On Myopic Uses of Elasticity Based Pricing Rules

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1. Introduction

We examine the effects of a myopic use of the so called inverse elasticity rule in pricing. By myopic, we mean ignoring that elasticity of marginal cost both may depend on price. It has been shown that myopic use for a single product will typically lead to price changes which are too large relative to the optimal price change (Fjell, 2003). We ask under what conditions will continued myopic use converge to the optimal price? Furthermore, we ask what are the effects of myopic use when the pricing rule is extended to related products, i.e. a product line?

2. Related literature

In academics, a well-known rule for marking up marginal cost to ensure optimal or profit maximizing price, is the so called inverse elasticity rule (e.g. Mansfield, 1994, Nicholson and Snyder, 1985, or Pindyck and Rubinfeld, 2001):

\[ p = \left( \frac{\varepsilon}{1 + \varepsilon} \right) mc \]  

(1)

where price elasticity is \( \varepsilon = \frac{\partial q}{\partial p} \frac{p}{q} \) and marginal cost is \( mc \). In Pindyck and Rubinfeld (2001) this rule is referred to as nothing less than “The Rule of Thumb for Pricing.” (p. 341). Presumably, if one could apply this rule, it would be superior to the marking up of some accounting cost. A common approach among practitioners is the marking up of some accounting cost, often full-cost (Lee, 1994). As Laitinen (2009) points out, to use the rule, one would at least need rough estimates of both marginal cost and elasticity.

In some cases, the application of this rule is described quite straightforward and it may appear as if all management needs to know to determine the profit maximizing price are the values of marginal
cost and price elasticity of demand (e.g. Mansfield, 1994). However, the use of marginal cost markup is not as straightforward as it may seem as the above relation only applies at the point of profit maximum. (Pindyck and Rubinfeld (2001, p. 334) point this out in a footnote). Applying the rule as if elasticity and marginal cost are given, will only yield the profit maximizing price if the price is already maximizing profit, and otherwise only if both marginal cost and elasticity are actually constant (see e.g. Fjell, 2003). Neither is necessarily, or even normally, the case.

As is sometimes noted, if elasticity of demand is constant, so will the markup be (Varian, 1996), and one will then at least indirectly obtain the correct price through marking up the going marginal cost. However, Monroe and Cox (2001) specifically warn against assuming constant elasticity. Krueger (2009) also cautions against treating elasticity as constant when analyzing the effects on an elasticity pricing rule for two-sided markets. He recommends that whenever possible, prices (or price ratios) should be traced back to true underlying parameters, rather than simply to elasticity.

If both elasticity and marginal cost depend on price, and if both vary considerably, one may need to know the entire demand and cost schedules to determine optimum output level (Pindyck and Rubinfeld, 2001). If so, the marginal cost markup rule will not offer any advantage over simply equating marginal revenue and marginal cost directly. However, as Fjell (2003) points out, local estimates of elasticity and marginal cost will at least provide the direction of the optimal price – up or down.

The (often) local property of the elasticity of demand and marginal cost, thus limits the usefulness of the rule and may also lead to erroneous pricing if elasticity and marginal cost vary. Fjell (2003) shows that when price is not optimal, a myopic application typically will lead to an excessive price change by increasing (or decreasing) price too much, in a sense “bypassing” the optimal price. We
ask will continued myopic use converge on the optimal price, or will it diverge? If it converges, then it is less problematic for management to apply such a rule myopically.

For a product line of related products, an analogous relationship for the profit maximizing price of product $i$ can be expressed as (Laitinen, 2009):

$$p_i = \frac{e_i}{1 + e_i} mc_i - \sum_{j \neq i} \left( p_j - mc_j \right) \left( e_{ij} \right) q_j \left( 1 + e_i \right) q_i$$

(2)

This rule reduces to the standard inverse elasticity rule for one product if products are independent, i.e. cross-elasticities are zero. The second term on the right hand side makes it more complex than for a single product, encompassing the effects of a price change on other products. Laitinen (2009) explores possibilities for reducing its complexities to simple rules of thumb for management when elasticities are constant. He suggests that the rule extracted for independent products can be adjusted for pricing of dependent products by a rough allocation of additional profit margin between the products in the line. We ask what is the effect of myopic use of the product line pricing expression if either marginal cost or elasticities (both cross- and own elasticities) are non-constant? (Footnote: The original relation for product line pricing was developed by Reibstein and Gatignon (1984). It appears to rest on an assumption of constant elasticities. Hence, our question is hypothetical in the sense that the relation may change if elasticities are non-constant.)

3. Analyses

We ask two questions. 1) for the single product case, will continued myopic use converge on the optimal price? And 2) for the product line case, what will a single myopic use fall short of, equal or bypass the optimal price (along the lines of Fjell, 2003)?
We assume that price is exceeding marginal cost for all products, i.e. \((p_j - mc_j) > 0\). We also assume that \(|\epsilon_{ji}| < |\epsilon_{ii}|\), i.e. at that the relative price effect is greater for the own products than related products. [we may consider relaxing this].

**Continued myopic use in the single product case**

Differentiating the first term on the right hand side (rhs) of equation (1), we get:

\[
mc \left( \frac{1}{1 + \frac{1}{\epsilon}} \right)^2 \frac{\partial \epsilon}{\partial p} dp + \frac{1}{1 + \frac{1}{\epsilon}} \frac{\partial mc}{\partial q} \frac{\partial q}{\partial p} dp \tag{3}
\]

This is the effect of a (single) price change on the myopic price analyzed in Fjell (2003). To briefly recap from Fjell (2003) we have that: If both elasticity and marginal cost are constant, i.e. \(\frac{\partial \epsilon}{\partial p} = 0\) and \(\frac{\partial mc}{\partial q} = 0\), then both terms will vanish and a myopic use will lead to the profit maximizing price.

However, if both are dependent on price, where \(\frac{\partial \epsilon}{\partial p} < 0\), then a myopic use of the rule will typically lead to an excessive price change, i.e. a bypass of the profit-maximizing price, when marginal cost is constant or increasing. The intuition is that an increase in price based on the rule, will increase the price sensitivity and hence reduce the markup on the marginal cost. This will thus reduce the myopic “profit maximizing” price. In other words, if current price is below (above) the profit maximizing price, the rule will suggest a price above (below) the profit maximizing price.

Only if marginal cost is sufficiently declining \(\frac{\partial mc}{\partial q} < 0\), then this marginal cost effect may counteract the elasticity effect sufficiently so that the price change may be too small to reach the optimal price. The intuition is that a price increase (decrease) leads to an increase (decrease) in
marginal cost which, all else equal, means that optimal price should be increased (beyond the myopic level). If this effect is large enough, it will more than off-set the elasticity effect, and we then have that the price change based on a myopic use will have been too small to reach the true, optimal price. In this case, continued myopic use would eventually converge on the optimal price. However, what if the price change is too large, i.e. we bypass the optimal price? Suppose a manager repeatedly adjusts the price using the elasticity based price-cost markup formula, will she then eventually reach the profit maximizing price?

To illustrate, consider a linear demand and constant marginal cost. The price elasticity varies along a linear demand. Let the demand and the marginal cost be \( q = A - Bp \) and \( MC = c \). Therefore, the profit is: \( \pi = pq - cq = (p - c)(A - Bp) \). The corresponding profit maximizing price is \( p^* = \frac{A + Bc}{2B} \).

Suppose the manager initially chooses a price \( p_0 \neq p^* \). Note that the elasticity for the demand \( q = A - Bp \) is: \( \epsilon = \frac{dq}{dp} \frac{p}{q} = -B \frac{p}{A - Bp} \). Hence, the manager changes the price from \( p_0 \) to \( p_1 \), where \( p_1 = \frac{\epsilon}{1 + \epsilon} * c \). Here the elasticity \( \epsilon \) is computed at price \( p_0 \). Hence,

\[
p_1 = \frac{\epsilon}{1 + \epsilon} * c = \left( \frac{\frac{-Bp_0}{A - Bp_0}}{\frac{-Bp_0}{A - Bp_0} + \frac{-Bp_0}{1 - (A - Bp_0)} c} \right) c = \left( \frac{-Bp_0}{A - Bp_1} \right) c
\]

If \( p_1 \neq p^* \), then the manager updates the price from \( p_1 \) to \( p_2 \), where \( p_2 = \left( \frac{-Bp_1}{A - Bp_1} \right) c \). The manager continues updating her price as \( p_i = \left( \frac{-Bp_{i-1}}{A - Bp_{i-1}} \right) c \), until one of the following happens.

1. If \( p_i = p_{i-1} \), then \( p_i = p_i = p^* \). In this case the manager reaches the profit maximizing price by repeatedly applying the elasticity based price-cost markup formula.
2. After few iterations, the adjusted price is more than \( \frac{A}{B} \) or \( \epsilon > -1 \). In this case, the iterative adjustment process diverges and the manager does not reach the profit maximizing price.
Numerical simulations suggest that for any difference between the profit maximizing price and the initial price; however small; the iterative price adjustment process always diverges. Below we provide a few numerical examples. In these examples, we use the notation $\varepsilon_i$ to denote price elasticity at price $p_i$.

Example 1: $A = 10$, $B = 1$, $c = 2$:

<table>
<thead>
<tr>
<th>Price</th>
<th>Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^* = 6$</td>
<td>$\varepsilon^* = -1.5$</td>
</tr>
<tr>
<td>$p_0 = 5.99$</td>
<td>$\varepsilon_0 = -1.49377$</td>
</tr>
<tr>
<td>$p_1 = 6.05051$</td>
<td>$\varepsilon_1 = -1.53197$</td>
</tr>
<tr>
<td>$p_2 = 5.75962$</td>
<td>$\varepsilon_2 = -1.35828$</td>
</tr>
<tr>
<td>$p_3 = 7.58228$</td>
<td>$\varepsilon_3 = -3.13613$</td>
</tr>
<tr>
<td>$p_4 = 2.93627$</td>
<td>$\varepsilon_4 = -0.415684$</td>
</tr>
<tr>
<td>$p_5 = -1.4228$</td>
<td>$\varepsilon_5 = 0.124558$</td>
</tr>
</tbody>
</table>

Additional examples are in the appendix. We now allow for non-linear demand $q = A + Bp + Cp^2$, where $A > 0$, $B > 0$. If $C > 0$ the demand is convex to the origin. If $C = 0$, the demand is linear, and if $C < 0$, then the demand is concave.

Example 5: $A = 10$, $B = 1$, $C = -1$, $c = 2$. (Note that the vertical intercept is 3.7016.)

<table>
<thead>
<tr>
<th>Price</th>
<th>Elasticity</th>
</tr>
</thead>
</table>

7
\[
\begin{array}{|c|c|}
\hline
p^* &= 2.91485 \\
\hline
\hat{\epsilon}^* &= -3.18614 \\
\hline
p_0 &= 2.9 \\
\hline
\hat{\epsilon}_0 &= -3.10022 \\
\hline
p_1 &= 2.95228 \\
\hline
\hat{\epsilon}_1 &= -3.41797 \\
\hline
p_2 &= 2.82714 \\
\hline
\hat{\epsilon}_2 &= -2.72179 \\
\hline
p_3 &= 3.16158 \\
\hline
\hat{\epsilon}_3 &= -5.31574 \\
\hline
p_4 &= 2.46342 \\
\hline
\hat{\epsilon}_4 &= -1.51266 \\
\hline
p_5 &= 5.9012 \\
\hline
\hat{\epsilon}_5 &= 3.36877 \\
\hline
\end{array}
\]

Example 6: \(A = 10, \ B = 1, \ C = -1, \ c = 2\). (Note that the vertical intercept is 3.7016.)

\[
\begin{array}{|c|c|}
\hline
\text{Price} & \text{Elasticity} \\
\hline
p^* &= 2.91485 \\
\hline
\hat{\epsilon}^* &= -3.18614 \\
\hline
p_0 &= 2.92 \\
\hline
\hat{\epsilon}_0 &= -3.21668 \\
\hline
p_1 &= 2.90225 \\
\hline
\hat{\epsilon}_1 &= -3.11303 \\
\hline
p_2 &= 2.94651 \\
\hline
\hat{\epsilon}_2 &= -3.38069 \\
\hline
p_3 &= 2.84009 \\
\hline
\hat{\epsilon}_3 &= -2.7843 \\
\hline
p_4 &= 3.12089 \\
\hline
\hat{\epsilon}_4 &= -4.83858 \\
\hline
p_5 &= 2.53108 \\
\hline
\hat{\epsilon}_5 &= -1.65278 \\
\hline
p_6 &= 5.06383 \\
\hline
\hat{\epsilon}_6 &= 4.3693 \\
\hline
\end{array}
\]
It appears that for a concave demand, numerical simulations suggest that for any difference between the profit maximizing price and the initial price; however small; the iterative price adjustment process always diverges.

The functional form used here does not work well with $C > 0$. We need to find out a better way to work with convex demands. In general, our conjecture is that for any difference between the profit maximizing price and the initial price; however small; the iterative price adjustment process will always diverge. However, for constant elasticity demand, e.g. $Q = Ap^B$, where $A$ is a positive scaling factor and $B$ is the constant elasticity of demand, we know that price converges (is reached in one iteration). Thus, we suspect that for a class of convex demands with non-constant elasticity, we might also find convergence. We, however, need analytic proof for the conjecture.

Next, we turn to the second term on the rhs in (1), i.e. the impact of related products.

**Effect of a single, myopic price change with related products**

We now ask, what is the effect of a single, myopic price change for a monopolist with a product line, i.e. with related products? We assume that products are only related on the demand side, i.e. non-zero cross-price elasticities. Further, we assume that they are unrelated on the cost side. Unlike Laitinen (2009), we assume that elasticities may depend on price, as well as marginal costs (indirectly).

Differentiating the second term on the rhs of equation (2), we get the effect of a price increase pertaining to related product(s):

$$
\begin{align*}
\frac{\partial mc_j}{\partial q_j} & \frac{\partial^2 \mu}{\partial p_i \partial q_j} \frac{q_i^j}{(1+\epsilon_{ii}) q_i} \\
& + (p_j - mc_j) \left( \epsilon_{jl} \frac{\partial \epsilon_{il}}{\partial p_i} - (1 + \epsilon_{il}) \frac{\partial \epsilon_{jl}}{\partial p_l} \right) \frac{1}{(1+\epsilon_{il})^2} \frac{q_j}{q_i}
\end{align*}
$$

(4)
\[ \epsilon_j \left( \frac{\varepsilon_j - \varepsilon_i}{-1(1+\varepsilon_i)} \right) \frac{a_j}{p_j q_i} \]

One might refer to these three terms as the marginal cost effect, the elasticity effect, and the relative quantities effect, respectively. Each term in (4) then refers back to each of the three factors of the second term on the rhs in (2).

The marginal cost effect

The marginal cost effect of the related product is zero if marginal cost is constant. For a non-constant marginal cost, the sign of the marginal cost effect is independent of whether the related product is a substitute or a complement (as the cross-elasticity is squared). Rather, it is solely dependent upon the whether the marginal cost is increasing or decreasing. Indeed, the sign is the opposite of the sign (or slope) of \( \frac{\partial mc_j}{\partial q_j} \). If the marginal cost of the related product is increasing in quantity, \( \frac{\partial mc_j}{\partial q_j} > 0 \) (positive slope), then its effect on prescribed price is negative. Thus, a myopic application of the rule will lead to bypass of the profit maximizing price, ceteris paribus. The intuition behind this is as follows. If marginal cost is increasing, then an increase in price will lead to increased demand for a substitute and a subsequent increase in its marginal cost, which in turn reduces the margin of the substitute. Ignoring the impact of this reduced margin, will lead to too high a price increase, i.e. bypass. If instead the related product is a complement, the price increase will lead to reduced sales, and subsequently to reduced marginal cost (and increased margin on the complement). Failing to take this into account, will lead to too large a price increase as we underestimate the lost profit margin of the complement. Again, bypass of the profit maximizing price.
If the related marginal cost is decreasing (in quantity), i.e. negative, then its effect on the prescribed profit maximizing price will be positive. Thus, an increase in price to the prescribed price will lead to a shortfall of the profit maximizing price.

Further to (3), we see that the same is true for the marginal cost effect on the price product, $i$. Hence, the marginal cost effects from product $i$ and from related product $j$ reinforce one another. As such, adding related products does not alter the results from the single product case.

The elasticity effect

If the elasticities are constant, then the elasticity effect is, of course, zero. When elasticities are non-constant, the sign of the elasticity effect from the related product depends on the sign of

$$(\varepsilon_{ji} \frac{\partial \varepsilon_{ui}}{\partial p_i} - (1 + \varepsilon_{ii}) \frac{\partial \varepsilon_{ji}}{\partial p_i}).$$

We already have by assumption that $\frac{\partial \varepsilon_{ii}}{\partial p_i} < 0$. [expand on this to justify]

For related products, we have that:

$$\varepsilon_{ji} = \frac{\partial q_j}{\partial p_i} \frac{p_i}{q_j}$$

$$\frac{\partial \varepsilon_{ji}}{\partial p_i} = \frac{\partial^2 q_j}{\partial p_i^2} \frac{p_i}{q_j} + \frac{\partial q_j}{\partial p_i} \left( \frac{q_j - p_i \frac{\partial q_j}{\partial p_i}}{q_j^2} \right)$$

$$\frac{\partial \varepsilon_{ji}}{\partial p_i} = \frac{\partial^2 q_j}{\partial p_i^2} \frac{p_i}{q_j} + \frac{\partial q_j}{\partial p_i} \left( 1 - \varepsilon_{ji} \right)$$

For complements, we assume an analogous effect, since an increase in price also will reduce the quantity demanded of the complement (through a leftward shift of its demand curve) while increasing product $i$’s price. Hence, if $\varepsilon_{ji} < 0$, we assume that $\frac{\partial \varepsilon_{ji}}{\partial p_i} < 0$. 


Thus, for a substitute, the elasticity effect will be negative and contribute to a bypass.

(continue discussion)

The relative quantities effect

The last term in (4) provides the effect from the relative quantity of the related product on prescribed price, i.e. from $\frac{q_j}{q_i}$ in (2). All else equal, the greater the sale of the related product relative to the product we are pricing, the greater the weight (or emphasis) should be placed on the profit of the related product. The sign of this effect depends both on whether the related product is a substitute or complement, as well as the magnitude of the cross-price elasticity relative to the own-price elasticity. This will be discussed in more detail later.

An analysis of various scenarios

We will now discuss some scenarios, based on magnitude and size of elasticities and marginal cost. For all scenarios, we consider the own price elasticity of demand to be elastic, i.e. $\epsilon_{ii} < -1$.

We consider the simplest scenario first, that of constant marginal costs and constant elasticities.

1) Constant marginal cost and constant elasticities

If all marginal costs and elasticities are constant, then
\[ \frac{\partial m_c_i}{\partial q_i} = \frac{\partial m_c_j}{\partial q_j} = \frac{\partial \varepsilon_{ij}}{\partial p_i} = \frac{\partial \varepsilon_{ji}}{\partial p_i} = 0 \]

We then have that the terms in (3) both vanish. Thus, for a single product, we then would get the optimal price from using the rule myopically (Fjell, 2003). However, this changes in the presence of related products. Further to (4) we get that the first two terms always vanish, but the third term remains, i.e. the relative quantities (or quantity weight) effect.

1a) Substitutes

For substitutes, \( \varepsilon_{ji} > 0 \), the effect of the last term in (4) will be positive. In other words, an increase (decrease) in price will also increase (decrease) the prescribed profit maximizing price, \( \frac{d\hat{p}_i}{dp_i} > 0 \). Thus, if current price is too low, the effect of increasing it towards optimum will result in an increase also in the prescribed optimal price. Hence, a myopic use of the rule in the presence of substitutes will mean that we will not change price enough. The intuition is that even if both marginal costs and elasticities are unaffected by a price change, the quantities are not. For substitutes, increase (decrease) in price will increase (decrease) the weight of the second term on the rhs of (2) through increasing (decreasing) the last factor of the last term in (2), \( \frac{q_j}{q_i} \) (which is a fraction). As price increases (decreases), the quantity of the substitute, \( q_j \), increases (decreases) whereas the opposite is the case for the product we are pricing, \( q_i \).

1b) Complements

For complements, an increase in price of the priced product will reduce both the own quantity sold and that of a complement. Hence, effect on the fraction will depend on which quantity changes is relatively larger. If the cross-price effect is small relative to the own-price effect, i.e. that \( |\varepsilon_{jl}| < |\varepsilon_{il}| \), then the value of the fraction will decrease and similarly the effect on the prescribed
price will be negative (bypass). Opposite if the cross-price effect is large relative to the own-price effect, i.e. that \( |\varepsilon_{ji}| > |\varepsilon_{ii}| \). In this case, an increase in own price will reduce the complement quantity so much more than the own quantity, that reduced weight on the former’s profit contribution will increase profit maximizing price. In other words, an increase in price will also increase prescribed price and we will have a shortfall.
Table 1 Summary of effects on prescribed (myopic) profit maximizing price from a change in price

<table>
<thead>
<tr>
<th>Scenario</th>
<th>( \frac{\partial m c_i}{\partial q_i} )</th>
<th>( \frac{\partial \varepsilon_{ii}}{\partial p_i} )</th>
<th>( \frac{\partial m c_j}{\partial q_j} )</th>
<th>( \frac{\partial \varepsilon_{ji}}{\partial p_i} )</th>
<th>( \frac{d p_i}{d p_i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) All constant</td>
<td>= 0</td>
<td>= 0</td>
<td>= 0</td>
<td>= 0 and ( \varepsilon_{ji} &gt; 0 )</td>
<td>&gt; 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>substitutes</td>
<td>Shortfall</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>= 0 and ( \varepsilon_{ji} &lt; 0 )</td>
<td>&lt; 0</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td>Complements and (</td>
<td>\varepsilon_{ji}</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>= 0 and ( \varepsilon_{ji} &lt; 0 )</td>
<td>&gt; 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Complements and (</td>
<td>\varepsilon_{ji}</td>
</tr>
<tr>
<td>2) Constant mc’s,</td>
<td>= 0</td>
<td>= 0</td>
<td>= 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>elastic</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>elasticities</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix:

\[
\varepsilon \equiv \frac{\partial q}{\partial p} \frac{p}{q}
\]

\[
\frac{\partial \varepsilon}{\partial p} = \frac{\partial^2 q}{\partial p^2} \frac{p}{q} + \frac{\partial q}{\partial p} \left( \frac{q - p \frac{\partial q}{\partial p}}{q^2} \right) = \frac{\partial^2 q}{\partial p^2} \frac{p}{q} + \frac{1}{q} \frac{\partial q}{\partial p} \left( 1 - \frac{\partial q}{\partial p} \frac{p}{q} \right)
\]

where the second term is always negative. A concave demand curve, \( \frac{\partial^2 q}{\partial p^2} \leq 0 \), is sufficient for

\[
\frac{\partial \varepsilon}{\partial p} < 0,
\]

i.e. that the absolute value of elasticity increases in price.

Example 2: Linear demand where \( A = 10, B = 1, c = 2 \):

<table>
<thead>
<tr>
<th>Price</th>
<th>Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p^* = 6 )</td>
<td>( \varepsilon^* = -1.5 )</td>
</tr>
<tr>
<td>( p_0 = 6.01 )</td>
<td>( \varepsilon_0 = -1.50627 )</td>
</tr>
<tr>
<td>( p_1 = 5.9505 )</td>
<td>( \varepsilon_1 = -1.46944 )</td>
</tr>
<tr>
<td>( p_2 = 6.26042 )</td>
<td>( \varepsilon_2 = -1.67409 )</td>
</tr>
<tr>
<td>( p_3 = 4.96694 )</td>
<td>( \varepsilon_3 = -0.986864 )</td>
</tr>
<tr>
<td>( p_4 = -150.25 )</td>
<td>( \varepsilon_4 = 0.937598 )</td>
</tr>
</tbody>
</table>

Example 3: Linear demand where \( A = 10, B = 1/2, c = 2 \).
<table>
<thead>
<tr>
<th>Price</th>
<th>Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^* = 11$</td>
<td>$\varepsilon^* = -1.22222$</td>
</tr>
<tr>
<td>$p_0 = 10.99$</td>
<td>$\varepsilon_0 = -1.21976$</td>
</tr>
<tr>
<td>$p_1 = 11.101$</td>
<td>$\varepsilon_1 = -1.24745$</td>
</tr>
<tr>
<td>$p_2 = 10.0826$</td>
<td>$\varepsilon_2 = -1.01665$</td>
</tr>
<tr>
<td>$p_3 = 122.111$</td>
<td>$\varepsilon_3 = -1.19587$</td>
</tr>
</tbody>
</table>

Example 4: Linear demand where $A = 10$, $B = 1/2$, $c = 2$.

<table>
<thead>
<tr>
<th>Price</th>
<th>Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^* = 11$</td>
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<td>$p_0 = 11.01$</td>
<td>$\varepsilon_0 = -1.22469$</td>
</tr>
<tr>
<td>$p_1 = 10.901$</td>
<td>$\varepsilon_1 = -1.19804$</td>
</tr>
<tr>
<td>$p_2 = 12.0989$</td>
<td>$\varepsilon_2 = -1.53129$</td>
</tr>
<tr>
<td>$p_3 = 5.7644$</td>
<td>$\varepsilon_3 = -0.404928$</td>
</tr>
</tbody>
</table>
4. References


