantly positive using CAPM model, raw returns, and Fama-French-Carhart 4 factor model. It is consistent with the result of Berk and van Binsbergen (2015) that the average mutual fund has skill to generate positive value added.

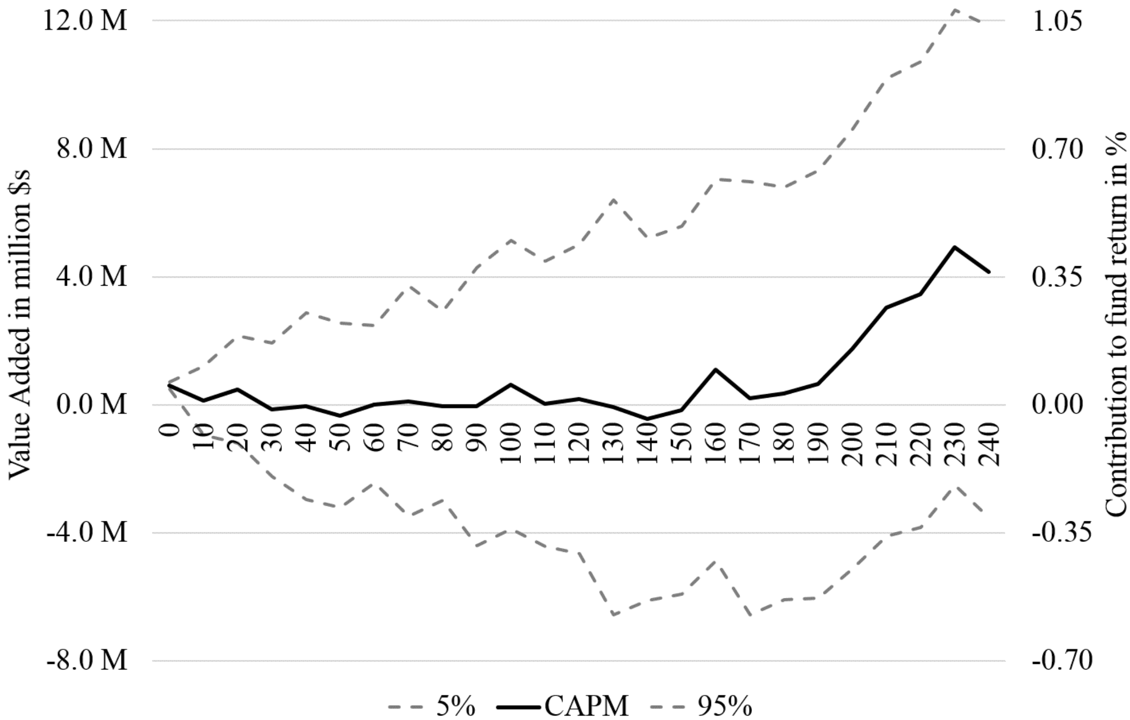


Figure 12: Value Added of Trades in One Year

This figure plots the value added from trades in the past 240 days (one year) and its 5% and 95% confidence intervals. Value added from trades in the past n (1 to 240) days is calculated using Eq. (30). CAPM abnormal return of each stock is used for this calculation. The right vertical axis is the corresponding contribution of trades to fund annual return, which is the value added divided by the TNA (total net assets) of the fund. Value Added is equally weighted across fund-day observations and the corresponding contribution to fund return is value-weighted. Confidence intervals are calculated based on robust standard errors clustered per day.

6 Conclusion

In this paper, we study a dynamic equilibrium with a stationary distribution of mutual funds under career concern, and empirically confirm our main theoretical predictions. Our model extends Berk and Green (2004) by allowing multiple investment opportunities with different investment horizons whose return decreases in competition, and also by endogenizing equilibrium distribution of fund perceived skills.

We provide three main findings. First, new fund managers tend to invest short-term because of trade-off between value added and growth options. As the competition among new fund managers reduces value added in short-term investment, old skilled fund managers optimally choose to extract value from long-term opportunities instead. Second, as a consequence of voluntary exits, steady state distribution of perceived skills, which is the distribution of surviving funds, naturally contains better skilled fund managers compared to the initial distribution. Furthermore, it becomes heavily concentrated around the mean because fund managers switches to long-term investment once their perceived skills are high enough, and it slows down the learning process. This shed light on empirical observations of having a large number of small funds and a small number of large funds. Third, long-term opportunities add more value than short term opportunities do in equilibrium. Because short term opportunities offer higher future growth options, fund managers are willing to accept lower current value added for short term opportunities. Competition makes short term opportunities less profitable (i.e., prices are more efficient). As a result, the value added of high-turnover funds is mainly from speeding up the learning of new fund managers' skills instead of extracting value from the short-term opportunities.

We then empirically test our model predictions using 59 years of US mutual funds datas, as well as a unique dataset of transactions and daily holdings for a small sample of US mutual funds. We find that new fund managers tend to invest short-term whereas old funds tend to invest long-term. There are more skilled new fund managers investing in short term, and vice versa. Competition among new fund managers reduce price inefficiency in short-term investment, resulting in lower value added.

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Appendix A

Proof of Theorem 1:

We first reformulate the problem for analytical convenience. Let $X \equiv \mathbb{R} \times \mathbb{R}_+$. We define a value function $\hat{V} : X \to X$ as a function of $\hat{\phi}$ and $\hat{\gamma}$ instead of $\hat{\phi}$ and τ where $\hat{\gamma} \equiv \gamma + \tau\omega$ is confidence on perceived talent (or, the inverse of conditional variance of the fund manager's talent). This formulation allows us to define the value function on a continuous domain, which is more convenient for the proof. Existence of a value function with this formulation guarantees existence of the corresponding value function in original formulation by simply setting $V(\hat{\phi}, \tau) \equiv \hat{V}(\hat{\phi}, \gamma + \tau\omega)$ for each $\tau = 0, 1, ...$

Let $\mathbf{C}(X)$ be the space of bounded continuous functions on X with the sup norm. We define an operator T on $\mathbf{C}(X)$ by

$$T\hat{V}(\hat{\phi}, \hat{\gamma}) \equiv \max\left\{\hat{V}_S(\hat{\phi}, \hat{\gamma}), \hat{V}_L(\hat{\phi}, \hat{\gamma})\right\}, \tag{A.1}$$

where $\hat{V}_i(\hat{\phi}, \hat{\gamma})$ denotes the value of choosing opportunity $i \in \{S, L\}$:

$$\hat{V}_i(\hat{\phi}, \hat{\gamma}) \equiv \Pi_i(\hat{\phi}) + \kappa(1 - d_i)\hat{V}(\hat{\phi}, \hat{\gamma}) + \kappa d_i \,\mathrm{E}\left[\max\left\{\hat{V}(\hat{\phi}', \hat{\gamma}'), 0\right\} \middle| \hat{\phi}, \hat{\gamma}\right], \tag{A.2}$$

and $\hat{\phi}'$ is the posterior of perceived talent in case of a successful belief update:

$$\hat{\phi}' \equiv \hat{\phi} + \left(\frac{\omega}{\hat{\gamma} + \omega}\right)(\xi - \hat{\phi}), \quad \text{and} \quad \hat{\gamma}' \equiv \hat{\gamma} + \omega.$$

We prove our first main result of the theorem.

Theorem A.6. *There exists a unique value function $\hat{V} \in \mathbf{C}(X)$ which solves $T\hat{V} = \hat{V}$.*

Proof. Suppose that $\hat{V} \in \mathbf{C}(X)$. Then, $\mathrm{E}\left[\max\left\{\hat{V}(\hat{\phi}', \hat{\gamma}'), 0\right\} \middle| \hat{\phi}, \hat{\gamma}\right]$ is bounded. Because $u(\cdot)$ is bounded, Π_i is bounded from Eq. (15). These findings together with Eq. (A.2) imply that $\hat{V}_S$ and $\hat{V}_L$ are bounded. Then, Eq. (A.1) implies that $T\hat{V}$ is bounded because the maximum of two bounded functions is bounded.

Likewise, because $\hat{V}$ is continuous in $\hat{\phi}$ and $\hat{\gamma}$ by the supposition that $\hat{V} \in \mathbf{C}(X)$, $\mathrm{E}\left[\max\left\{\hat{V}(\hat{\phi}', \hat{\gamma}'), 0\right\} \middle| \hat{\phi}, \hat{\gamma}\right]$ is continuous in $\hat{\phi}$ and $\hat{\gamma}$. From Eq. (12), it is immediate that q^* is continuous in $\hat{\phi}$ on $[0, \infty)$ because $C'(\cdot) > 0, C''(\cdot) > 0$. Therefore, Eq. (15) implies that Π_i is continuous in $\hat{\phi}$ (and it is also continuous in $\hat{\gamma}$ trivially). These findings together with Eq. (A.2) imply that $\hat{V}_S$ and $\hat{V}_L$ are continuous in $\hat{\phi}$ and $\hat{\gamma}$. Then, $T\hat{V}$ is continuous in $\hat{\phi}$ and $\hat{\gamma}$ because the maximum of two continuous functions is continuous.

Therefore, T maps $\mathbf{C}(X)$ to $\mathbf{C}(X)$. It is straight forward to show that the monotonicity and the discounting conditions are satisfied for the Blackwell's sufficient conditions. Because $\mathbf{C}(X)$ is a complete normed space, the contraction mapping theorem implies that T has a unique fixed point on $\mathbf{C}(X)$, i.e., there exists a unique value function $\hat{V}^*$ in $\mathbf{C}(X)$. □

We now turn to our second main result that $\hat{V}$ strictly increases in $\hat{\phi}$, and strictly decreases in $\hat{\gamma}$. For notational convenience, we let $\hat{V}_1 \equiv d\hat{V}/d\hat{\phi}$, $\hat{V}_2 \equiv d\hat{V}/d\hat{\gamma}$. Let $\mathbf{C}'(X)$ be the set of bounded, continuous functions on X that are non-decreasing in $\hat{\phi}$ (i.e., $\hat{V}_1 \geq 0$), are non-increasing in $\hat{\gamma}$ (i.e., $\hat{V}_2 \leq 0$). We also let $\mathbf{C}''(X) \subset \mathbf{C}'(X)$ be the set of functions that are strictly increasing in $\hat{\phi}$ (i.e., $\hat{V}_1 > 0$), are strictly decreasing in $\hat{\gamma}$ (i.e., $\hat{V}_2 < 0$).

Because the sufficient statistic for the fund's performance ξ follows a conditional normal distribution with mean $\hat{\phi}$ and variance $1/\hat{\gamma} + 1/\omega$ given $\hat{\phi}$ and $\hat{\gamma}$, we can represent

$$\hat{\phi}'\Big|_{\hat{\phi},\hat{\gamma}} = \left[\hat{\phi} + \left(\frac{\omega}{\hat{\gamma}+\omega}\right)(\xi - \hat{\phi})\right]\Bigg|_{\hat{\phi},\hat{\gamma}} = \hat{\phi} + \left(\frac{\omega}{\hat{\gamma}+\omega}\right)\left(\frac{1}{\hat{\gamma}} + \frac{1}{\omega}\right)\theta = \hat{\phi} + \frac{1}{\hat{\gamma}}\theta, \tag{A.3}$$

where θ is a random variable follows the standard normal distribution. Then, we can obtain the conditional expectation of continuation value of managing the fund as follows:

$$\begin{aligned} \mathrm{E}\left[\max\left\{\hat{V}(\hat{\phi}',\hat{\gamma}'),0\right\}\Big|\hat{\phi},\hat{\gamma}\right] &= \int_{-\infty}^{\infty} \max\left\{\hat{V}\left(\hat{\phi}',\hat{\gamma}+\omega\right),0\right\} n(\theta)d\theta \\ &= \int_{-\infty}^{\infty} \max\left\{\hat{V}\left(\hat{\phi}+\frac{1}{\hat{\gamma}}\theta,\hat{\gamma}+\omega\right),0\right\} n(\theta)d\theta \\ &= \int_{\bar{\theta}(\hat{\phi},\hat{\gamma})}^{\infty} \hat{V}\left(\hat{\phi}+\frac{1}{\hat{\gamma}}\theta,\hat{\gamma}+\omega\right) n(\theta)d\theta, \end{aligned} \tag{A.4}$$

where $n(\cdot)$ is the standard normal density function, and $\bar{\theta}(\hat{\phi},\hat{\gamma})$ is the cutoff threshold of the realization of the standard normal variable θ where the fund manager is indifferent between continuing the operation and exiting given $\hat{\phi},\hat{\gamma}$ such that

$$\hat{V}\left(\hat{\phi} + \frac{1}{\hat{\gamma}}\bar{\theta}(\hat{\phi},\hat{\gamma}),\hat{\gamma}+\omega\right) = 0. \tag{A.5}$$

We first prove the following lemma which will be useful later in the proof.

Lemma A.4. *At any given $\hat{\gamma}$, the cutoff threshold $\bar{\theta}$ decreases in $\hat{\phi}$.*

Proof. From Eq. (A.5), we can let

$$\psi(\hat{\phi},\bar{\theta}) \equiv \hat{V}\left(\hat{\phi}+\frac{1}{\hat{\gamma}}\bar{\theta}(\hat{\phi},\hat{\gamma}),\hat{\gamma}+\omega\right).$$

For $\hat{\phi}$ and $\bar{\theta}$ that satisfy $\psi(\hat{\phi},\bar{\theta})=0$, the implicit function theorem implies that

$$\frac{d\bar{\theta}}{d\hat{\phi}} = -\frac{\frac{\partial\psi}{\partial\hat{\phi}}}{\frac{\partial\psi}{\partial\bar{\theta}}} = -\frac{\frac{\partial\hat{V}\left(\hat{\phi}',\hat{\gamma}+\omega\right)}{\partial\hat{\phi}'}}{\frac{1}{\hat{\gamma}}\frac{\partial\hat{V}\left(\hat{\phi}',\hat{\gamma}+\omega\right)}{\partial\hat{\phi}'}} = -\hat{\gamma}<0,$$

which finishes the proof. □

Now, we prove that $T[\mathbf{C}'(X)] \subseteq \mathbf{C}''(X)$ in several steps.

Lemma A.5. *For any $\hat{V}\in\mathbf{C}'(X)$, $T\hat{V}(\hat{\phi},\hat{\gamma})$ is strictly increasing in $\hat{\phi}$ at any given level of $\hat{\gamma}$, .*

Proof. Pick a value function $\hat{V}\in\mathbf{C}'(X)$. At any given level of $\hat{\gamma}$, we first show that $\mathrm{E}\left[\max\left\{\hat{V}(\hat{\phi}',\hat{\gamma}'),0\right\}\middle|\hat{\phi},\hat{\gamma}\right]$ is strictly increasing in $\hat{\phi}$. Using the Leibniz integral rule, we have[8]

$$\begin{aligned}
&\frac{d\left(\mathrm{E}\left[\max\left\{\hat{V}(\hat{\phi}',\hat{\gamma}'),0\right\}\middle|\hat{\phi},\hat{\gamma}\right]\right)}{d\hat{\phi}}\\
&=\int_{\bar{\theta}(\hat{\phi},\hat{\gamma})}^{\infty}\hat{V}_1\left(\hat{\phi}+\frac{1}{\hat{\gamma}}\theta,\hat{\gamma}+\omega\right)n(\theta)d\theta-\hat{V}\left(\hat{\phi}+\frac{1}{\hat{\gamma}}\bar{\theta}(\hat{\phi},\hat{\gamma}),\hat{\gamma}+\omega\right)n(\bar{\theta}(\hat{\phi},\hat{\gamma}))\frac{d\bar{\theta}(\hat{\phi},\hat{\gamma})}{d\hat{\phi}} \qquad (A.6)\\
&=\int_{\bar{\theta}(\hat{\phi},\hat{\gamma})}^{\infty}\hat{V}_1\left(\hat{\phi}+\frac{1}{\hat{\gamma}}\theta,\hat{\gamma}+\omega\right)n(\theta)d\theta\geq 0,
\end{aligned}$$

where the second equality is due to the definition of $\bar{\theta}(\hat{\phi},\hat{\gamma})$ in Eq. (A.5), and the last inequality is because $\hat{V}_1\geq 0$ by the supposition that $\hat{V}\in\mathbf{C}'(X)$. From Eq. (15), using the Envelope theorem, we have

$$\frac{d\Pi_i(\hat{\phi})}{d\hat{\phi}} = u'\left(d_i\hat{\phi}g_i(Q_i)q_i^*(\hat{\phi})-C^*(q_i^*)-F\right)d_ig_i(Q_i)q_i^*(\hat{\phi})>0, \qquad (A.7)$$

[8] Note that $\Pi_L-\Pi_S$ is continuously differentiable with respect to $\hat{\phi}$, in which case the value function $\hat{V}$ is also continuously differentiable with respect to $\hat{\phi}$.

which implies Π_i is strictly increasing in $\hat{\phi}$. Therefore, $T\hat{V}$ is strictly increasing in $\hat{\phi}$ (which proves that $\hat{V}_1$ strictly increases in $\hat{\phi}$). □

Lemma A.6. *For any $\hat{V} \in \mathbf{C}'(X)$, $T\hat{V}(\hat{\phi}, \hat{\gamma})$ is strictly decreasing in $\hat{\gamma}$ at any given level of $\hat{\phi}$.*

Proof. Similarly as in the proof of Lemma A.5, we can obtain its first-order derivative of $\mathrm{E}\left[\max\left\{\hat{V}(\hat{\phi}', \hat{\gamma}'), 0\right\} \middle| \hat{\phi}, \hat{\gamma}\right]$ with respect to $\hat{\gamma}$:

$$\begin{aligned}
&\frac{d\left(\mathrm{E}\left[\max\left\{\hat{V}(\hat{\phi}', \hat{\gamma}'), 0\right\} \middle| \hat{\phi}, \hat{\gamma}\right]\right)}{d\hat{\gamma}} \\
&= \int_{\bar{\theta}(\hat{\phi},\hat{\gamma})}^{\infty} \left[-\frac{1}{\hat{\gamma}^2}\hat{V}_1\left(\hat{\phi} + \frac{1}{\hat{\gamma}}\theta, \hat{\gamma} + \omega\right) + \hat{V}_2\left(\hat{\phi} + \frac{1}{\hat{\gamma}}\theta, \hat{\gamma} + \omega\right)\right] n(\theta) d\theta \\
&\quad - \hat{V}\left(\hat{\phi} + \frac{1}{\hat{\gamma}}\bar{\theta}(\hat{\phi}, \hat{\gamma}), \hat{\gamma} + \omega\right) n(\bar{\theta}(\hat{\phi}, \hat{\gamma})) \\
&= \int_{\bar{\theta}(\hat{\phi},\hat{\gamma})}^{\infty} \left[-\frac{1}{\hat{\gamma}^2}\hat{V}_1\left(\hat{\phi} + \frac{1}{\hat{\gamma}}\theta, \hat{\gamma} + \omega\right) + \hat{V}_2\left(\hat{\phi} + \frac{1}{\hat{\gamma}}\theta, \hat{\gamma} + \omega\right)\right] n(\theta) d\theta < 0,
\end{aligned} \tag{A.8}$$

where the second equality is due to the definition of $\bar{\theta}(\hat{\phi}, \hat{\gamma})$ in Eq. (A.5), and the last inequality is because $\hat{V}_1 > 0$ (it is already shown), and $\hat{V}_2 \leq 0$ by the supposition ($\hat{V} \in \mathbf{C}'(X)$). Furthermore, Π_i is independent of $\hat{\gamma}$, $\hat{V}(\hat{\phi}', \hat{\gamma}')$ is non-increasing in $\hat{\gamma}$. Therefore, $T\hat{V}$ is strictly decreasing in $\hat{\gamma}$. □

From Lemmas A.5 and A.6, we have $T[\mathbf{C}'(X)] \subseteq \mathbf{C}''(X) \subseteq \mathbf{C}'(X)$. Because $\mathbf{C}'(X)$ is a closed subset of the complete normed space $\mathbf{C}(X)$, and T is a contraction mapping that has a unique fixed point $\hat{V}^*$ on $\mathbf{C}(X)$ (Theorem A.6), we have $\hat{V}^* = T\hat{V}^* \in \mathbf{C}''(X)$, which finishes the proof.[9]

Proof of Theorem 2:

Recall, from Section 4.1, that the optimal choice of investment is zero whenever the perceived type is less than or equal to zero, i.e., $q^*(\hat{\phi}) = 0$ for any $\hat{\phi} \leq 0$. Then, there will be no opportunity of updating the perceived talent, which means any level of $\hat{\phi}$ less than or equal to zero is an absorbtion. From Eq. (9), it is immediate that $V(0, \tau) < 0$ for all τ.

[9]For a detailed proof of the final step of this proof, see Corollary 1 to Theorem 3.2 in Stokey and Lucas (1989).

Now, fix τ at any given level. There exist at least a set of funds which find it optimal to invest a positive amount because the expected return diverges to infinity without any investment ($g_i(0) = \infty$ for all $i \in \{S, L\}$). Then, there should exist $\hat{\phi}^{**} > 0$ such that $V(\hat{\phi}^{**}, \tau) > 0$. Therefore, there exists a unique threshold of $\hat{\phi}_E(\tau) \in (0, \hat{\phi}^{**})$ at which V changes its sign because V is continuous and increases in $\hat{\phi}$ (due to Theorem 1).

Furthermore, because V decreases in τ (due to Theorem 1), this in turn implies that the threshold $\hat{\phi}_E(\tau)$ is increasing in τ.

Proof of Theorem 3:

Fix $\hat{\gamma}$ at any given level. Using the envelope theorem, differentiating the R.H.S of Eq. (16) with respect to $\hat{\phi}$ reveals that the R.H.S is increasing in $\hat{\phi}$, i.e.,

$$\begin{aligned}\frac{d(\Pi_L(\hat{\phi}) - \Pi_S(\hat{\phi}))}{d\hat{\phi}} =& u'\left(d_L\hat{\phi}g_L(Q_L)q_L^*(\hat{\phi}) - C^*(q_L^*) - F\right) d_L g_L(Q_L) q_L^* \\ &- u'\left(d_S\hat{\phi}g_S(Q_S)q_S^*(\hat{\phi}) - C^*(q_S^*) - F\right) d_S g_S(Q_S) q_S^* > 0,\end{aligned}$$

because $q_L^* > q_S^*$ whenever $d_L g_L(Q_L) - d_S g_S(Q_S) > 0$, which is immediate from Eq. (12).

When $\hat{\phi} = 0$, the L.H.S is positive whereas the R.H.S is zero. When $\hat{\phi}$ is arbitrarily large, the L.H.S converges to zero whereas the R.H.S is still positive. Therefore, there exists a threshold below which the L.H.S is greater than the R.H.S.

In case the concavity of the value function is large enough, we can show that the threshold is unique. From Eqs. (A.4) and (A.5), using the Leibniz integral rule, we can obtain the derivative of the L.H.S of Eq. (16) with respect to $\hat{\phi}$:

$$
\begin{aligned}
&\frac{d\left[(d_S - d_L)\kappa\left(\mathrm{E}\left[\max\left\{V(\hat{\phi}', \tau+1), 0\right\}\middle|\hat{\phi}, \tau\right] - V(\hat{\phi}, \tau)\right)\right]}{d\hat{\phi}} \\
&= (d_L - d_S)\kappa\Bigg\{\int_{\bar{\theta}(\hat{\gamma},\hat{\phi})}^{\infty} V_1(\hat{\phi}', \tau+1)n(\theta)d\theta\left(\frac{\hat{\gamma}}{\hat{\gamma}+\omega}\right) - V_1(\hat{\phi}, \tau) \\
&\quad - V\left(\hat{\phi} + \frac{1}{\gamma+\tau\omega}\bar{\theta}(\hat{\gamma}, \hat{\phi}), \tau+1\right) n(\bar{\theta}(\hat{\gamma}, \hat{\phi}))\frac{d\bar{\theta}(\hat{\phi})}{d\hat{\phi}}\Bigg\} \\
&= (d_L - d_S)\kappa\left\{\int_{\bar{\theta}(\hat{\gamma},\hat{\phi})}^{\infty} V_1(\hat{\phi}', \tau+1)n(\theta)d\theta\left(\frac{\hat{\gamma}}{\hat{\gamma}+\omega}\right) - V_1(\hat{\phi}, \tau)\right\} \\
&< (d_L - d_S)\kappa\left\{\int_{\bar{\theta}(\hat{\gamma},\hat{\phi})}^{\infty} V_1(\hat{\phi}', \tau+1)n(\theta)d\theta - V_1(\hat{\phi}, \tau+1)\right\} \\
&< (d_L - d_S)\kappa\left\{\int_{-\infty}^{\infty} V_1(\hat{\phi}', \tau+1)n(\theta)d\theta - V_1(\hat{\phi}, \tau+1)\right\} < 0,
\end{aligned}
$$

where the last inequality is true in case $-\hat{V}_{11}/\hat{V}_1$ is sufficiently large for any $\hat{\phi} \geq \hat{\phi}_E(\tau)$.

Because the L.H.S is strictly decreasing, there exists a unique threshold above and below which the sign of difference between the L.H.S and the R.H.S switches. The R.H.S does not change with τ, but the L.H.S decreases in τ. Therefore, the threshold decreases in τ.

Proof of Lemma 3:

We first calculate the transition function for the case of exit. From Theorem 2, the optimal voluntary exit becomes a function of state variable $\hat{\phi}, \tau$, which is captured by $I(\hat{\phi}, \tau)$. Then, given state $\hat{\phi}, \tau$, the transition function for the case of exit is

$$Z(\hat{\phi}' = \hat{\phi}, \tau' = E|\hat{\phi}, \tau) = (1-\kappa) + \kappa(1 - I(\hat{\phi}, \tau)) = 1 - \kappa I(\hat{\phi}, \tau).$$

Conditioning on no exit, the probability of payoff realization is determined by the choice of investment opportunity. From Theorem 3, the choice of investment is a function of state variable $\hat{\phi}, \tau$. Therefore, the probability of payoff realization can be represented as a function of state variable $d(\hat{\phi}, \tau)$. Then, given state $\hat{\phi}, \tau$, the transition function for the case of no update is

$$Z(\hat{\phi}' = \hat{\phi}, \tau' = \tau | \hat{\phi}, \tau) = \kappa I(\hat{\phi}, \tau)(1 - d(\hat{\phi}, \tau)).$$

Now, we work on the case for the belief update conditioning on no exit and payoff realization. Similarly as in Eq. (A.3), we can represent the conditional distribution of $\hat{\phi}'$ given $\hat{\phi}$ and τ as

$$\hat{\phi}'\Big|_{\hat{\phi},\tau} = \hat{\phi} + \frac{1}{\gamma + (\tau+1)\omega}\theta.$$

where θ is a random variable follows the standard normal distribution. Then, $\hat{\phi}'$ is obtained if

$$\theta = (\gamma + (\tau+1)\omega)(\hat{\phi}' - \hat{\phi}).$$

Then, given state $\hat{\phi}, \tau$, the transition function for the case of update is

$$Z(\hat{\phi}', \tau' = \tau + 1 | \hat{\phi}, \tau) = \kappa I(\hat{\phi}, \tau)(1 - d(\hat{\phi}, \tau)) n \left((\gamma + (\tau+1)\omega)(\hat{\phi}' - \hat{\phi}) \right).$$

Finally, all other states than those states in the above can not be reached, which implies the value of the transition function should be zero.

Proof of Theorem 4:

We provide a sketch of the proof for unique existence of stationary distribution. We first state a stronger condition (henceforth condition M) that implies Doeblin's condition (see, for example, Stokey and Lucas [1989] for further discussion on the condition). Let $Z^N(A|s) \equiv Z$ be the probability of transition from state $s = (\hat{\phi}, \tau)$ to a set A in N steps.

Condition M. There exists $\epsilon > 0$ and an integer $N > 1$ such that for any $A \in \mathbb{R} \times \mathcal{T}$, either $Z^N(A|s) \geq \varepsilon$, for all $s \in S$, or $Z^N(A^c|s) \geq \varepsilon$, all $s \in \mathbb{R} \times \mathcal{T}$.

Let $\varepsilon \equiv \kappa(1 - d_S) = \kappa \min(1 - d_S, 1 - d_L)$. From Lemma 3 and Eq. (18), it is immediate that $Z^N(\hat{\phi}, E | \hat{\phi}, \tau) \geq \varepsilon$ for all $\hat{\phi}, \tau$. Because, for any $A \subset S$, it is either $\hat{\phi}, E \in A$ or $\hat{\phi}, E \in A^c$, we have either $Z^N(A|\hat{\phi}, \tau) \geq Z^N(\hat{\phi}, E|\hat{\phi}, \tau) \geq \varepsilon$ or $Z^N(A^c|\hat{\phi}, \tau) \geq Z^N(\hat{\phi}, E|\hat{\phi}, \tau) \geq \varepsilon$.

Then, due to Theorem 11.12 in Stokey and Lucas (1989), there exists a unique stationary distribution ν that solves the functional equation in Eq. (19).

Proof of Theorem 5:

We prove the theorem in several steps in the following:

Tables

Table 1: Parameter Values used in Numerical Analysis

Variable	Value	Interpretation
d_S	1	payoff rate (turnover) of short-term opportunity
d_L	.9	payoff rate (turnover) of long-term opportunity
a_S	1.2	adjustment cost for short-term opportunity
a_L	1	adjustment cost for long-term opportunity
b_S	1	return scale for short-term opportunity
b_L	1.3	return scale for for long-term opportunity
ϕ_0	.2	prior mean of talent ϕ
γ	15	prior precision of talent ϕ
ω	4	precision of idiosyncratic noise ϵ
κ	.95	probability of fund survival
F	.08	fixed cost of operation per period

Table 2: Summary Statistics

This table shows summary statistics for our sample of actively managed US equity mutual funds from January 1961 to 2019 for the full sample, year 1961 to 1990, and 1991 to 2019 separately. Panel A reports the mean and standard deviation of fund size, turnover from the CRSP mutual fund database, fund age, and fund manager tenure from the MorningStar at the fund-year level. Panel B reports the net fund returns, CAPM alphas, Vanguard alphas estimated using Vanguard benchmark per month in percentage per month, expense ratios per year, and value added based on CAPM and Vanguard benchmark. Fund size and value added are reported in millions dollars adjusted by inflation into January 1, 2020 dollars. All numbers are equally weighted.

	Full sample		1961 - 1990		1991 - 2019	
Num. of funds	3,390		714		3,356	
	Mean	Std	Mean	Std	Mean	Std
Panel A: Fund characteristics (per fund-year)						
Fund size (in mill $s)	1,374	5,874	222	962	1,567	6,313
Turnover	0.81	0.82	0.71	0.67	0.81	0.96
Age	12.9	11.6	17.2	14.7	12.7	11.4
Manager tenure	5.7	5.1	5.9	6.7	5.7	5.0
Panel B: Return, expenses, alphas, and value added (per fund-month)						
Net return (in %)	0.78	15.31	0.57	6.56	0.79	15.61
Expense ratio (yearly, in %)	1.22	0.53	1.03	0.62	1.24	0.52
CAPM gross alpha (in %)	0.07	4.10	-0.09	2.92	0.08	4.15
Vanguard gross alpha (in %)	0.04	3.73	0.03	2.44	0.04	3.77
CAPM value added (in mill $s)	7.26	530.55	29.54	125.20	7.14	531.99
Vanguard value added (in mill $s)	0.54	126.18	-0.33	24.02	0.57	128.36

Table 3: Fund Turnover and Manager Tenure

This table reports the regression results of annual fund turnover on manager tenure as in Eq. (22), for all funds and funds in each size quintile separately. Panel A reports the benchmark regression results. Panel B reports the robustness checks including (1) regressing return volatility (measured by the standard deviation of fund monthly returns per year), instead of turnover, on manager tenure to rule out the risk-taking story, (2) fund fixed effects, (3) controlling for fund age, and (4) controlling for return volatility. Fund turnover is reported in the CRSP mutual fund database, and the manager tenure is from the Morningstar. Robust standard errors are clustered at fund level. Sig. lvl: *** 0.01, ** 0.05, and * 0.1

Panel A: Regressions of fund turnover on manager tenure

Dependent Var.	Turnover	by Fund Size Quintiles				
	(all)	1 (small)	2	3	4	5 (large)
Tenure	-0.019***	-0.005	-0.027***	-0.030***	-0.020***	-0.017***
	(-8.09)	(-0.80)	(-5.82)	(-10.99)	(-4.08)	(-4.94)
ln(TNA)	-0.044***	-0.050**	-0.072***	-0.050**	-0.058**	-0.082***
	(-8.86)	(-2.27)	(-2.76)	(-2.06)	(-1.99)	(-5.63)
cons	1.149***	1.083***	1.308***	1.241***	1.270***	1.409***
	(38.96)	(14.72)	(11.07)	(9.38)	(6.67)	(12.77)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	42,586	7,361	8,052	8,307	8,664	9,465
Adjusted R^2	0.061	0.025	0.040	0.071	0.055	0.112

Panel B: Robustness Checks

Dependent Var.	Volatility	Turnover		
	(1)	(2)	(3)	(4)
Tenure	-0.000	-0.011***	-0.019***	-0.019***
	(-0.20)	(-5.86)	(-8.02)	(-8.24)
ln(TNA)	-0.000***	-0.053***	-0.043***	-0.043***
	(-3.26)	(-9.12)	(-8.25)	(-8.84)
Turnover	0.003***			
	(8.66)			
Age			-0.001	
			(-0.72)	
Volatility				9.096***
				(10.88)
cons	0.045***	1.150***	1.149***	0.724***
	(74.46)	(39.73)	(38.97)	(15.72)
Year FE	Yes	No	Yes	Yes
Fund FE	No	Yes	No	No
Observations	42,209	42,586	42,583	42,209
Adjusted R^2	0.623	0.539	0.061	0.089

Table 4: Flow-Performance Sensitivity by Fund Turnover

This table reports the regression results of quarterly fund flows on funds' return ranks in the past quarter (1), year (2), and three years (3), as described in Eq. (24). Ret Rank is the percentile of the fund's return among all the funds, which is zero for the lowest and one for highest. Turnover is the average turnover in the past quarter, year, and three years as reported in the CRSP database. Age is the number of years since the fund's starting date. ln(TNA) is the ln value of the fund's total net asset at the end of last quarter. Robust standard errors are clustered per quarter. Sig. lvl: *** 0.01, ** 0.05, and * 0.1

Dependent Variable: Fund Flow (a % of TNA)			
	(1)	(2)	(3)
	Last-Quarter Ret	Last-Year Ret	Last-3-Year Ret
Ret Rank * Turnover	0.009***	0.007***	0.005*
	(2.91)	(2.73)	(1.72)
Turnover	-0.008***	-0.005***	-0.003**
	(-7.09)	(-5.00)	(-2.04)
Ret Rank * Age	-0.002***	-0.003***	-0.002***
	(-6.50)	(-10.31)	(-11.61)
Age	-0.000***	0.000**	0.000***
	(-3.91)	(2.60)	(3.96)
Ret Rank * ln(TNA)	-0.003***	-0.002**	0.000
	(-3.39)	(-2.53)	(0.54)
ln(TNA)	-0.006***	-0.004***	-0.004***
	(-8.21)	(-7.29)	(-9.95)
Ret Rank	0.100***	0.149***	0.123***
	(11.57)	(17.19)	(17.02)
Constant	0.024***	-0.026***	-0.033***
	(4.82)	(-6.05)	(-10.83)
Quarterly FE	Yes	Yes	Yes
Observations	150,407	142,473	121,428
Adjusted R^2	0.101	0.125	0.113

Table 5: Value Added of Trades in One Year

This table reports the value added from trades in the past 240 days (one year) and the corresponding contribution to fund annual return. Value added from trades in the past n (1 to 240) days is calculated using equation (30). We use CAPM abnormal return, raw return, and Fama-French-Carhart 4-factor abnormal return of each stock for this calculation separately. Day 0 is for value added of trades on the same day. Value added of all holdings (value added of fund) is also reported, and value added from holdings 240 days ago is calculated as value added of all holdings minus value added of trades within 240 days. The corresponding contribution to fund annual return is the value added divided by the fund TNA (total net assets). Value Added is equally weighted across fund-day observations and the corresponding contribution to fund return is value-weighted. Robust standard errors are clustered per day. Sig. lvl: *** 0.01, ** 0.05, and * 0.1

	Value Added of Trades (in million $s)			Contribution to Fund Return (in %)		
Day	CAPM	Raw Ret	FFC 4 factor	CAPM	Raw Ret	FFC 4 factor
0	0.6	0.6	0.6	0.06	0.06	0.06
10	0.1	0.1	0.2	0.01	0.01	0.02
20	0.5	0.0	0.7	0.04	0.00	0.06
30	-0.2	-0.7	0.4	-0.01	-0.07	0.03
40	0.0	-0.9	0.9	0.00	-0.08	0.08
50	-0.3	-1.4	0.7	-0.03	-0.13	0.07
60	0.0	-1.0	1.2	0.00	-0.10	0.11
70	0.1	-0.9	1.0	0.01	-0.08	0.09
80	0.0	-0.6	0.7	0.00	-0.06	0.07
90	-0.1	-0.4	0.4	0.00	-0.03	0.04
100	0.6	0.7	1.1	0.06	0.06	0.10
110	0.0	-0.6	0.8	0.00	-0.05	0.07
120	0.2	0.0	0.6	0.02	0.00	0.05
130	-0.1	-0.1	0.3	-0.01	-0.01	0.03
140	-0.4	-0.4	-0.1	-0.04	-0.03	-0.01
150	-0.2	0.1	-0.1	-0.02	0.01	0.00
160	1.1	2.0	0.5	0.10	0.18	0.04
170	0.2	0.4	0.0	0.02	0.04	0.00
180	0.4	0.5	-0.6	0.03	0.05	-0.06
190	0.6	1.1	0.3	0.06	0.10	0.03
200	1.8	2.6	1.0	0.16	0.24	0.09
210	3.0	2.7	2.1	0.28	0.25	0.20
220	3.4	2.1	2.0	0.32	0.20	0.19
230	4.9	4.2	2.7	0.46	0.39	0.25
240	4.2	2.9	2.1	0.38	0.27	0.20
Daily Holdings	27.5***	42.0***	13.8***	2.55***	3.89***	1.28***
Holdings > 240 days	24.0**	39.7**	12.3**	2.22**	3.68**	1.13**