A Model of Voluntary Managerial Disclosure*

Patrick Hummel
Google

John Morgan
University of California, Berkeley

Phillip C. Stocken
Dartmouth College

March 8, 2018

Abstract

We study voluntary disclosure in a persuasion model with a wide range of manager and investor preferences and where the quality of the manager's knowledge is private information. When manager preferences over stock prices are relatively insensitive to fundamentals, moderate states are disclosed and extremes are suppressed. When the manager's sensitivity is similar to the investor's, disclosure is the mirror image—extreme states are disclosed and moderate states are suppressed. The latter disclosure policy appears to be more consistent with extant empirical findings.

Keywords: voluntary managerial disclosure, verifiable information, second-order uncertainty

JEL Codes: C72, D80, D83, L21, M41, M52

*We are grateful for comments from Mark Bagnoli, Davide Cianciaruso, Ron Dye, Henry Friedman, Ilan Guttman, Bob Magee, Nikki Skinner, Joel Sobel, and Sri Sridharan. We also thank workshop and conference participants at the Carnegie Mellon University, Lehigh University, Northwestern University, Rice University, UCLA, and University of Houston.

1 Introduction

Firm voluntary disclosure has a substantial impact on capital markets (Penman, 1980; Healy and Palepu, 2001; Bhattacharya *et al.,* 2003). Indeed, earnings forecasts, which represent a key voluntary disclosure by management, provide about 55% of accounting-based information and account for approximately 16% of the quarterly stock return variance for the average firm (Beyer *et al.,* 2010). Such forecasts may be quantitative, i.e. a projected earnings per share amount, which represent 78% of all announcements, or qualitative (Skinner, 1994). Quantitative announcements have the key feature that their veracity may be checked by comparison with a subsequent earnings report (e.g., Lev and Penman 1990; Rogers and Stocken 2005). In addition, if a management forecast is not offered in good faith, then the firm may be exposed to penalties under the anti-fraud provisions of the federal securities laws.

Recognizing the legal and ethical constraints managers face when reporting, the seminal work examining firm voluntary disclosure models it as a persuasion game (e.g., Jovanovic 1982; Verrecchia 1983; Dye, 1985). In a persuasion game, the manager is free to obscure or offer ambiguous information but cannot offer reports directly contradicting his private information. In short, reports cannot be shown to have been false *ex post*. Another common feature in this work is the modeling of preferences. Investors seek to match the stock price to the fundamental value of the firm whereas the manager seeks to maximize the firm's stock price irrespective of fundamentals.

Yet this view of managerial disclosure incentives has proven controversial. For instance, empirical works studying disclosure, such as Ajinkya and Gift (1984) and Hassell and Jennings (1986), suggest that managers issue forecasts to align investors' expectations with their own, the so-called "expectations adjustment hypothesis." Buttressing the view that the interests of managers and investors are often correlated, Fuller and Jensen (2002) point out that an overvalued stock can be as damaging to a firm as an undervalued stock since it often leads to dysfunctional firm behavior. Indeed, Warren Buffett's leadership at Berkshire Hathaway also seems consistent with this view. As far back as in the 1988 Berkshire Hathaway Annual Report, Buffett writes:

> "We do not want to maximize the price at which Berkshire shares trade. We wish instead for them to trade in a narrow range centered at intrinsic business value... [We] are bothered as much by significant overvaluation as significant undervaluation."

Institutional considerations also argue for alignment. To comply with the "disclose-or-abstain" principle for insider trading, it is important for managers to align investor beliefs with their own (Li *et al.,* 2016). Furthermore, the risk of lawsuits prompted by a stock price decline in response to negative earnings announcements causes "managers [to] behave as if they bear large costs when investors are surprised by large negative earning news" (Skinner, 1994, p. 39).

Both state-invariant manager preferences and the perfectly aligned preferences under the expectation adjustment hypothesis represent extreme cases. The truth likely lies somewhere in between—certainly the manager cares that stock prices somewhat reflect fundamentals but would probably also prefer that shares trade a bit above, rather than below, this value. Unfortunately, no formal analysis of this case exists.

We remedy this. We analyze a model that has the flexibility to accommodate a wide range of views concerning the relationship between managerial and shareholder preferences. We consider a model in which a manager (i.e., sender) sends a message to a representative investor (i.e., receiver) about the state, such as the fundamental value of the firm. The receiver then takes an action, such as buying or selling the firm's stock, that affects the payoffs of both parties.

An important feature of the firm voluntary disclosure environment is that the manager might be endowed with new information that is relevant for pricing the firm's stock and that investors are uncertain about whether the manager is endowed with such information. Following the extant literature, we model this uncertainty by assuming that, with some commonly known probability, the manager possesses new information pertinent to the firm's stock price. Otherwise the manager has no information not already incorporated into the firm stock price. Thus the investor labors under second-order uncertainty as she knows neither the realized state nor the manager's knowledge of the state.

Together preferences and second-order uncertainty give rise to predictions about what information managers disclose and what they withhold. While the extant literature has shown that information loss is inevitable when the manager seeks to maximize the firm's stock price, we show that this phenomenon holds for general preferences, even if the manager and the investor sometimes agree about the ideal action. This observation, however, reveals little about exactly what information is disclosed under such uncertainty.

We focus in particular on two special cases, one in which preferences satisfy the *gradual slope ordering property*, and the other in which preferences satisfy the *steep slope ordering property*. The gradual slope ordering property has the feature that, compared to investors, the manager's ideal actions are relatively insensitive to the state, consistent with the motive of stock price maximization irrespective of fundamentals. By contrast, the steep slope ordering property has the opposite feature—the manager's desired stock price is relatively sensitive to fundamentals—and is thus consistent with the expectations adjustment hypothesis.

We first show that, under the gradual slope ordering property, there is an equilibrium in which full revelation occurs on an interval of states and complete nondisclosure occurs elsewhere. Under the traditional view where managers all seek the highest stock price irrespective of fundamentals, disclosure takes the simple form of suppressing bad news and disclosing good news. More generally, in equilibrium the manager suppresses "extreme" news and discloses "moderate" news, where the precise bounds depend on the particulars of the preferences.

Disclosure under the steep slope ordering property is markedly different. In equilibrium, the manager fully discloses extreme states (both high and low) while suppressing information about moderate states. Again, the precise bounds dividing extreme and moderate states depend on the players' preferences.

The distinction in the nature of disclosure, depending on whether the manager's preferences are relatively sensitive or not, provides a novel testable hypothesis: When the manager is relatively insensitive to fundamentals, very bad news is suppressed, but when he is relatively sensitive, such news is disclosed. We examine this hypothesis in light of the existing empirical evidence concerning the nature of managerial disclosure.

The empirical evidence seems more consistent with the preferences satisfying the steep slope ordering property, including those underlying the expectations adjustment hypothesis. Firms are more likely to disclose extremely bad news than moderately bad news (e.g., Skinner, 1994; Kasznik and Lev, 1995; Kothari *et al.*, 2009). While none of these works offers a formal test of our hypothesis, their results are suggestive.

While the nature of the disclosure of bad news is our most important finding, we offer a number of other important theoretical results. For instance, when one allows for the possibility of agreement between the two parties in certain states, the situation can diverge even further from the extant literature. In particular, multiple equilibria can arise for a dense set of parameters, including equilibria which entail the partial revelation of information where the manager merely discloses that the state lies in some range. Thus, limiting attention to point forecasts or complete non-disclosure, as is the usual modeling approach, has the unfortunate consequence of ruling out certain equilibria that are consistent with the empirical presence of range forecasts (e.g., Skinner, 1994; Jensen and Plumlee, 2015).

Our work also suggests other possible empirical tests. Both the Sarbanes-Oxley Act of 2002 and the Dodd-Frank Act of 2010 had the express purpose of better aligning the preferences of managers and investors. Thus, a panel data set of managerial disclosure should reveal greater disclosure of very bad news after this legislation came into force. This prediction awaits formal testing.

We add to the extant literature on persuasion under second-order uncertainty, most notably Dye (1985), Shin (1994a), and their successors.[1] This earlier work assumed that the sender either reveals truthfully or remains silent; in addition, it assumed that the receiver wishes to match her action to the state while the sender wants to induce the highest rationalizable action. By contrast, we characterize equilibria for general message spaces and preference structures, including the possibility that the sender and receiver may agree as to the optimal action. We then apply this characterization to better understand voluntary disclosure by managers.

This work is but one strand of the vast persuasion literature. Most of this literature abstracts away from second-order uncertainty, instead assuming that the sender is

[1] See, e.g., Jung and Kwon (1988), Penno (1997), Dye (1998), Pae (2005), Shin (2006), Guttman *et al.* (2014), and Hummel *et al.* (2016).

In addition, Shin (1994b), Bhattacharya and Mukherjee (2013), and Bhattacharya *et al.* (2015) add multiple senders, but retain the same preference structure as earlier work.

always informed and that this is common knowledge. Milgrom (1981) and Grossman (1981) showed that, under the same preference structure as in Dye (1985), the sender fully reveals information in equilibrium. Seidmann and Winter (1997) extended this result to general preferences while other work extended this result to general message spaces.[2] We add to this literature by incorporating second-order uncertainty on top of general messages and preferences. Second-order uncertainty destroys the possibility of full revelation, so we analyze what information is disclosed in equilibrium.

The paper proceeds as follows: Section 2 describes the model. Section 3 characterizes the equilibria that prevail depending on the relative slopes of the manager's and investor's bliss lines when second-order uncertainty is present. Section 4 discusses the results within the context of the related empirical literature. Section 5 concludes.

2 The Model

Managerial disclosure about new or updated products represents an archetypal situation of the model. Studies determining the impact of new product introductions sometimes produce inconclusive outcomes. When study outcomes are particularly clear, the manager is well informed about how the product introduction will affect fundamental value. But when studies produce inconclusive or ambiguous results, the opposite is the case—the manager knows little more about fundamentals than the investors themselves. Investors lack access to marketing or scientific information and must rely on managerial disclosure to update their views as to the firm's value. Thus, they listen carefully to the disclosures the manager chooses to make (or not make) and price the firm's stock accordingly.

[2] See, e.g., Okuno-Fujiwara *et al.* (1990), Koessler (2003), Mathis (2008), and Hagenbach *et al.* (2014).

To capture the main economic elements of such settings, we consider a model featuring two players, a manager/sender S and a representative investor/receiver R. It is common knowledge that some payoff relevant state variable θ is drawn from an atomless distribution $F(\theta)$ with corresponding probability density function $f(\theta)$ and support $[\underline{\theta}, \bar{\theta}]$ including, possibly, the entire real line. The random variable θ represents the fundamental value of the firm at a given point in time.

With some probability $p \in (0, 1)$, the manager is knowledgeable about the state; otherwise, he knows no more than the investor. The variable p captures the probability of a conclusive test result or study outcome, whereas its complement represents a situation where reports to the manager were inconclusive or ambiguous. The manager's knowledge state is private as are the details he has received concerning the fundamental value of the firm. Thus, the investor faces two levels of uncertainty (sometimes called second-order uncertainty)—she knows neither the true fundamental value of the firm nor the extent of the manager's knowledge expertise, i.e., whether he is better informed than the investor.

Throughout, we refer to uncertainty about the manager's *knowledge* to describe situations where the investor cannot determine whether the manager is informed about the state. We refer to uncertainty about the manager's *information* to describe situations where the investor cannot discern the state, θ, conditional on the manager being informed about its realization.

After receiving the report, the manager sends a message m to the investor, where m indicates some information about the state θ. This message can be thought of as forward-looking information, such as a management earnings forecast, that is voluntary disclosed. As usual in persuasion games, messages are constrained to be non-falsifiable *ex post.* Restricting the message space in this fashion captures the institutional reality that if a firm does not offer forward-looking information in good faith, then it may be exposed to severe penalties under the anti-fraud provisions of the federal securities laws.[3] In this light, the message m is a (possibly degenerate) subset $m \subseteq [\underline{\theta}, \bar{\theta}]$ that contains the true state θ.[4] Furthermore, an uninformed manager must send the message $m = [\underline{\theta}, \bar{\theta}]$ to ensure that his message is never false *ex post*; we denote this message as $m = \emptyset$.

After receiving the message m, the investor selects an action $y \in \Re$ based on her preferences. Let $U_i(y, \theta)$ denote the payoff of player $i \in \{R, S\}$ when action y is chosen in state θ. We assume that, for every θ, payoffs are continuous and single-peaked in y, with a unique payoff-maximizing action, $y_i(\theta)$, which we term i's *bliss action*. In our setting, $y_S(\theta)$ represents the manager's preferred stock price and $y_R(\theta)$ reflects the investor's preferred stock price given the fundamental value of the firm. We assume $y_S(\theta)$ is continuous and weakly increasing in θ, whereas $y_R(\theta)$ is continuous and strictly increasing in θ; thus, the preferences of both parties are at least weakly correlated with the state. It is convenient to let $y(\emptyset)$ denote the investor's optimal action (stock price) under her prior beliefs about firm fundamentals.

Bliss actions alone are not sufficient to define preferences; they merely indicate the payoff-maximizing value of y for agent i in state θ. The manager and investor also care about the loss associated with not choosing their bliss action, i.e., the payoff when $y \neq y_i(\theta)$. Depending on the setting, or even the state itself, the loss function

[3] Specifically, the anti-fraud provisions of Section 10(b) of the Securities Exchange Act of 1934 and the SEC promulgated Rule 10b-5.

[4] Examples of feasible messages include $\{\theta\}$, $[\theta_1, \theta_2]$, $[\theta_1, \theta_2] \cup [\theta_3, \theta_4]$ and $[\underline{\theta}, \bar{\theta}]$ for any $\theta_1, ..., \theta_4$ satisfying $\underline{\theta} \leq \theta_1 < \theta < \theta_2 < \theta_3 < \theta_4 \leq \bar{\theta}$. In the context of management earnings forecasts, this assumption allows for the possibility of point forecasts (e.g., earnings are expected to be \$1.00 per share), range forecasts (e.g., earnings are expected to be between \$0.90 and \$1.10 per share), lower bound forecasts (e.g., earnings are expected to be less than \$1.00 per share), or upper bound forecasts (e.g., earnings are expected to be above \$1.00 per share), all of which are commonly observed (see Skinner, 1994).

may be steep or flat, symmetric or asymmetric. We impose a fairly minimal amount of structure on preferences, assuming that payoffs satisfy the *distance property;* that is, the payoffs to each party are proportional to the distance between the chosen action and the bliss action. Formally,

Definition 1 *The preferences of player $i \in \{S, R\}$ satisfy the* ***distance property*** *if and only if $U_i(y, \theta) = u_i(|y - y_i(\theta)|; \theta)$ for some strictly decreasing function u_i.*

The distance property implies that only the magnitude of the difference between a player's ideal action and the selected action affects payoffs while the direction (i.e., whether the selected value of y is above or below the bliss action) does not. It does not imply that losses from errors of a given magnitude are the same across states. For instance, the marginal cost of missing the player's bliss action may be very high in state θ'; whereas, in state θ'', a player's payoff may be relatively insensitive to the distance between the chosen action and the bliss action.

Nearly all of the extant applied literature on voluntary disclosure imposes some version of the distance property on investor preferences. Typically, the investor is assumed to set the stock price equal to the firm's expected value conditional on public information (including managerial disclosures). This coincides with optimizing behavior under a quadratic loss function.

There is less agreement as to the manager's preferences. Under the canonical model of managerial disclosure in Dye (1985), the manager seeks to maximize the firm's stock price irrespective of the state. Since $y_R(\bar{\theta})$ is the highest rationalizable action of the investor, the manager's bliss function is, in effect, $y_S(\theta) = y_R(\bar{\theta})$ for all θ. Since the investor never chooses an action greater than $y_S(\theta)$, the distance property is trivially satisfied. On the other hand, Fuller and Jensen (2002, p. 42), among others, point out that "an overvalued stock can be as dangerous to a company as an undervalued stock." While they do not formalize managerial preferences implied by this statement, a specification where the manager suffers just as much if a firm is overvalued by some amount as if it is undervalued by the same amount, would be consistent with this view. Clearly, any such specification satisfies the distance property.

Disagreement between the two parties as to the bliss action constitutes the key barrier to information transmission. We model this by assuming that $y_S(\theta) \neq y_R(\theta)$ except at finitely many *agreement points*, which occur when the bliss lines cross.[5] Formally,

Definition 2 *State* $\theta' \in (\underline{\theta}, \bar{\theta})$ *is an* ***agreement point*** *if and only if* $y_R(\theta') = y_S(\theta')$ *and there is a neighborhood of* θ'*,* N*, such that* $sign(y_S(\theta_a) - y_R(\theta_a)) \neq sign(y_S(\theta_b) - y_R(\theta_b))$ *for all* $\theta_a, \theta_b \in N$ *satisfying* $\theta_a < \theta' < \theta_b$*.*

Since, for a given state, the manager and investor (generically) disagree about the "optimal" stock price, the manager has an incentive to persuade, or perhaps even mislead, the investor about the firm's fundamental value, θ. Investors, recognizing this, seek to "decode" the manager's message so as to avoid being misled. Importantly, such decoding also includes dealing with *non-disclosure*. When the manager remains silent, the investor must parse out the chances that the manager is truly uninformed compared to the chances and circumstances where non-disclosure is strategic. Finally, we assume a genericity condition with respect to agreement points: throughout the analysis we implicitly rule out knife-edge cases where the action associated with an agreement point coincides with the investor's optimal action under her prior beliefs.

[5] An agreement point represents a state where the sender and receiver share a bliss action but where this action does not represent a tangency point between their bliss lines.

We use the following solution concept to characterize the results: The investor uses Bayes' rule wherever possible in formulating beliefs. We further restrict beliefs such that, if the manager sends the (possibly degenerate) message m, then the investor must believe that the state lies somewhere in m, even if m lies off the equilibrium path. Given her beliefs, the investor chooses an action maximizing expected payoffs. The manager chooses messages optimally given the investor's anticipated response.

Throughout, we will often be concerned with situations where the manager fully discloses his information, θ, to a representative investor. He might do this by sending the message $m = \theta$; however, there are a continuum of economically equivalent, but more complicated, messaging strategies that achieve the same end. For instance, in a setting where the manager fully discloses only sufficiently high states, a strategy where $m(\theta) = \left[\theta, \min\left\{\theta + h, \bar{\theta}\right\}\right]$ for arbitrary $h > 0$ yields the same information content as full disclosure and satisfies incentive compatibility. Throughout, we ignore action-equivalent, but more complicated, equilibria in favor of straightforward messaging where the manager chooses $m = \theta$ under full disclosure.

3 Analysis

We now turn to analyzing the nature of information disclosure under second-order uncertainty. Throughout we apply this to a situation of a possibly uninformed manager strategically revealing the future prospects of the firm. The model, however, can be construed much more broadly. For instance, our findings would apply equally well to an expert testifying in court and trying to influence the judge's decisions.

When the manager's knowledge about the future prospects of the firm is uncertain, non-disclosure is always on the equilibrium path since uninformed managers have no recourse but to send the null message. This, in turn, limits the ability of investors to strategically respond to uninformative messages so as to create incentives for full disclosure by informed managers. To see why such constraints on actions destroy full revelation, notice that the null message necessarily produces an action that differs generically from the fully-revealing action and which cannot be strategically manipulated for incentive purposes. Since the manager and investor disagree as to the bliss action under full revelation, it then follows that the manager can profitably deviate by sending the null message in some states θ. Thus, by placing non-disclosure *on* the equilibrium path, unraveling arguments cease to work. Formally,

Theorem 1 *Full revelation is never an equilibrium.*

In a managerial disclosure setting, Theorem 1 implies that the manager sometimes withholds information from investors. Dye (1985) proved a special case of this theorem for situations where investors choose an action equal to the expected state and the manager always prefers to induce the highest rationalizable action. Theorem 1 may be seen as the second-order uncertainty analog to Seidmann and Winter (1997), who showed that the unraveling logic producing full revelation generalizes across preference structures. This theorem shows how the *failure* of full revelation exhibits a similar preference independence property.

While Theorem 1 shows that full revelation is not an equilibrium under second-order uncertainty, one might suspect that as the manager's knowledge becomes precise ($p \to 1$), information loss vanishes. Below, we show that this intuition is correct, but only for certain preference structures. Specifically, we highlight a condition on preferences, which we call *conservatism*, that proves pivotal to the result:

Condition 1 *A manager is* ***conservative*** *in state* θ *if there exists an agreement point,* θ'*, such that* $U_S(y_R(\theta'), \theta) > U_S(y_R(\theta), \theta)$*. If, for every agreement point* θ' *and (almost) all states* θ*,* $U_S(y_R(\theta'), \theta) < U_S(y_R(\theta), \theta)$*, a manager is* ***not conservative.***

As we show elsewhere, the absence of a conservative manager is necessary and sufficient for the fully revealing equilibrium to be unique when there is no second-order uncertainty (Hummel *et al.*, 2018). Using this finding, we offer the "dual" to Theorem 1: So long as the manager is never conservative, full revelation occurs as second-order uncertainty vanishes. Formally,

Remark 1 *If the manager is not conservative at any agreement point, then all equilibria converge to full revelation in the limit as* $p \to 1$*.*

Since one can no longer focus on an equilibrium characterized by full revelation when the knowledge state is private, it remains to determine the nature of equilibrium in these settings. We do this next.

3.1 Gradual Slope Ordering Property

Early persuasion games focused on firms' voluntary disclosure of product quality (e.g., Grossman, 1981; Milgrom, 1981). In these models, firms prefer to be seen as selling a high quality product, regardless of true product quality. Consumers, on the other hand, wish to discover true product quality. Similarly, studies examining voluntary disclosure of financial information typically assume that the manager wants to convince investors that the firm has the highest possible value whereas investors seek to determine the firm's fundamental value (e.g., Dye, 1985; Shin, 1994a; Pae, 2005). The bliss lines are identical in both settings; that is, $y_S(\theta) = \bar{\theta}$ for the firm/manager and $y_R(\theta) = \theta$ for consumers/investors.

Moreover, even if the manager's primary goal is to maximize the stock price, the manager may nonetheless prefer a slightly higher stock price when firm fundamentals are stronger. This possibility is captured by preferences where $y_S(\theta)$ is slightly increasing in θ, but the slope for $y_S(\theta)$ is significantly lower than that for $y_R(\theta)$. These situations all share the feature that a manager's bliss line is relatively insensitive to the state compared to the investor's bliss line. These preferences, as well as many others, satisfy the *gradual slope ordering property*, which holds when the slope of the manager's bliss line is less than half that of the investor's bliss line. Formally,

Definition 3 *The bliss lines* $y_S(\theta)$ *and* $y_R(\theta)$ *satisfy the* gradual slope ordering property *if for all* $\theta' > \theta$*, we have* $y_S(\theta') - y_S(\theta) < \frac{1}{2}(y_R(\theta') - y_R(\theta))$.

In addition to the scenarios above, this definition also applies in the opposite case where the manager prefers that the stock price be as low as possible. For instance, in a study of whether chief executives manage the timing of their voluntary disclosures around stock option awards, Aboody and Kasznik (2000) document that chief executives are more likely to voluntarily preempt negative earnings news than positive earnings news in anticipation of a stock option grant. By lowering the firm's stock price and thereby the strike price of the options they anticipate been awarded, they raise the probability of being able to profitably exercise their stock options. These preferences again yield bliss actions that satisfy the gradual slope ordering property, but here $y_S(\theta) = \underline{\theta}$ and $y_R(\theta) = \theta$, contrary to the above setting.

When there are no agreement points, we show that all equilibria under such a formulation are of the following form: There is a convex region in which disclosure occurs and a convex region in which no disclosure occurs. The ordering of these two regions depends on whether the investor's preferences are above or below the manager's—when below, the equilibria resemble those in much of the extant literature, where "good news" (high values of θ) is revealed whereas "bad news" (low values of θ) is suppressed. When the investor prefers higher actions, disclosure takes the opposite form where "bad news" is revealed and "good news" is suppressed.

When agreements points occur, equilibrium may take on a different form. Generically, an equilibrium in this case consists of non-disclosure both for *low and high* values of θ and disclosure for values of θ surrounding the agreement point. Such behavior stands in sharp contrast to the formulation in the extant literature. Indeed, regardless of whether there is an agreement point, we show that under the gradual slope ordering property, there exists an equilibrium in which disclosure takes place over an interval $[\theta_1, \theta_2]$ and non-disclosure occurs elsewhere. We call this a *convex disclosure equilibrium.* Against this background, we establish the following result:

Proposition 1 *Under the gradual slope ordering property, there exists a convex disclosure equilibrium.*

A generic example of Proposition 1 is illustrated in Figure 1; the solid lines in the figure depict the investor's equilibrium actions following the report of an informed manager. There is always a cutoff at θ_1 where the investor's bliss action when the manager discloses equals the action the investor would choose when the manager remains silent. The possibility of agreement, which is illustrated in state θ' in the figure, causes equilibrium to differ in fundamental ways from earlier characterizations. Rather than dividing the state space into good news, which is disclosed, and bad news, which is withheld, a second cutoff can arise in the presence of agreement points. In

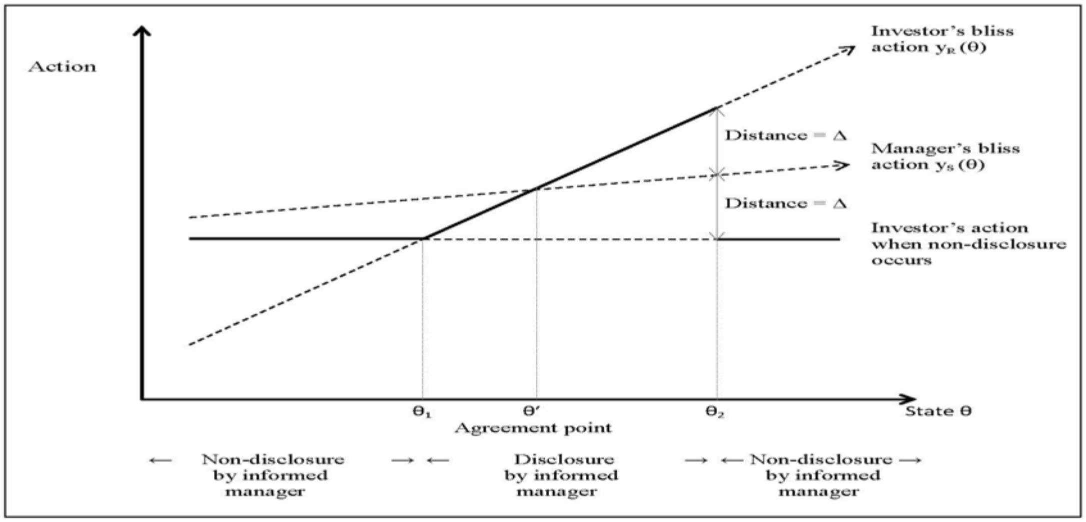

Figure 1: Convex Disclosure Equilibrium: *Under the gradual slope ordering property, there exists a convex disclosure equilibrium in which disclosure takes place over the interval* $[\theta_1, \theta_2]$ *and non-disclosure occurs elsewhere.*

the figure, this cutoff occurs at θ_2 where the distance between the investor's and manager's bliss actions equals the distance between the manager's bliss action and the investor's action when the manager remains silent. In Figure 1, this distance is denoted as Δ.

As is evident from Figure 1, when $\theta < \theta_1$, the manager's bliss action is closer to the investor's action that arises when the manager remains silent than it is to the investor's bliss action when the manager reveals his private information. The same is true when $\theta > \theta_2$. Moreover, the gradual slope ordering property implies that for more extreme values of θ, there will be an even larger gap between the manager's and the investor's bliss lines than there is between the manager's bliss line and the non-disclosure action. Thus, if the manager prefers non-disclosure at a moderate value of θ, the manager will also prefer non-disclosure at a more extreme value of θ.

Together these cutoffs imply the suppression of "extreme" news, consisting of both extremely high and extremely low states, rather than the simple good versus bad news dichotomy that arises when agreement points are absent.

A remarkable feature of this analysis is to show that, by adding an agreement point, equilibrium disclosure presents the investor with the perplexing situation that silence from the manager may indicate very good news about the firm's prospects, or very bad news (or no news). Thus, unlike the more standard situations where agreement points are absent, an investor's action following non-disclosure is fraught with considerable risk.

Uniqueness

Unlike the nearest antecedents to our work, notice that Proposition 1 merely states that a convex disclosure equilibrium exists while saying nothing about uniqueness. This follows from the more general preferences we consider. In particular, we allow for the possibility that the manager and investor might agree as to the optimal stock price in some states. Agreement points (or lack thereof) turn out to be critical to obtaining both uniqueness and the usual equilibrium formulation where bad news is suppressed and good news is disclosed. Formally,

Proposition 2 *Under the gradual slope ordering property with no agreement points, there is a unique equilibrium. In this equilibrium, full disclosure occurs over some interval* $[\theta_1, \theta_2]$ *and non-disclosure results otherwise. Moreover, in this equilibrium either* $\theta_1 = \underline{\theta}$ *or* $\theta_2 = \bar{\theta}$*, but not both.*

Antecedent works mainly concentrate on the comparative static implications of the chance that the manager is informed and generally find that, *ceteris paribus*, a more informed manager discloses more information. The same analysis can be conducted more generally as an implication of Proposition 2. Perhaps more interesting is the relationship between disclosure and preferences of the two parties. Proposition 2 has the striking implication that, so long as its required conditions hold, equilibrium information disclosure is *independent* of the particulars of the manager's preferences. The intuition is the following: the point marking the boundary between "good news" and "bad news" occurs when the investor's action is the same regardless of disclosure. Since neither action depends on the manager's preferences, equilibrium is undisturbed when these preferences change.

Strategic Range Forecasts

Managers often issue range forecasts, rather than point estimates, when offering earnings' guidance. For instance, Kasznik and Lev (1995) show that roughly 25% of all management earnings forecasts take this form. Empirical studies (e.g., Baginski *et al.* 1993; Bonsall *et al.* 2013; Feng and McVay, 2010) mainly attribute such forecasts to a manager's lack of knowledge—the manager is vague because this is the best information available. Yet the extant theory literature hints at the possibility that strategic motives might drive range forecasts, i.e., that the manager is strategically withholding information so as to trigger a particular price response.[6]

The fundamental insight of the extant literature is to highlight circumstances where information withholding can be optimal, but by limiting messages to full revelation or no revelation, the incentives for partial information revelation cannot be analyzed. Non-trivial range forecasts are ruled out by fiat in Dye (1985), Jung and Kwon (1988), Shin (1994a, 1994b, 2006), Einhorn (2007), and Bhattacharya and Mukherjee (2013), as in these papers the message spaces are limited to full revelation

[6]In most such work, the manager strategically withholds information by issuing a null message, interpreted as silence. The null message, however, could be reinterpreted as a trivial range forecast where the range of possibilities consists of the entire state space.

or no revelation.

An important contribution of this manuscript is to generalize the set of messages available to the manager, including range forecasts. This permits the study of incentives for partial revelation. Our next result identifies conditions under which strategic range forecasts occur in equilibrium, even when managers are *perfectly* informed. This is not to discount the possibility that range forecasts might reflect lack of information, but only to show that they need not occur for this reason.

Proposition 3 *Under the gradual slope ordering property with an agreement point θ', there exists an equilibrium characterized by cutoffs $\theta_1 < \theta'_1 < \theta'_2 < \theta_2$, where $\theta' \in (\theta'_1, \theta'_2)$, such that partial disclosure occurs when $\theta \in [\theta'_1, \theta'_2]$, full disclosure occurs when $\theta \in [\theta_1, \theta_2] \,/\, [\theta'_1, \theta'_2]$, and non-disclosure occurs when $\theta \notin (\theta_1, \theta_2)$.*

Together Propositions 2 and 3 illustrate the importance of preferences that accommodate agreement points and message spaces that allow for partial disclosure. Uniqueness turns on the absence of the first condition while strategic range forecasts require both conditions. Both propositions require the gradual slope ordering property.

How reasonable are these conditions? The gradual slope ordering property holds when the manager's ideal share price is relatively insensitive to the fundamental value, thus leading the manager to reduce share price volatility. Such smoothing incentives are widely observed in practice, especially in the management of earnings across reporting periods.[7] Agreement points might arise when the manager and investor initially agree about the appropriate firm valuation, such as with a newly hired manager whose incentives are calibrated to the status quo.

[7] The voluminous literature documenting earnings smoothing includes DeFond and Park (1997), Graham *et al.* (2005), Jung *et al.* (2013), Leuz *et al.* (2003), and Rountree *et al.* (2008).

As Proposition 3 shows, range forecasts are important to a manager seeking to influence investor beliefs. While partial disclosure via range forecasts works for small changes in fundamentals, larger changes require different tactics depending on their magnitude: extreme events are met with silence, whereas moderately large changes provoke full disclosure in equilibrium.

To understand the intuition behind Proposition 3, note that the gradual slope ordering property implies that the manager prefers the stock price $y_R(\theta')$ to $y_R(\theta)$ after observing a fundamental value θ near the agreement point θ'. By strategically choosing an interval around θ', say $[\theta'_1, \theta'_2]$, and issuing a range forecast whenever θ lies in this interval, investors optimally respond by setting the stock price to $y_R(\theta')$. The manager could have induced a price $y_R(\theta)$ by fully disclosing rather than issuing a range forecast, but gains no benefit by this deviation. Accordingly, there exists an equilibrium in which only partial disclosure occurs in the neighborhood of the agreement point. Ironically, the effect of an agreement point in this situation is to *reduce* disclosure by the manager.

3.2 Steep Slope Ordering Property

While it may seem natural to assume that the manager wants to maximize the share price regardless of fundamentals, the expectations adjustment hypothesis takes a different view, instead positing that the manager's ideal share price roughly corresponds to fundamentals. Indeed, Berkshire Hathaway explicitly aims for this outcome. Preferences satisfying the *steep slope ordering property*, where the slope of the manager's bliss line is more than half that of the investor's bliss line, describe these situations.[8]

[8] The reason a cutoff of 1/2 is used in defining whether the bliss lines satisfy the gradual slope ordering property or the steep slope ordering property is the following: If the slope of the manager's bliss line is more than half that of the investor's, then the difference between the manager's bliss

Formally,

Definition 4 *The bliss lines* $y_S(\theta)$ *and* $y_R(\theta)$ *satisfy the* steep slope ordering property *if for all* $\theta' > \theta$*, we have* $y_S(\theta') - y_S(\theta) > \frac{1}{2}(y_R(\theta') - y_R(\theta))$.

When preferences satisfy the steep slope ordering property, *all* equilibria share the feature that extremes are disclosed while non-disclosure occurs over a convex region of moderate states. Notice that disclosure in this equilibrium is the mirror image of that under the gradual slope ordering property. We call such an equilibrium a *convex non-disclosure equilibrium.* Formally,

Proposition 4 *Under the steep slope ordering property, there exists a convex non-disclosure equilibrium. Moreover, every equilibrium is a convex non-disclosure equilibrium.*

A generic convex non-disclosure equilibrium is illustrated in Figure 2; the solid lines in the figure depict the investor's equilibrium actions following the report of an informed manager. There is a cutoff at θ_1 where the manager is indifferent between full disclosure and non-disclosure, i.e., the actions induced under each circumstance are equidistant to the manager's bliss line. In Figure 2, this distance is denoted as Δ. The steep slope ordering property implies that, for $\theta < \theta_1$, the manager prefers to disclose, which may be readily seen in the figure. There is also another cutoff at θ_2 where the investor's bliss action when the manager discloses is identical to her action when the manager remains silent. For $\theta > \theta_2$, the manager again will prefer to disclose.

action and the investor's bliss action changes less rapidly as a function of the state than the difference between the manager's bliss action and the action taken upon non-disclosure. But if the slope of the manager's bliss line is less than half that of the investor's then the opposite holds. Hence, 1/2 represents the critical value dividing the two cases.

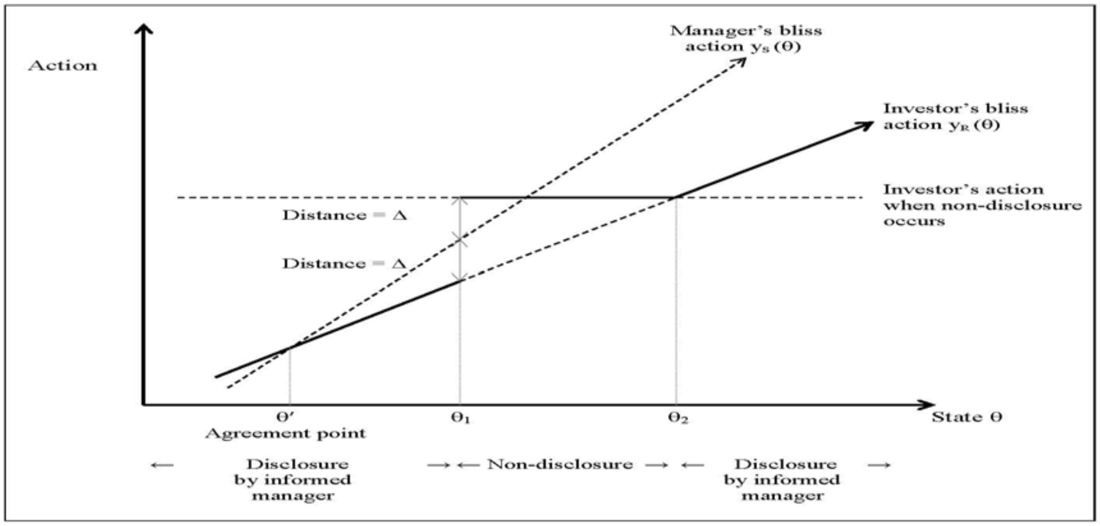

Figure 2: Convex Non-Disclosure Equilibrium: *Under the steep slope ordering property, there exists a convex non-disclosure equilibrium in which non-disclosure occurs in the interval* $[\theta_1, \theta_2]$ *and disclosure occurs elsewhere.*

This proposition has the substantive interpretation that the manager will disclose good news or extremely bad news in equilibrium but withhold moderately bad news. The nature of disclosure under the steep slope ordering property is thus quite different from the disclosure under the gradual slope ordering property. Furthermore, unlike the situation of convex disclosure equilibria, where equilibrium was unique in the absence of agreement points, no such property obtains for convex non-disclosure equilibria. An example illustrating the possibility of multiple convex non-disclosure equilibria is offered in Example 1 at the end of the appendix.

Why does the steep slope ordering property change the nature of equilibrium disclosure so starkly? The differences are most readily seen in extreme states. Under the gradual slope ordering property, the investor prefers more extreme actions than the manager, at least near one of the endpoints. Hence, the manager resorts to non-disclosure, inducing a moderate action in response. Under the steep slope ordering property, preferences are not as misaligned at the extremes. Indeed, the manager may prefer more extreme actions than the investor. Hence, moderating investor actions is unattractive compared to disclosure.

An interesting insight from our analysis is that disclosure hinges on the relative sensitivities of the sender's and receiver's ideal actions to the state rather than the amount of disagreement. Thus, our characterization contrasts strikingly with the main results in the cheap talk literature, where the amount of disagreement between the two parties plays the main role (e.g., Crawford and Sobel, 1982).

4 Comparing Theory and Empirics

Our previous analysis indicates that the form of managerial disclosure depends crucially on the relative sensitivities of the manager and investor's ideal stock prices to the fundamental value of the firm. Consider the case where the manager's ideal stock price is always greater than the investor's ideal stock price. When the manager's preferences are relatively insensitive to fundamentals, the manager withholds extremely bad news but discloses good news and moderately bad news. By contrast, if the manager's preferences are roughly as sensitive to firm fundamentals as the investor's, the manager withholds moderately bad news but discloses extremely bad news and good news.

Which of these equilibrium predictions is more consistent with empirical evidence? The previous literature focused on equilibria where good news is disclosed and extremely bad news is suppressed.[9] Accordingly, one might expect such disclosure

[9] See, for instance, Dye (1985), Jovanovic (1982), and Verrecchia (1983).

patterns to be common empirically. This turns out not to be the case. In examining the existing empirical literature on voluntary managerial disclosure, we find a consistent pattern that differs from these equilibria in which only extremely bad news is withheld.

The evidence that the empirical literature offers is largely consistent with the view that firms are more likely to disclose extremely bad news than moderately bad news. For example, Skinner (1994) partitions voluntary management forecasts into different pools depending on the type of news disclosed (e.g., extremely bad news, moderately bad news, no news, extremely good news, etc.), and finds that voluntary disclosure of extremely bad news is more common than voluntary disclosure of news that is only moderately bad. Similarly, Kasznik and Lev (1995) find that firms facing larger earnings disappointments are more likely to voluntarily provide quantitative and earnings-related warnings than firms with more moderate earnings disappointments. Finally, Kothari, *et al.* (2009) find that bad news is more extreme than good news, and accordingly, the market responds more negatively to the release of bad news than positively to the release of good news. They rationalize this finding by positing that firms withhold moderately bad news up to some threshold, but then release the news if the bad news is extremely bad, again consistent with the notion that firms are more likely to disclose extremely bad news than only moderately bad news.[10]

Unlike the extant literature, which assumes one view about the relationship between managerial and shareholder incentives or the other, our model has the flexi-

[10] On this note, recently, Nintendo Co. shares experienced the largest one-day plunge since 1990 when the company disclosed that the financial benefits from the worldwide hit Pokemon Go will be limited. The stock sank 18 percent, the maximum one-day move allowed by the Tokyo exchange, reducing its market capitalization by $6.7 billion. See "Nintendo Slumps By Most Since 1990 on Dashed Pokemon Go Hopes" (Bloomberg (July 24, 2016)) for more details.

bility to accommodate a wide range of views. Moreover, our rubric of distinguishing between gradual and steep slope ordering offers guidelines that managers can use when voluntarily releasing information to investors and factors investors might consider when using a firm's voluntary disclosure and earnings forecasts. The empirical literature suggests that the steep slope ordering property more faithfully describes the correlation between the preferences of managers and investors in the voluntary disclosure environment than the gradual slope ordering property.

The extent to which the incentives of managers and investors are correlated is a function of the corporate governance environment. Following the financial scandals around the turn of the century, the U.S. Congress enacted the Sarbanes-Oxley Act of 2002, which established new corporate governance requirements for public firms. The New York Stock Exchange and the NASDAQ also altered their listing requirements; they now require, for instance, that listed companies have a majority of outside directors (i.e., directors with no employment ties to the company) as opposed to inside directors (i.e., directors who are employees or officers of the company). If these changes have strengthened the correlation between the incentives of managers and investors, then our analysis predicts that firms are more likely to disclose extreme news and withhold moderate news *ceteris paribus*. This prediction awaits empirical testing.

5 Conclusion

First-generation persuasion models established that preferences were irrelevant to information disclosure—full revelation was always an equilibrium. Second-generation models showed that this conclusion held only when the quality of the sender's information was common knowledge. When information quality was private, how preferences affect disclosure had not been characterized. We have filled this gap by characterizing how preferences affect the qualitative features of equilibrium disclosure.

While this represents a useful contribution, it is not the main point of the paper. Motivated by a long-running academic debate, we sought to understand how manager and investor preferences affect voluntary disclosure. We have found that when the manager's ideal stock price is relatively insensitive to fundamentals, extreme news is suppressed and moderate news is disclosed. When sensitivity is relatively similar, disclosure takes the opposite form: extreme news is disclosed and moderate news is suppressed.

Neither researchers nor analysts agree as to the preferences of managers and investors. Extant theory models study a situation of extreme preference divergence—investors want the stock price to match fundamentals while managers prefer higher prices regardless of fundamentals. An alternative view, mainly driven by institutional considerations, stresses that managers and investors both want the stock price to align with fundamentals to some considerable degree. Preferences are flexible in our model, so each view represents a special case.

By explaining how disclosure and preferences are linked, a manager might use our analysis as a rough guide for voluntarily releasing information. It also highlights key factors for investors to consider when interpreting this information. The analysis might also be used to "recover" preferences from disclosure data. For instance, empirical evidence on disclosure seems more consistent with managers and investors having relatively similar sensitivity to firm fundamentals. Structurally estimating preference sensitivity from disclosure data, as well as subsequent firm performance, could improve on our observations, perhaps even recovering sensitivity parameters.

No model perfectly describes reality, and there are many factors missing from our analysis. Like most of the existing literature, we have abstracted away from dynamic considerations, but acknowledge that reputational concerns might sometimes be important. Further, the possible quality of manager information is extreme in our model—the manager either learns the fundamentals perfectly or learns nothing at all. Reality is much messier, and we make this assumption purely for tractability. The possible quality of the manager's information might well affect equilibrium disclosure, but how it does so remains an open question.

Although we have characterized equilibria under fairly general preferences, we do not cover all cases. For example, a manager may be relatively sensitive to fundamentals when the firm is doing well and relatively insensitive when performing poorly. Such preferences satisfy neither ordering property. Finally, manager preferences might derive from the solution to a larger contracting problem where both moral hazard and disclosure are key considerations. Future work might extend our analysis to capture these trade-offs.

6 Appendix

Proof of Theorem 1: Suppose to the contrary that full revelation is an equilibrium. We will derive a contradiction by constructing a profitable deviation. Clearly there exists a state θ' where $y_R(\theta') = y(\emptyset)$. Since generically $y_S(\theta') \neq y_R(\theta')$ and $y_R(\theta)$ is continuous and strictly increasing, there is a positive measure of states θ near θ' where the manager will prefer to report that he is uniformed and induce the action $y(\emptyset)$ than report the true value of θ and induce the action $y_R(\theta)$. Thus full revelation is not an equilibrium. ■

Proof of Remark 1: Suppose by means of contradiction that there exists a set of equilibria corresponding to a given value of p, $Q(p)$, such that some equilibria do not converge to full revelation in the limit as $p \to 1$. There are two possible ways this might arise.

The first possibility is that there is some value of θ such that the manager sends a partially informative message in some equilibrium $Q(p)$. If the manager sends such a message, then the investor will know that the manager is informed. However, we know from Proposition 3 in Hummel *et al.* (2018) that if a manager is informed but not conservative, and the manager sends a partially informative message for some positive measure of values of θ, then the manager can profitably deviate. Thus this possibility may not arise in any equilibrium $Q(p)$.

The second possibility is that there is some positive measure of values of θ for which the manager sends the message $m = \emptyset$ even for values of p arbitrarily close to 1. Specifically, if $\Pr(m = \emptyset|Q(p))$ denotes the probability that an informed manager sends the message $m = \emptyset$ (unconditional on θ), then $\limsup_{p\to 1} \Pr(m = \emptyset|Q(p)) > 0$. This implies that there is some $\delta > 0$ such that there is an infinite sequence of values of p, $\{p_n\}$, with $\lim_{n\to\infty} p_n = 1$ that satisfies $\Pr(m = \emptyset | Q(p_n)) > \delta$ for all n.

To prove that this possibility may not arise, let $\Theta(p)$ denote the set of values of θ for which the manager sends the message $m = \emptyset$ under the equilibrium $Q(p)$. Note that in the limit as $p \to 1$ along the sequence $\{p_n\}$, the action the investor chooses upon receiving the message $m = \emptyset$ will become arbitrarily close to the action the investor would have chosen if the manager were always informed and the manager sends the message $m = \emptyset$ if and only if $\theta \in \Theta(p)$. However, we know from Proposition 3 in Hummel *et al.* (2018) that if a manager is informed but not conservative, and the manager sends the message $m = \emptyset$ for a positive measure of θ, then the manager can profitably deviate by fully revealing θ. Since the manager's payoffs for values of p sufficiently close to 1 in the sequence $\{p_n\}$ are arbitrarily close to those the manager would obtain if the manager were informed with certainty, the manager can profitably deviate by fully revealing θ in some cases where the manager would have sent the message $m = \emptyset$ under the equilibrium $Q(p)$. Thus $Q(p)$ cannot be an equilibrium for some value of p that is sufficiently close to 1. ■

6.1 Proofs under the Gradual Slope Ordering Property

Throughout this subsection, let $y(\emptyset; D = [\theta_1, \theta_2])$ denote the equilibrium action in response to the null message when the informed manager discloses over the interval $D = [\theta_1, \theta_2]$.

Proof of Proposition 1: The gradual slope ordering property implies that there is no more than one agreement point. By Proposition 2 of Hummel *et al.* (2018), we know that if the manager always sends the messages $m = \theta$ or $m = \emptyset$, then the investor's beliefs off the equilibrium path can be chosen in such a way that the manager would never have an incentive to send a partially informative message. Thus it suffices to prove that there exists a convex disclosure equilibrium to the game in which the only messages available to the manager are $m = \theta$ or $m = \emptyset$. As a result, the proof restricts attention to incentive compatibility of full versus no disclosure.

First, assume there are no agreement points. Suppose $y_S(\theta) > y_R(\theta)$ for all θ; the opposite case is analogous. Define θ^* as follows: An informed manager sends the message $m = \emptyset$ for $\theta < \theta^*$ and sends the message $m = \theta$ for $\theta \geq \theta^*$. Following the message $m = \emptyset$, the investor's action $y\left(\emptyset; D = \left[\theta^*, \bar{\theta}\right]\right)$ maximizes her expected payoff conditional on the message coming from a manager who is uninformed with probability $(1-p)/(1-p+pF(\theta^*))$, and from a manager who is informed and where the state is $\theta < \theta^*$ with the remaining probability. The value of θ^* is defined to satisfy $y\left(\emptyset; D = \left[\theta^*, \bar{\theta}\right]\right) = y_R(\theta^*)$.

To establish that such a θ^* exists, notice that when $\theta^* \to \underline{\theta}$ or $\theta^* \to \bar{\theta}$, the action $y\left(\emptyset; D = \left[\theta^*, \bar{\theta}\right]\right)$ reflects the optimal action conditional on the manager being uninformed and, therefore, $\lim_{\theta^* \to \underline{\theta}} y\left(\emptyset; D = \left[\theta^*, \bar{\theta}\right]\right) > \lim_{\theta^* \to \underline{\theta}} y_R(\theta^*)$ and $\lim_{\theta^* \to \bar{\theta}} y\left(\emptyset; D = \left[\theta^*, \bar{\theta}\right]\right) < \lim_{\theta^* \to \bar{\theta}} y_R(\theta^*)$. Since $y\left(\emptyset; D = \left[\theta^*, \bar{\theta}\right]\right)$ is continuous in θ^*, it follows that a value of θ^* satisfying $y\left(\emptyset; D = \left[\theta^*, \bar{\theta}\right]\right) = y_R(\theta^*)$ exists.

Next, we show the manager can do no better than to send the message $m = \emptyset$ for all $\theta < \theta^*$ and $m = \theta$ for all $\theta \geq \theta^*$. For $\theta \geq \theta^*$, notice that $y_S(\theta) > y_R(\theta) \geq y\left(\emptyset; D = \left[\theta^*, \bar{\theta}\right]\right)$; therefore disclosure is preferred to non-disclosure by an informed manager in this state.

For $\theta < \theta^*$, when not disclosing, a manager earns $U_S(|y_R(\theta^*) - y_S(\theta)|)$, and when disclosing, a manager earns $U_S(|y_R(\theta) - y_S(\theta)|)$. We claim that for all $\theta < \theta^*$, $|y_R(\theta) - y_S(\theta)| > |y_R(\theta^*) - y_S(\theta)|$.

Case 1: $y_R(\theta^*) < y_S(\theta)$. Then $|y_R(\theta) - y_S(\theta)| > |y_R(\theta^*) - y_S(\theta)|$ holds if and only if $y_S(\theta) - y_R(\theta) > y_S(\theta) - y_R(\theta^*)$ or $y_R(\theta) < y_R(\theta^*)$, and since $\theta < \theta^*$, this condition holds.

Case 2: $y_R(\theta^*) > y_S(\theta)$. Then $|y_R(\theta) - y_S(\theta)| > |y_R(\theta^*) - y_S(\theta)|$ holds if and only if $y_S(\theta) - y_R(\theta) > y_R(\theta^*) - y_S(\theta)$. To establish this inequality, observe that $y_S(\theta) - y_R(\theta) > y_S(\theta) - y_S(\theta^*) + y_R(\theta^*) - y_R(\theta) > y_S(\theta) - y_S(\theta^*) + 2(y_S(\theta^*) - y_S(\theta)) = y_S(\theta^*) - y_S(\theta) > y_R(\theta^*) - y_S(\theta)$, where the first inequality follows because $y_S(\theta) > y_R(\theta)$ while the second inequality follows from the gradual slope ordering property. This establishes that non-disclosure is preferred to disclosure, and completes the proof for the case where there are no agreement points.

Next, assume there is a single agreement point occurring in state θ'. Suppose that $y(\emptyset) < y_R(\theta')$ (the situation where $y(\emptyset) > y_R(\theta')$ follows an analogous line of proof). We will show that there is an interval $[\theta_1, \theta_2]$ where disclosure occurs. In the remaining states, an informed manager chooses not to disclose.

To construct $[\theta_1, \theta_2]$, we require (1) $\theta_1 < \theta' < \theta_2$, (2) $y_R(\theta_1) = y(\emptyset; D = [\theta_1, \theta_2])$, and (3) $|y_R(\theta_2) - y_S(\theta_2)| \leq |y(\emptyset; D = [\theta_1(\theta_2), \theta_2]) - y_S(\theta_2)|$ with equality if $\theta_2 < \bar{\theta}$. To see that such a construction is possible, fix $\theta_2 > \theta'$ and find a value $\theta_1(\theta_2)$ solving condition (2). Notice that, for θ_1 sufficiently small, $y_R(\theta_1) < y(\emptyset; D = [\theta_1, \theta_2])$ while for θ_1 close to θ', $y_R(\theta_1) > y(\emptyset; D = [\theta_1, \theta_2])$. Therefore a solution $\theta_1(\theta_2)$ exists. Similarly, by varying θ_2, one can show that there exists a value of $\theta_2 > \theta'$ satisfying condition (3). Therefore, such a construction is feasible.

When $\theta < \theta_1$, we claim the manager prefers non-disclosure. To establish this claim, we show that $|y(\emptyset; D = [\theta_1, \theta_2]) - y_S(\theta)| \leq |y_R(\theta) - y_S(\theta)|$. The combination of $\theta < \theta'$ and the gradual slope ordering property implies that $y_S(\theta) > y_R(\theta)$. Thus, $|y_R(\theta) - y_S(\theta)| = y_S(\theta) - y_R(\theta) = y_S(\theta) - y_R(\theta_1) + y_R(\theta_1) - y_R(\theta) > y_S(\theta) - y_S(\theta_1) + y_R(\theta_1) - y_R(\theta) > y_S(\theta) - y_S(\theta_1) + 2(y_S(\theta_1) - y_S(\theta)) = y_S(\theta_1) - y_S(\theta) >$

$y_R(\theta_1) - y_S(\theta) = y(\emptyset; D = [\theta_1, \theta_2]) - y_S(\theta)$, where the first and third inequalities follow because $y_S(\theta) > y_R(\theta)$ for $\theta < \theta'$ while the second inequality follows from the gradual slope ordering property.

Next, when $\theta > \theta_2$, we also claim the manager prefers non-disclosure. To establish this claim, we show that $|y(\emptyset; D = [\theta_1, \theta_2]) - y_S(\theta)| \leq |y_R(\theta) - y_S(\theta)|$. In this case $y_S(\theta) < y_R(\theta)$ for all $\theta > \theta'$. Therefore, $|y_R(\theta) - y_S(\theta)| = y_R(\theta) - y_S(\theta) = y_R(\theta_2) - y_S(\theta_2) + (y_R(\theta) - y_R(\theta_2)) - (y_S(\theta) - y_S(\theta_2)) \geq y_R(\theta_2) - y_S(\theta_2) + 2(y_S(\theta) - y_S(\theta_2)) - (y_S(\theta) - y_S(\theta_2)) = y_R(\theta_2) - y_S(\theta_2) + y_S(\theta) - y_S(\theta_2) = y_S(\theta_2) - y(\emptyset; D = [\theta_1(\theta_2), \theta_2]) + y_S(\theta) - y_S(\theta_2) = y_S(\theta) - y(\emptyset; D = [\theta_1(\theta_2), \theta_2])$, where the weak inequality follows from the gradual slope ordering property and the penultimate equality follows from condition (3).

Finally, for $\theta \in (\theta_1, \theta_2)$, we claim the manager prefers to reveal. To establish this claim, we show that $|y_R(\theta) - y_S(\theta)| \leq |y(\emptyset; D = [\theta_1, \theta_2]) - y_S(\theta)|$. We consider two cases: $\theta < \theta'$ and $\theta > \theta'$. When $\theta < \theta'$, $y_S(\theta) > y_R(\theta) > y_R(\theta_1) = y(\emptyset; [\theta_1, \theta_2])$. Alternatively, when $\theta > \theta'$, we know that $y_R(\theta) - y_S(\theta) = y_R(\theta_2) - y_S(\theta_2) + (y_R(\theta) - y_R(\theta_2)) - (y_S(\theta) - y_S(\theta_2)) \leq y_R(\theta_2) - y_S(\theta_2) + 2(y_S(\theta) - y_S(\theta_2)) - (y_S(\theta) - y_S(\theta_2)) = y_R(\theta_2) - y_S(\theta_2) + y_S(\theta) - y_S(\theta_2) = y_S(\theta_2) - y(\emptyset; D = [\theta_1(\theta_2), \theta_2]) + y_S(\theta) - y_S(\theta_2) = y_S(\theta) - y(\emptyset; D = [\theta_1(\theta_2), \theta_2])$, where the inequality follows from the gradual slope ordering property and because $\theta < \theta_2$. In either case, the required inequality holds. ■

Lemma 1 *Under the gradual slope ordering property with no agreement points, there is a unique convex disclosure equilibrium.*

Proof. We prove this result for the case in which $y_S(\theta) > y_R(\theta)$ for all θ. The proof for the case where $y_S(\theta) < y_R(\theta)$ for all θ is analogous and thus omitted.

Recall from the proof of Proposition 1 that under the gradual slope ordering property with no agreement points, a convex disclosure equilibrium consists of a value of θ^* that solves $y_R(\theta^*) = y\left(\emptyset; D = \left[\theta^*, \bar{\theta}\right]\right)$ such that full revelation occurs when $\theta \geq \theta^*$ and non-disclosure occurs otherwise. We will show that, for any such θ^*, it must be the case that $\frac{dy_R(\theta^*)}{d\theta^*} > \frac{d}{d\theta^*} y\left(\emptyset; D = \left[\theta^*, \bar{\theta}\right]\right)$. Recall that $y\left(\emptyset; D = \left[\theta^*, \bar{\theta}\right]\right)$ is the argument y that maximizes

$$\frac{pF(\theta^*)}{1-p+pF(\theta^*)} \frac{1}{F(\theta^*)} \int_{\underline{\theta}}^{\theta^*} U_R(y,\theta) f(\theta)\, d\theta + \frac{1-p}{1-p+pF(\theta^*)} \int_{\underline{\theta}}^{\bar{\theta}} U_R(y,\theta) f(\theta)\, d\theta$$

Our assumptions imply $y\left(\emptyset; D = \left[\theta^*, \bar{\theta}\right]\right)$ satisfies the first-order condition, $\Psi(y,\theta^*) \equiv$

$$\frac{p}{1-p+pF(\theta^*)} \int_{\underline{\theta}}^{\theta^*} \frac{\partial U_R(y,\theta)}{\partial y} f(\theta)\, d\theta + \frac{1-p}{1-p+pF(\theta^*)} \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_R(y,\theta)}{\partial y} f(\theta)\, d\theta = 0$$

where $\partial\Psi(y,\theta^*)/\partial y < 0$ since y is a maximum.

Using the Implicit Function Theorem and that $\frac{\partial U_R(y,\theta)}{\partial y}|_{\theta=\theta^*} = 0$, we have

$$\begin{aligned}
&\frac{dy\left(\emptyset; D = \left[\theta^*, \bar{\theta}\right]\right)}{d\theta^*} \\
=\ & -\frac{\partial\Psi(y,\theta^*)}{\partial\theta^*} \Big/ \frac{\partial\Psi(y,\theta^*)}{\partial y} \\
=\ & \frac{pf(\theta^*)}{1-p+pF(\theta^*)} \left[\frac{\frac{p}{1-p+pF(\theta^*)} \int_{\underline{\theta}}^{\theta^*} \frac{\partial U_R(y,\theta)}{\partial y} f(\theta)\, d\theta + \frac{(1-p)}{1-p+pF(\theta^*)} \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_R(y,\theta)}{\partial y} f(\theta)\, d\theta}{\partial\Psi(y,\theta^*)/\partial y}\right] \\
=\ & \frac{pf(\theta^*)}{1-p+pF(\theta^*)} \left[\frac{\Psi(y,\theta^*)}{\partial\Psi(y,\theta^*)/\partial y}\right] = 0
\end{aligned}$$

Since $\frac{dy_R(\theta^*)}{d\theta^*} > 0$, it then follows that $\frac{dy_R(\theta^*)}{d\theta^*} > \frac{d}{d\theta^*} y\left(\emptyset; D = \left[\theta^*, \bar{\theta}\right]\right)$ at any intersection point. Hence, there is a unique solution, θ^*. ■

Lemma 2 *Under the gradual slope ordering property with no agreement points, every equilibrium is a convex disclosure equilibrium.*

Proof. First, we rule out partial disclosure in any equilibrium. If the manager sends a partially informative message, then the investor will know that the manager is informed. However, we know from Proposition 3 in Hummel *et al.* (2018) that if a manager is informed, there are no agreement points, and the manager sends a partially informative message for some positive measure of values of θ, then the manager can profitably deviate. Thus partial disclosure may not arise in any equilibrium.

Next, we consider the situation where disclosure regions are non-convex as a result of non-disclosure. Then there exist states θ' and θ'' where $\theta' < \theta''$ such that disclosure occurs in equilibrium in each of these states, but, for some $t \in (0,1)$, non-disclosure occurs in state $\theta''' = t\theta' + (1-t)\theta''$. Disclosure in states θ' and θ'' implies that $|y(\emptyset; D) - y_S(\theta')| \geq |y_R(\theta') - y_S(\theta')|$ and $|y(\emptyset; D) - y_S(\theta'')| \geq |y_R(\theta'') - y_S(\theta'')|$, where D denotes the set of states in which disclosure occurs. To show that non-disclosure will not occur in state θ''' we show that $|y(\emptyset; D) - y_S(\theta''')| < |y_R(\theta''') - y_S(\theta''')|$ cannot occur. We consider two separate cases:

Case 1: Suppose $y_S(\theta) > y_R(\theta)$ for all $\theta \in (\theta', \theta'')$. Then $|y(\emptyset; D) - y_S(\theta''')| < |y_R(\theta''') - y_S(\theta''')|$ can only hold if $y(\emptyset; D) > y_R(\theta''')$, and hence, $y(\emptyset; D) - y_S(\theta''') < y_S(\theta''') - y_R(\theta''')$, or equivalently, $2y_S(\theta''') - y_R(\theta''') > y(\emptyset; D)$. By the gradual slope ordering property, this implies $2y_S(\theta') - y_R(\theta') > y(\emptyset; D)$, which may be rewritten as $y_S(\theta') - y_R(\theta') > y(\emptyset; D) - y_S(\theta')$. But this contradicts our previous finding that $|y(\emptyset; D) - y_S(\theta')| \geq |y_R(\theta') - y_S(\theta')|$ (regardless of whether $y(\emptyset; D) > y_S(\theta')$) because if $y(\emptyset; D) < y_S(\theta')$, then we have $y_S(\theta') > y(\emptyset; D) > y_R(\theta')$. Thus, $|y(\emptyset; D) - y_S(\theta''')| < |y_R(\theta''') - y_S(\theta''')|$ cannot hold in this case.

Case 2: Suppose $y_S(\theta) < y_R(\theta)$ for all $\theta \in (\theta', \theta'')$. The proof establishing that $|y(\emptyset; D) - y_S(\theta''')| < |y_R(\theta''') - y_S(\theta''')|$ cannot hold is analogous to Case 1. ■

Proof of Proposition 2: This result follows immediately from Proposition 1, and

Lemmas 1 and 2. ■

Proof of Proposition 3: The proof is by construction. Let θ' be an agreement point. Recall that there exists a convex disclosure equilibrium with disclosure interval $[\theta_1, \theta_2]$ satisfying $\theta_1 < \theta'$ and $\theta_2 > \theta'$. Consider an interval $I' = [\theta_1', \theta_2']$, where $\theta' \in [\theta_1', \theta_2'] \subset [\theta_1, \theta_2]$, and the manager sends message $m = I'$ in equilibrium with the resulting action $y(I') = y_R(\theta')$. To see that such a construction is feasible, notice that, by continuity of the investor's bliss line, there exists a continuum of pairs (θ_1', θ_2') that induce $y(I') = y_R(\theta')$. Moreover, these pairs can be made arbitrarily close to θ' and hence $[\theta_1', \theta_2'] \subset [\theta_1, \theta_2]$. Finally, since the non-disclosure region remained unchanged by this amendment, the equilibrium conditions for (θ_1, θ_2) are undisturbed.

It remains to show that the manager cannot profitably deviate from such a strategy. By the arguments in the proof of Proposition 1, we know that disclosure is preferred to non-disclosure in the region $[\theta_1, \theta_2]$ and vice versa. Furthermore, the gradual slope ordering property implies that there exists an interval sufficiently close to θ' where the manager prefers the action $y_R(\theta')$ to the disclosure action. Thus sending the message I' in the interval $[\theta_1', \theta_2']$ is preferred to full disclosure. ■

6.2 Proofs under the Steep Slope Ordering Property

Throughout this subsection, we use the following definitions: Let $y(\emptyset; ND = [\theta_1, \theta_2])$ denote the equilibrium action following the null message when the *non*-disclosure interval is $ND = [\theta_1, \theta_2]$. Let $\theta_\emptyset$ be the state θ solving $y_R(\theta) = y(\emptyset)$, where $y(\emptyset)$ is the investor's optimal action given her prior beliefs. Define θ_L to be the largest agreement point $\theta' < \theta_\emptyset$, if such a point exists, and $\theta_L = \underline{\theta}$ otherwise. Likewise, define θ_H to be the smallest agreement point $\theta'' > \theta_\emptyset$, if such a point exists, and $\theta_H = \bar{\theta}$

otherwise. To prove Proposition 4, the following lemmas are helpful.

Lemma 3 *Under the steep slope ordering property, when $y_S(\theta) > y_R(\theta)$ for all $\theta \in (\theta_L, \theta_H)$, there exists a convex non-disclosure equilibrium with non-disclosure region $[\theta_1, \theta_2] \subseteq [\theta_L, \theta_H]$ solving*

$$|y(\emptyset; ND = [\theta_1, \theta_2]) - y_S(\theta_1)| \leq |y_S(\theta_1) - y_R(\theta_1)| \quad (1)$$

$$y(\emptyset; ND = [\theta_1, \theta_2]) = y_R(\theta_2) \quad (2)$$

where (1) holds with equality if $\theta_1 > \theta_L$.

Proof. We first show that there exists some θ_1 and θ_2 satisfying conditions (1) and (2). To see this, fix $\theta_1 < \theta_\emptyset$. Since $y_R(\theta_\emptyset) > y(\emptyset; ND = [\theta_1, \theta_\emptyset])$ and $y_R(\theta_1) < y(\emptyset; ND = [\theta_1, \theta_1])$, it follows from the Intermediate Value Theorem that there exists some $\theta_2 \in (\theta_1, \theta_\emptyset)$ satisfying $y(\emptyset; ND = [\theta_1, \theta_2]) = y_R(\theta_2)$. Let $\theta_2(\theta_1)$ denote this value of θ_2. For values of θ_1 close to $\theta_\emptyset$, we have $y_S(\theta_1) > y(\emptyset; ND = [\theta_1, \theta_2(\theta_1)]) > y_R(\theta_1)$, and thus $|y(\emptyset; ND = [\theta_1, \theta_2(\theta_1)]) - y_S(\theta_1)| < |y_S(\theta_1) - y_R(\theta_1)|$. But this implies that when $\lim_{\theta_1 \to \theta_L} |y(\emptyset; ND = [\theta_1, \theta_2]) - y_S(\theta_1)| > \lim_{\theta_1 \to \theta_L} |y_S(\theta_1) - y_R(\theta_1)|$, there exists some $\theta_1 \in (\theta_L, \theta_\emptyset)$ such that $|y(\emptyset; ND = [\theta_1, \theta_2(\theta_1)]) - y_S(\theta_1)|$ $= |y_S(\theta_1) - y_R(\theta_1)|$. Consequently, there exists some θ_1 and θ_2 satisfying conditions (1) and (2).

It remains to show that for such θ_1 and θ_2, it is incentive compatible for the manager not to disclose if and only if $\theta \in [\theta_1, \theta_2]$. First, consider $\theta > \theta_2$. If $y_S(\theta) > y_R(\theta)$, then $y_S(\theta) > y_R(\theta) > y(\emptyset; ND = [\theta_1, \theta_2])$. It follows immediately that disclosure is strictly preferred to non-disclosure. Conversely, if θ is such that $y_S(\theta) \leq y_R(\theta)$, define θ'' to be the largest agreement point where $\theta'' < \theta$. (Since $y_S(\theta) > y_R(\theta)$ in the region $[\theta_L, \theta_H]$, such an agreement point θ'' must exist for $y_S(\theta) \leq y_R(\theta)$ to hold.) For

$\theta > \theta''$, we have $y_R(\theta) - y_S(\theta) = y_R(\theta) - y_R(\theta'') - (y_S(\theta) - y_S(\theta'')) < y_S(\theta) - y_S(\theta'') < y_S(\theta) - y(\emptyset; ND = [\theta_1, \theta_2])$, where the first equality follows from $y_S(\theta'') = y_R(\theta'')$, the first inequality follows from the steep slope ordering property, and the second inequality follows because $y(\emptyset; ND = [\theta_1, \theta_2]) = y_R(\theta_2) < y_R(\theta'') = y_S(\theta'') \leq y_S(\theta)$. Therefore, the manager prefers disclosure in this region. Thus, for all $\theta > \theta_2$, disclosure is preferred.

Next, consider $\theta \in (\theta_1, \theta_2)$. We claim that $|y(\emptyset; ND = [\theta_1, \theta_2]) - y_S(\theta)| < |y_S(\theta) - y_R(\theta)|$. For θ close to θ_2, $y_S(\theta) \geq y(\emptyset; ND = [\theta_1, \theta_2]) > y_R(\theta)$ and hence non-disclosure is strictly preferred to disclosure. For θ close to θ_1, $y(\emptyset; ND = [\theta_1, \theta_2]) > y_S(\theta) > y_R(\theta)$. It follows that $y_S(\theta) - y_R(\theta) = y_S(\theta_1) - y_R(\theta_1) + \{(y_S(\theta) - y_S(\theta_1)) - (y_R(\theta) - y_R(\theta_1))\} > y_S(\theta_1) - y_R(\theta_1) - \{y_S(\theta) - y_S(\theta_1)\} \geq y(\emptyset; ND = [\theta_1, \theta_2]) - y_S(\theta_1) - \{y_S(\theta) - y_S(\theta_1)\} = y(\emptyset; ND = [\theta_1, \theta_2]) - y_S(\theta)$, where the first inequality follows from the steep slope ordering property, and the second inequality follows from the equilibrium properties of θ_1 and θ_2. Since this exhausts the space of possibilities for $\theta \in (\theta_1, \theta_2)$, we have shown that the manager prefers non-disclosure to disclosure in this region.

For $\theta < \theta_1$, if $y_S(\theta) > y_R(\theta)$, a similar argument shows $y_S(\theta) - y_R(\theta) = y_S(\theta_1) - y_R(\theta_1) + \{(y_S(\theta) - y_S(\theta_1)) - (y_R(\theta) - y_R(\theta_1))\} < y_S(\theta_1) - y_R(\theta_1) - \{y_S(\theta) - y_S(\theta_1)\} = y(\emptyset; ND = [\theta_1, \theta_2]) - y_S(\theta_1) - \{y_S(\theta) - y_S(\theta_1)\} = y(\emptyset; ND = [\theta_1, \theta_2]) - y_S(\theta)$, so disclosure is preferred to non-disclosure in this region. Conversely, if $y_S(\theta) \leq y_R(\theta)$, then $y_S(\theta) \leq y_R(\theta) < y(\emptyset; ND = [\theta_1, \theta_2])$, and thus disclosure is preferred. ■

Lemma 4 *Under the steep slope ordering property, partial disclosure can never arise in equilibrium.*

Proof. Suppose, contrary to the lemma, that there exists an equilibrium with partial disclosure. Fix a message $m = I$, where I is some non-degenerate subset of $\left[\underline{\theta}, \overline{\theta}\right]$. Define a set of positive measure $\Theta(m)$ such that the manager sends the message m in equilibrium if and only if $\theta \in \Theta(m)$. Let $C(\Theta(m))$ be the convex hull of $\Theta(m)$. Let θ' be the largest agreement point in $C(\Theta(m))$ such that the set $\{\theta : \theta > \theta' \text{ and } \theta \in \Theta(m)\}$ has positive measure if such an agreement point exists, and let $\theta' = \inf C(\Theta(m))$ otherwise.

Case 1: Suppose that for (almost) all $\theta > \theta'$ in $C(\Theta(m))$, $y_R(\theta) < y_S(\theta)$. Then, since the investor believes that the state θ is contained in $C(\Theta(m))$ following the equilibrium message m, it then follows that $y(m) = y_R(\theta'')$ for some value of θ'' strictly in the interior of $C(\Theta(m))$, and furthermore, that there exists a positive measure of values of $\theta \in \Theta(m)$ such that $\theta > \max\{\theta', \theta''\}$. For such values of θ, the manager can profitably deviate by revealing truthfully, thus inducing an action $y_R(\theta)$ satisfying $y_S(\theta) > y_R(\theta) > y(m)$.

Case 2: Suppose that for (almost) all $\theta > \theta'$ in $C(\Theta(m))$, $y_R(\theta) > y_S(\theta)$. As in the previous case, the putative equilibrium action $y(m)$ lies strictly in the interior of $C(\Theta(m))$. If $y(m) \leq y_R(\theta')$, then for a positive measure of states θ where $\theta \in \Theta(m)$ and $\theta > \theta'$, we have $U_S(y_R(\theta), \theta) > U_S(y_R(\theta'), \theta) \geq U_S(y(m), \theta)$. Hence, for these states θ, the manager prefers full revelation to $y_R(\theta')$.

If $y(m) > y_R(\theta')$, then either there exists a positive measure of values of $\theta \in \Theta(m)$ where $\theta < \theta'$ or there exists a positive measure of values of $\theta \in \Theta(m)$ where $\theta \in \left(\theta', y_R^{-1}(y(m))\right)$. In the former case, $y_S(\theta) < y_R(\theta') < y(m)$ for any such θ, and hence, $U_S(y_R(\theta), \theta) > U_S(y_R(\theta'), \theta) > U_S(y(m), \theta)$. In the latter case, $y_S(\theta) < y_R(\theta) < y(m)$ for any such θ, and hence $U_S(y_R(\theta), \theta) > U_S(y(m), \theta)$. Thus, in either case, a positive measure of senders can profitably deviate by revealing truthfully. Since this exhausts all possibilities, the result follows. ■

Because of Lemma 4, in proving Proposition 4 we can assume without loss of generality that the sender will either fully disclose or not disclose in equilibrium. With these preliminary results in mind, we now prove Proposition 4:

Proof of Proposition 4: The proof has two parts: first, we establish the existence of a convex non-disclosure equilibrium, and second, we prove that every equilibrium is a convex non-disclosure equilibrium.

When there are no agreement points and $y_S(\theta) > y_R(\theta)$, existence follows from Lemma 3. The case where $y_S(\theta) < y_R(\theta)$ is analogous. When there is one agreement point, θ', where for $\theta > \theta'$, $y_S(\theta) > y_R(\theta)$ and $y(\emptyset) > y_R(\theta')$, then, setting $\theta_L = \theta'$ and $\theta_H = \bar{\theta}$, we can invoke Lemma 3 to show existence. The case where $y(\emptyset) < y_R(\theta')$ is analogous. Conversely, when for $\theta < \theta'$, $y_S(\theta) > y_R(\theta)$ and $y(\emptyset) < y_R(\theta')$, then, setting $\theta_L = \underline{\theta}$ and $\theta_H = \theta'$, we can invoke Lemma 3. The case where $y(\emptyset) > y_R(\theta')$ is analogous. Where there are multiple agreement points, define θ' and θ'' to be adjacent agreement points relative to $y(\emptyset)$ as set out in Lemma 3. When $y_S(\theta) > y_R(\theta)$ in (θ', θ''), the result follows immediately. An analogous argument shows existence when $y_S(\theta) < y_R(\theta)$ in (θ', θ'').

Next we prove that every equilibrium is a convex non-disclosure equilibrium. Suppose, to the contrary, that non-disclosure regions are non-convex. Then there exist states θ' and θ'' where $\theta' < \theta''$ such that non-disclosure occurs in equilibrium in each of these states, but, for some $t \in (0,1)$, disclosure occurs in state $\theta''' = t\theta' + (1-t)\theta''$. Non-disclosure in states θ' and θ'' implies that $|y(\emptyset; ND) - y_S(\theta')| \leq |y_R(\theta') - y_S(\theta')|$ and $|y(\emptyset; ND) - y_S(\theta'')| \leq |y_R(\theta'') - y_S(\theta'')|$, where ND denotes the set of states in which there is non-disclosure. To show that non-disclosure occurs in θ''', we show that $|y(\emptyset; ND) - y_S(\theta''')| > |y_R(\theta''') - y_S(\theta''')|$ cannot occur. We prove this for three separate cases.

Case 1: Suppose that for all $\theta \in (\theta', \theta'')$, $y_S(\theta) > y_R(\theta)$. This implies that $y_R(\theta') \leq y(\emptyset; ND)$ and $y_R(\theta'') \leq y(\emptyset; ND)$, since if either of these inequalities were reversed, we would have $y_S(\theta) > y_R(\theta) > y(\emptyset; ND)$, implying the manager would prefer to disclose. It follows that $y_R(\theta''') < y(\emptyset; ND)$ since $y_R(\theta''') < y_R(\theta'')$.

Non-disclosure at θ' implies that $y(\emptyset; ND) - y_S(\theta') \leq y_S(\theta') - y_R(\theta')$ or, equivalently $2y_S(\theta') - y_R(\theta') \geq y(\emptyset; ND)$. And disclosure at θ''' implies $y(\emptyset; ND) - y_S(\theta''') \geq y_S(\theta''') - y_R(\theta''')$ or, equivalently $2y_S(\theta''') - y_R(\theta''') \leq y(\emptyset; ND)$. However, by the steep slope ordering property, $2y_S(\theta''') - y_R(\theta''') > 2y_S(\theta') - y_R(\theta') \geq y(\emptyset; ND)$. Thus, disclosure cannot occur at $\theta = \theta'''$.

Case 2: Suppose that, for all $\theta \in (\theta', \theta'')$, $y_S(\theta) < y_R(\theta)$. The proof of this case is analogous to that of Case 1.

Case 3: Suppose that some $\theta''' \in (\theta', \theta'')$ is an agreement point (possibly one of many). We will show that in equilibrium there cannot exist non-disclosure intervals $[\theta'_L, \theta''_L]$ and $[\theta'_H, \theta''_H]$ such that $\theta''_L < \theta''' < \theta'_H$. Suppose to the contrary that such intervals exist. There are four cases to consider.

Case 3(a): Suppose that, for all $\theta \in [\theta'_L, \theta''_L] \cup [\theta'_H, \theta''_H]$, $y_S(\theta) > y_R(\theta)$. Then it must be that $|y(\emptyset; ND) - y_S(\theta'_L)| \leq |y_R(\theta'_L) - y_S(\theta'_L)|$ and $|y(\emptyset; ND) - y_S(\theta''_H)| \leq |y_R(\theta''_H) - y_S(\theta''_H)|$.

When $y_S(\theta''') > y(\emptyset; ND)$, it follows that, since $y_S(\theta''') = y_R(\theta''') > y(\emptyset; ND)$, then for $\theta \in [\theta'_H, \theta''_H]$, we have that $y_S(\theta) > y_R(\theta) > y(\emptyset; ND)$ and hence disclosure is strictly preferred in the interval $[\theta'_H, \theta''_H]$, which is a contradiction.

Conversely, when $y_S(\theta''') \leq y(\emptyset; ND)$, then $y(\emptyset; ND) - y_S(\theta''') > y_S(\theta''') - y_R(\theta''')$ or, equivalently, $2y_S(\theta''') - y_R(\theta''') < y(\emptyset; ND)$. From the steep slope ordering property, it follows that $2y_S(\theta) - y_R(\theta) < y(\emptyset; ND)$ for $\theta \in [\theta'_L, \theta''_L]$. Hence $|y_R(\theta) - y_S(\theta)| < |y(\emptyset; ND) - y_S(\theta)|$ and disclosure is strictly preferred in states $\theta \in [\theta'_L, \theta''_L]$, which is a contradiction.

Case 3(b): Suppose that $y_S(\theta) < y_R(\theta)$ for all $\theta \in [\theta'_L, \theta''_L] \cup [\theta'_H, \theta''_H]$. A proof analogous to Case 3(a) establishes a contradiction.

Case 3(c): Suppose that $y_S(\theta) < y_R(\theta)$ for $\theta \in [\theta'_L, \theta''_L]$ while $y_S(\theta) > y_R(\theta)$ for $\theta \in [\theta'_H, \theta''_H]$. When $y_S(\theta''') > y(\emptyset; ND)$, it then follows immediately that, since $y_S(\theta''') = y_R(\theta''') > y(\emptyset; ND)$, then $y_S(\theta) > y_R(\theta) > y(\emptyset; ND)$ for $\theta \in [\theta'_H, \theta''_H]$, and therefore, disclosure is strictly preferred in the interval $[\theta'_H, \theta''_H]$, which is a contradiction. In contrast, when $y_S(\theta''') \leq y(\emptyset; ND)$, then, since $y_S(\theta''') = y_R(\theta''') \leq y(\emptyset; ND)$, it follows that $y_S(\theta) < y_R(\theta) < y(\emptyset; ND)$ for $\theta \in [\theta'_L, \theta''_L]$. Consequently, disclosure is strictly preferred in the interval $[\theta'_L, \theta''_L]$, which is a contradiction.

Case 3(d): Suppose that $y_S(\theta) > y_R(\theta)$ for $\theta \in [\theta'_L, \theta''_L]$ while $y_S(\theta) < y_R(\theta)$ for $\theta \in [\theta'_H, \theta''_H]$. The proof is analogous to the proof where in Case 3(c).

Since this exhausts all of the possibilities, the proof is complete. ■

Example 1 *This example illustrates that under the steep slope ordering property, there may exist multiple equilibria, even if there are no agreement points. Suppose the state is uniformly distributed on* $[-50, 50]$*, the investor suffers quadratic losses in the difference between her action and the state, and the manager is informed with probability* $p = 3/4$ *and has a bliss line:*

$$y_S(\theta) = \begin{cases} \theta + 1 & \text{if } \theta \geq -2.06 \\ 0.528\,41\theta + 0.028\,525 & \text{if } \theta < -2.06 \end{cases} \tag{3}$$

It may be readily verified that there is a convex non-disclosure equilibrium in which non-disclosure occurs over the interval $[-2.06, -0.060]$ *as well as another convex non-*

disclosure equilibrium in which non-disclosure occurs over the interval $[-2.0776, -0.061]$ *and disclosure occurs elsewhere. Further, if we modify preferences for sufficiently high values of* θ *such that one or more agreement points arise (while maintaining the steep slope ordering property), the equilibria we identified remain undisturbed because adding agreement points does not alter the incentives to disclose.*

References

[1] Aboody, D., and R. Kasznik. 2000. CEO stock option awards and the timing of corporate voluntary disclosures. Journal of Accounting and Economics 29 (1): 73–100.

[2] Ajinkya, B. B., and M. J. Gift. 1984. Corporate managers' earnings forecasts and symmetrical adjustments of market expectations. Journal of Accounting Research 22 (2): 425–44.

[3] Baginski, S. P., E. J. Conrad, and J. M. Hassell. 1993. The effects of management forecast precision on equity pricing and on the assessment of earnings uncertainty. The Accounting Review 68 (4): 913–927.

[4] Bhattacharya, N., E. L. Black, T. E. Christensen, C. R. Larson. 2003. Assessing the relative informativeness and permanence of pro forma earnings and GAAP operating earnings. Journal of Accounting and Economics 36 (1–3): 285–319.

[5] Bonsall, S. B., Z. Bozanic, and P. E. Fischer. 2013. What do management earnings forecasts convey about the macroeconomy? Journal of Accounting Research 51 (2): 225–266.

[6] Buffett, W. 1988. Chairman's letter. Berkshire Hathaway Annual Report (http://www.berkshirehathaway.com/letters/1988.html).

[7] Beyer, A., D. A. Cohen, T. Z. Lys, B. R. Walther. 2010. The financial reporting environment: review of the recent literature. Journal of Accounting and Economics 50 (2–3): 296–343.

[8] Bhattacharya, S., M. Goltsman, and A. Mukherjee. 2015. On the optimality of diverse expert panels in persuasion games. University of London working paper.

[9] Bhattacharya, S., and A. Mukherjee. 2013. Strategic information revelation when experts compete to influence. RAND Journal of Economics 44 (3): 522–544.

[10] Crawford, V. P., and J. Sobel. 1982. Strategic information transformation. Econometrica 50 (6): 1431–1451.

[11] DeFond, M. L., and C. W. Park. 1997. Smoothing income in anticipation of future earnings. Journal of Accounting and Economics 23 (2): 115–139.

[12] Dye, R. A. 1985. Disclosure of nonproprietary information. Journal of Accounting Research 23 (1): 123–145.

[13] Dye, R. A. 1998. Investor sophistication and voluntary disclosures. Review of Accounting Studies 3 (3): 261–287.

[14] Einhorn, E. 2007. Voluntary disclosure under uncertainty about the reporting objective. Journal of Accounting and Economics 43 (2-3): 245–274.

[15] Feng, M., and S. McVay. 2010. Analysts' incentives to overweight management guidance when revising their short-term earnings forecasts. The Accounting Review 85 (5): 1617-1646.

[16] Fuller, J., and M. C. Jensen. 2002. Just say no to Wall Street: putting a stop to the earnings game. Journal of Applied Corporate Finance 14 (4): 41–46.

[17] Giovannoni , F., and D. J. Seidmann. 2007. Secrecy, two-sided bias and the value of evidence. Games and Economic Behavior 59 (2): 296–315.

[18] Graham, J. R., C. R. Harvey, and S. Rajgopal. 2005. The economic implications of corporate financial reporting. Journal of Accounting and Economics 40 (1–3): 3–73.

[19] Grossman, S. 1981. The role of warranties and private disclosure about product quality. Journal of Law and Economics 24: 461–483.

[20] Guttman, I., I. Kremer, and A. Skrzypacz. 2014. Not only what but also when: a theory of dynamic voluntary disclosure. American Economic Review 104 (8): 2400–2420.

[21] Hagenbach, J., F. Koessler, and E. Perez-Richeti. 2014. Certifiable pre-play communication: full disclosure. Econometrica 82 (3): 1093–1131

[22] Hassell, J. M., and R. H. Jennings. 1986. Relative forecast accuracy and the timing of earnings forecast announcements. Accounting Review 61 (1): 58–75.

[23] Healy, P. M. and K. G. Palepu. 2001. Information asymmetry, corporate disclosure, and the capital markets: a review of the empirical disclosure literature. Journal of Accounting and Economics 31 (1–3): 405–440.

[24] Hummel, P., J. Morgan, and P. C. Stocken. 2016. Optimal firm (non-)disclosure. University of California, Berkeley working paper.

[25] Hummel, P., J. Morgan, and P. C. Stocken. 2018. Equilibrium identification in persuasion games. University of California, Berkeley working paper.

[26] Jensen, T. K., and M. Plumlee (2015). Analysts' use of range information provided in management earnings forecasts. Iowa State University and University of Utah working paper.

[27] Jovanovic, B. 1982. Truthful disclosure of information. Bell Journal of Economics 13 (1): 36–44.

[28] Jung, B., N. Soderstrom, and Y. S. Yang. 2013. Earnings smoothing activities of firms to manage credit ratings. Contemporary Accounting Research 30 (2): 645–676.

[29] Jung, W. O., and Y. K. Kwon. 1988. Disclosure when the market is unsure of information endowment of managers. Journal of Accounting Research 26, 146–153.

[30] Kasznik, R., and B. Lev. 1995. To warn or not to warn: management disclosures in the face of an earnings surprise. Accounting Review 70 (1): 113–134.

[31] Koessler, F. 2003. Persuasion games with higher-order uncertainty. Journal of Economic Theory 110 (2): 393–399.

[32] Kothari, S. P., S. Shu, and P. D. Wysocki. 2009. Do managers withhold bad news? Journal of Accounting Research 47 (1): 241–276.

[33] Leuz, C., D. Nanda, and P. D. Wysocki. 2003. Earnings management and investor protection: an international comparison. Journal of Financial Economics 69 (3): 505–527.

[34] Lev, B., and S. H. Penman. 1990. Voluntary forecast disclosure, nondisclosure, and stock prices. Journal of Accounting Research 28 (1): 49–76.

[35] Li, E. X., C. E. Wasley, and J. L. Zimmerman. 2016. The disclose or abstain incentive to issue management guidance. Journal of Law, Finance, and Accounting 1 (1): 187–233.

[36] Mathis, J. 2008. Full revelation of information in sender–receiver games of persuasion. Journal of Economic Theory 143(1): 571–584.

[37] Milgrom, P. 1981. Good news and bad news: representation theorems and applications. Bell Journal of Economics 12 (2): 380–391.

[38] Okuno-Fujiwara, M., A. Postlewaite, and K. Suzumura. 1990. Strategic information revelation. Review of Economic Studies 57 (1): 25–47.

[39] Pae, S. 2005. Selective disclosures in the presence of uncertainty about information endowment. Journal of Accounting and Economics 39 (3): 383–409.

[40] Penman S. H. 1980. An empirical investigation of the voluntary disclosure of corporate earnings forecasts. Journal of Accounting Research 18 (1): 132–160.

[41] Penno, M. C. 1997. Information quality and voluntary disclosure. The Accounting Review 72 (2): 275–284.

[42] Rogers, J. L., and P. C. Stocken. 2005. Credibility of management forecasts. The Accounting Review 80 (4): 1233–1260.

[43] Rountree, B., J. P. Weston, and G. Allayannis. 2008. Do investors value smooth performance? Journal of Financial Economics 90 (3): 237–251.

[44] Seidmann, D., and E. Winter. 1997. Strategic information transmission with verifiable messages. Econometrica 65 (1): 163–170.

[45] Shin, H. S. 1994a. News management and the value of firms. RAND Journal of Economics 25 (1): 58–71.

[46] Shin, H. S. 1994b. The burden of proof in a game of persuasion. Journal of Economic Theory 64 (1): 253–264.

[47] Shin, H. S. 2006. Disclosure risk and price drift. Journal of Accounting Research 44 (2): 351–379.

[48] Skinner, D. J. 1994. Why firms voluntarily disclose bad news. Journal of Accounting Research 32 (1): 38–60.

[49] Verrecchia, R. E. 1983. Discretionary disclosure. Journal of Accounting and Economics 5 (3): 179–194.