Friction in Related Party Trade when a Rival is also a Customer

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Abstract

Related parties in vertical relationships routinely have competing objectives. While conventional wisdom suggests that such frictions can be alleviated by centralized control, this paper demonstrates that decentralization and the tensions that arise in transfer pricing can help coordinate the decisions of affiliated firms. In particular, a vertically integrated central planner may find it difficult to convince a wholesale customer that it will not subsequently encroach on its retail territory, thereby necessitating wholesale price concessions to the wary customer. However, under decentralized control, related parties in the supply chain exhibit strife manifested in limited related-party pricing discounts. In this case, the upstream affiliate's customer realizes retail competition will not be as cutthroat, thereby inducing a greater willingness to pay in the wholesale arena. Such wholesale profit gains can outweigh the costs of transfer pricing distortions that arise in the retail realm. Further, in our setting, wherein input sales both to downstream affiliates and rivals are an issue, arm's length (parity) pricing restrictions can have the upside of solidifying commitments to less favorable related party pricing.
1. Introduction

Familial connection notwithstanding, related parties often exhibit substantial friction in trade. A common manifestation of this friction is when a parent's upstream affiliate excessively charges the parent's downstream affiliate who, in response, underprocures inputs. Such inefficiencies in the retail market often prompt calls for centralized planning or the use of marginal-cost transfer prices.

One practical aspect missing in such analyses is the fact that upstream affiliates often are also suppliers to rivals of downstream affiliates. For example, soft-drink producers, cereal manufacturers, and gasoline refiners have long supplied key inputs both to their downstream affiliates and to retail competitors. Such supply arrangements have proliferated substantially in recent years due to the presence of online sales arms affiliated with manufacturers who also sell through independent retail outlets (e.g., Tedeschi 2005).1

This paper reexamines the effects of related party frictions in light of the prevalence of input sales to rivals. While this issue would seemingly only exacerbate coordination issues, the paper demonstrates that frictions in decentralized entities can actually prove helpful. In particular, we show that when a vertically-integrated producer (VIP) sells inputs to its rival, it cannot resist the *ex post* temptation to encroach excessively on its wholesale customer's retail business. With such behavior imminent, the wholesale customer requires substantial concessions *ex ante* to purchase inputs.

A decentralized structure where transfer pricing is a salient consideration alters the nature of interactions. That is, with related party (transfer) prices above marginal cost, the parent firm is able to convey less aggressive retail encroachment which, in turn, engenders higher wholesale prices. To be sure, decentralization imposes costs consistent with the

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1 Other prominent instances reside in franchising and telecommunications. Franchisors typically have both company-owned and independently-owned franchises. In the telecommunications industry, cable, internet, and phone providers are actively engaged in buying and selling capacity both to their own affiliates and to unaffiliated rivals.
traditional view in the retail realm: the affiliated retail arm's market share is depressed and the unaffiliated rival's market share is expanded. However, the boost in wholesale profitability brought by decentralization can outweigh the costs due to ceding retail market share.

Briefly stated, the paper demonstrates benefits of decentralized arrangements in the presence of dual distribution channels. Importantly, the linchpin for such decentralization benefits is the imposition of (related party) transfer prices above marginal cost. While the initial analysis considers the effects of decentralization for centrally imposed transfer prices, the analysis is also extended to consider the case in which the upstream and downstream entities themselves determine the appropriate pricing. The results indicate that as long as neither of the affiliated parties is too influential in setting prices, a decentralized structure is preferred. Further, when power is doled out to the parties in a judicious manner, ceding control of all decisions to the separate entities can replicate the parent's preferred arrangement. Not only can such decentralization achieve the desired outcome, but it can do so without the parent knowing the precise details of the retail market or the relative efficiency of the two retail operators.

As a final consideration, we also address the results when regulatory restrictions preclude related party favoritism in pricing. While such arm's length (parity) requirements on input pricing can potentially reduce the attractiveness of decentralization, the paper demonstrates that decentralization and the attendant transfer pricing distortions can still be preferred. Further, if the parent finds it difficult to credibly convey related party prices to external parties, such restrictions can further solidify the implicit commitments to limited encroachment that are the source of decentralization benefits.

Turning to related literature, this paper lies at the nexus of the literatures on concurrent (dual) distribution and strategic delegation. Extant work has emphasized that dual distribution arrangements, wherein a manufacturer provides inputs to a downstream competitor, stand to offer benefits of better reaching heterogeneous consumers, effectively
monitoring independent distributors, and signaling product profitability (Dutta et al. 1995; Gallini and Lutz 1992; Vinhas and Anderson 2005). However, the downside of dual distribution arrangements lies in concerns of excessive supplier encroachment and the related inability to "direct traffic" in the channel (Kalnins 2004; Vinhas and Anderson 2005). This paper posits that the concerns of dual distribution can be minimized by a degree of related party conflict associated with decentralization and transfer pricing.

In this vein, the premise behind the benefits of decentralization herein mirrors those in the literature on strategic delegation (e.g., Fershtman and Judd 1987; Sklivas 1987; Vickers 1985) and, in particular, strategic transfer pricing (e.g., Alles and Datar 1998; Goex and Schiller 2006). This literature has examined benefits of a central planner ceding control to its affiliates and the role of transfer prices that deviate from marginal cost on downstream competitive interactions. In the context of the Cournot model studied herein, such analyses advocate transfer prices below marginal cost so as to convey a strong competitive posture in retail markets. In contrast, we demonstrate that when the rival is also a wholesale customer, the parent opts to erode retail profits via transfer prices above marginal cost in order to boost wholesale profits.

Finally, we note that the paper also relates to the literature in accounting on transfer pricing for inputs which are also sold in imperfect external markets. Most notably, Baldenius and Reichelstein (2006) demonstrate that eliminating distortions in intra-company trade entails offering related-party discounts. In this paper, we consider the case in which the input sold externally ends up in competition with the input transferred internally. In such a circumstance, it is demonstrated that eliminating distortions is not preferred, yet related-party discounts can still arise.

The remainder of the paper proceeds as follows. Section 2 outlines the basic model. Section 3 presents the key results: section 3.1 identifies outcomes under centralized control; section 3.2 identifies outcomes under decentralized control; section 3.3 compares centralization and decentralization; section 3.4 considers the outcome when pricing is an
outcome of negotiation between the related parties; section 3.5 addresses the role of arm's length (parity) pricing restrictions. Section 4 concludes the analysis.

2. Model

A vertically-integrated producer (VIP) consists of two entities, an upstream subsidiary and a downstream subsidiary. The upstream subsidiary (U) is the sole supplier of a key input to the downstream subsidiary (D) as well as an independent downstream rival (R). The two downstream parties engage in (Cournot) competition in the final good market. The inverse demand function for the final good produced by firm i is

\[ p_i = a - q_i - kq_j, \]

for \( i, j = D, R; i \neq j \), where \( p_i \) denotes the retail price for firm i's good, and \( q_i \) and \( q_j \) denote the product quantities of firms i and j, respectively. The parameter \( k \in (0,1) \) represents the degree of substitution among the competing products, where the limiting values of \( k = 0 \) and \( k = 1 \) correspond to the cases of independent products and perfect substitutes, respectively.\(^2\)

Each unit of the final product for D requires one unit of U's input, whereas each unit of output for R requires \( \tau > 0 \) units of input. In effect, \( \tau = 1 \) represents equally efficient firms, while \( \tau \neq 1 \) reflects differences in input efficiency among the competing retail firms: \( \tau < 1 \) (\( \tau > 1 \)) indicates that R (D) is the more efficient producer (e.g., Yoshida 2000). We normalize U's production cost to 0, and let \( c \) denote each firm's per unit selling cost, \( a > c \); the resulting demand intercept net of (downstream) cost is \( \alpha, \alpha = a - c \).

With this basic setting, we seek to compare the outcomes under centralization and decentralization, as well as investigate the role of transfer pricing and effects of arm's length restrictions in decentralized arrangements. The ensuing analysis employs backward induction to identify the (subgame perfect) equilibria.

\(^2\) This standard inverse demand formulation is derived from a representative consumer with quadratic utility (e.g., Singh and Vives 1984).
3. Results

The typical view is that a centralized decision-maker privy to all pertinent information can generate greater firm profit than under a decentralized structure. However, a VIP who can also supply a rival faces a problem of effectively balancing wholesale and retail profits. Somewhat surprisingly, this need to balance profits on two fronts puts decentralization and the ensuing intra-firm conflicts in a more favorable light. To demonstrate this, we first establish the outcome under centralization.

3.1. Centralization

With centralized production by the VIP, the sequence of events is as follows. First, the VIP establishes its per-unit (wholesale) price for $R$, denoted $w_R$. Second, the VIP and $R$ simultaneously choose retail quantities for their products, after which demand and profits are realized.

In this case, the VIP chooses its retail quantity, $q_D$, to maximize firm-wide profit, $\Pi(q_D,q_R,w_R)$, given $R$'s chosen retail quantity, $q_R$, and the stated wholesale price, $w_R$. Formally, the VIP's problem is:

$$\max_{q_D} \Pi(q_D,q_R,w_R) \Leftrightarrow \max_{q_D} w_R q_R + [a - q_D - kq_R]q_D - cq_D. \quad (1)$$

In $\Pi(q_D,q_R,w_R)$ from (1), the first term, $w_R q_R$, reflects the VIP's wholesale profit, while the second term, $[a - q_D - kq_R]q_D - cq_D$, reflects its retail profit. Notice, however, that the VIP's choice of $q_D$ in (1) is made with an eye only on retail profits. That is, the VIP takes wholesale profit as given when choosing its retail output. As we will see shortly, this feature of ex post behavior under centralization can make it unappealing ex ante.

In a similar fashion, given its input price, $w_R$, and the VIP's chosen quantity, $q_D$, $R$ chooses retail quantity $q_R$ to maximize its profit, or:

$$\max_{q_R} [a - q_R - kq_D]q_R - [\tau w_R + c]q_R. \quad (2)$$
Solving the first-order conditions associated with (1) and (2) jointly yields equilibrium quantities as a function of the wholesale price in the centralized regime (indicated by the superscript c):

\[ q_D^c(w_R) = \frac{\alpha[2 - k] + k\tau w_R}{4 - k^2} \quad \text{and} \quad q_R^c(w_R) = \frac{\alpha[2 - k] - 2\tau w_R}{4 - k^2}. \]  

(3)

As can be expected, each firm’s retail quantity is increasing in product demand and decreasing in selling costs (reflected by \(\alpha\), the net intercept). Further, the VIP’s (R’s) quantity is increasing (decreasing) in the wholesale price charged to R. Intuitively, the VIP's (R's) competitive position is strengthened (weakened) by an increase in R's production cost. However, due to differentiation, R can still sell to customers outside the VIP's reach. This means the VIP may opt not to foreclose its rival but instead seek profits both in the retail and wholesale arenas. More precisely, from (3), the VIP's wholesale pricing problem, which seeks to maximize the sum of wholesale and retail profits, is as follows:

\[ \max_{w_R} \Pi(q_D^c(w_R), q_R^c(w_R), w_R). \]  

(4)

Solving the first-order condition of (4) and then substituting the ensuing wholesale price into quantities in (3) and profit in (4) provides the equilibrium outcome under centralization, presented in the following lemma. (All proofs are provided in the Appendix.)

**Lemma 1.** Under centralization, the equilibrium wholesale price, quantities, and VIP profit, respectively are:

\[ w_R^c = \frac{\alpha[8 - 4k^2 + k^3]}{2\tau[8 - 3k^2]}; \quad q_D^c = \frac{\alpha[2 - k][4 + k]}{2[8 - 3k^2]}; \quad q_R^c = \frac{2\alpha[1 - k]}{[8 - 3k^2]}; \quad \text{and} \quad \Pi^c = \frac{\alpha^2[6 - k][2 - k]}{4[8 - 3k^2]}.

As confirmed in the lemma, in the absence of perfect retail substitutability (\(k < 1\)), the VIP opts not to foreclose the rival, instead seeking to sell in both wholesale and retail markets. As we next consider, the extent of its profitability in each market is altered under decentralization.
3.2. Decentralization

With decentralized production, the VIP relies on its downstream subsidiary to determine production and thus must determine the transfer price which governs intra-company trade. In this case, the sequence of events is as follows. First, the VIP establishes the wholesale price for \( R, w_R \), and the intra-company transfer price, \( w_D \). Second, \( D \) and \( R \) simultaneously choose retail quantities for their products to maximize their respective entities’ profits. Finally, consumer demand and entity profits are realized.

In this case, \( R \)'s retail decision is again as in (2). \( D \) chooses its quantity, \( q_D \), to maximize its profit, \( \Pi_D(q_D,q_R,w_D,w_R) \), given \( R \)'s chosen quantity (\( q_R \)) and input prices (\( w_D \) and \( w_R \)).

\[
\text{Max}_{q_D} \Pi_D(q_D,q_R,w_D,w_R) \Leftrightarrow \text{Max}_{q_D} [a - q_D - kq_R]q_D - [w_D + c]q_D. \tag{5}
\]

Solving the first-order conditions associated with (2) and (5) jointly yields equilibrium quantities as a function of the wholesale price and transfer price in the decentralized regime (indicated by the superscript \( d \)):

\[
q_D^d(w_D,w_R) = \frac{\alpha[2 - k] - 2w_D + kw_R}{4 - k^2} \quad \text{and} \quad q_R^d(w_D,w_R) = \frac{\alpha[2 - k] - 2\tau w_R + kw_D}{4 - k^2}. \tag{6}
\]

Comparing (6) and (3), one can readily confirm that marginal cost transfer pricing (\( w_D = 0 \)) permits decentralization to replicate centralization, consistent with the typical view. And, as \( w_D \) increases, \( D \)'s production is distressed while \( R \)'s production is increased, reflecting the fact that \( D \) internalizes higher transfer costs which, in turn, emboldens \( R \). Given the induced demand in (6), the VIP's input pricing problem is as follows:

\[
\text{Max}_{w_D,w_R} \Pi(q_D^d(w_D,w_R),q_R^d(w_D,w_R),w_R). \tag{7}
\]
Solving the first-order conditions of (7) and then substituting the ensuing input prices into quantities in (6) and profit in (7) provides the equilibrium outcome under decentralization, summarized in the following Lemma.

**Lemma 2.** Under decentralization, the equilibrium transfer price, wholesale price, quantities, and VIP profit, respectively are:

\[
\begin{align*}
  w^d_D &= \frac{\alpha(1-k)k}{2[2-k^2]}; \quad w^d_R = \frac{\alpha}{2}; \quad q^d_D = \frac{\alpha(2-k)}{2[2-k^2]}; \quad q^d_R = \frac{\alpha(1-k)}{2[2-k^2]}; \quad \text{and} \quad \Pi^d = \frac{\alpha^2(3-2k)}{4[2-k^2]};
\end{align*}
\]

Given the equilibrium outcomes in each regime, we next compare centralization to decentralization.

### 3.3. Centralization vs. Decentralization

The standard view of decentralized decision making is that it is best when it can approximate outcomes under a fully-rational central planner. Such thinking gives rise to the textbook prescription of marginal-cost transfer pricing to alleviate attendant production distortions. However, in the case of a VIP selling products to a retail rival, an additional consideration arises. With centralized decision making, the VIP rationally ignores wholesale profit when choosing retail output. Knowing this will be the case, the rival expects intense competition and procures fewer inputs for a given wholesale price. The VIP's only remedy to its *ex post* retail aggression is to offer wholesale price concessions *ex ante*.

Decentralization permits another avenue to convey a less-aggressive posture *ex post*. In particular, when the transfer price is above marginal cost, the VIP conveys to its wholesale customer that its downstream arm will be less aggressive in retail competition, thereby enhancing the customer's demand. And, since decentralization is able to restore demand, the VIP can rely less on wholesale price cuts to boost demand, which thus generates greater wholesale profit. This feature of decentralization is confirmed by a
comparison of input prices and output quantities in Lemmas 1 and 2, summarized in the following Proposition.

PROPOSITION 1.

(i) The transfer price under decentralization is set above marginal cost, i.e., \( w_D^d > 0 \).

(ii) The firm's retail output is lower under decentralization, i.e., \( q_D^d < q_D^c \).

(iii) The rival's retail output is higher under decentralization, i.e., \( q_R^d > q_R^c \).

(iv) The wholesale price is higher under decentralization, i.e., \( w_R^d > w_R^c \).

Parts (i), (ii), and (iii) are consistent with the standard view that decentralization generates detrimental productive effects. In particular, transfer pricing above marginal cost in (i) yields inefficient production in (ii), the effects of which are compounded by additional aggression by the rival in (iii). The resulting downside of decentralization is manifest in lower retail profits. In particular, the VIP's equilibrium retail profit in regime \( i \) is:

\[
\hat{\Pi}^i = [a - q_D^i - kq_R^i]q_D^i - cq_D^i.
\]

And, comparing retail profit across regimes yields:

\[
\hat{\Pi}^d - \hat{\Pi}^c = -\frac{\alpha^2 k^2 [2 - k] [1 - k] [4 + 2k - k^2]}{4[2 - k^2] [8 - 3k^2]^2} < 0.
\] (8)

From (8), decentralization with transfer prices above marginal cost imposes a strict loss in retail profits. However, the offsetting tension, which is absent in traditional discussions of attendant transfer pricing distortions, resides in the wholesale realm. The lower retail output by the firm (in (ii)) and the concomitant surge in retail output by the rival (in (iii)) stand to increase wholesale profit. Further, by convincing the rival that the VIP will take a less aggressive competitive posture, decentralization permits a higher wholesale price (in (iv)). Thus, in the wholesale market, decentralization affords two-fold benefits: both higher prices and more purchases. The result is a strong boost in wholesale profit. More precisely, the VIP's equilibrium wholesale profit in regime \( i \) is:

\[
\hat{\Pi}^i = w_R^i \tau q_R^i.
\]

And, comparing wholesale profit across regimes yields:
\[ \tilde{\Pi}^d - \tilde{\Pi}^c = \frac{\alpha^2 k^2 [1 - k] [16 - 8k - 7k^2 + 4k^3]}{4[2 - k^2][8 - 3k^2]^2} > 0. \] (9)

What remains to be seen is the net effect of decentralization. Mathematically, this can be determined by comparing VIP profits in Lemmas 1 and 2 or, equivalently, summing the expressions in (8) and (9). Intuitively, one can infer a positive net effect of decentralization from the fact that the VIP intentionally deviates from marginal-cost transfer pricing. That is, since marginal-cost transfer pricing can replicate centralization, the fact that the VIP prefers a higher transfer price indicates that the potential gains (at the margin) in the wholesale market exceed the potential losses in the retail market. Consider the case of \( \tau = 1 \), wherein the two retail firms are equally efficient. In this case, \( 0 < w_D^d < w_R^d \), indicating that the optimal balancing of the two markets entails transfer pricing above marginal cost, yet still there is preferential pricing provided to the related party. In this sense, decentralization with a modest transfer price can better balance profitability in the two markets than centralization, a result formally confirmed in the next proposition.

**PROPOSITION 2.** VIP profit is higher under decentralization than under centralization.

While decentralization provides a strict benefit for all \( k \in (0, 1) \), the magnitude of the benefit is intrinsically linked to the extent of competitive overlap (intensity) between \( D \) and \( R \). Recall that the benefit of decentralization arises because it permits the VIP to convey reduced retail competitiveness so as to restore wholesale demand. At the extreme case of \( k = 0 \), the parties' retail products are independent so there is no such competitiveness to speak of. As \( k \) increases from 0, retail competitiveness plays a more prominent role and thus decentralization can benefit wholesale profits more. However, for \( k \) sufficiently large, the cost of softened competitiveness in the retail market becomes so pronounced that the preferred decentralization arrangement entails lower transfer prices and thus behavior closer to that under centralization. In fact, at the other extreme of \( k = 1 \), the competitive costs of softened competition are so strong that the VIP prefers to foreclose its rival under either
regime, thereby rendering centralization and decentralization equivalent. As a result, the attractiveness of decentralization and transfer prices above marginal cost is most pronounced for intermediate values of $k$. This feature of the result is confirmed in Figure 1.

![Figure 1](image-url)

**Figure 1.** The net benefit of decentralization in terms of wholesale profit ($\Pi^d - \Pi^c$), retail profit, ($\hat{\Pi}^d - \hat{\Pi}^c$), and total profit ($\Pi^d - \Pi^c$) as a function of $k$.

Broadly speaking, the reason decentralization proves helpful mirrors that in other settings of strategic delegation: decentralization engenders a decision-maker whose priorities differ from those of the firm which, in turn, convinces a strategic party that the firm will take a certain course of action. In other manifestations of strategic delegation (e.g., Fershtman and Judd 1987; Sklivas 1987; Vickers 1985) and strategic transfer pricing (e.g., Alles and Datar 1998; Goex and Schiller 2006), the firm seeks to convey a certain posture in retail markets so as to increase profits in such markets. In this instance, however, the posture conveyed in the retail market is to the detriment in that market and only useful for its ramifications in the wholesale market.

More specifically, in the standard strategic delegation literature, a firm seeks an aggressive retail arm when facing Cournot competition. Thus, a commonly noted friction under decentralization–internal pricing above marginal cost–makes the outcome particularly
undesirable for the firm. In contrast, when the firm seeks to balance its interests in multiple markets (here, wholesale and retail), pricing above marginal cost is precisely the kind of friction that aids the firm in softening its retail market stance. Decentralization permits the VIP to convince its wholesale customer that it will not wile away its customer's retail profitability by encroaching excessively. This, in turn, boosts the customer's wholesale demand and its willingness to pay.

Given this essential insight, it is worth stressing that the presumption that decentralization entails $D$ being entirely fixated on its own entity's profit is not critical. In fact, all that is important is that $D$'s priorities overweight its own entity's profit relative to the VIP. To see this, consider the outcome if $D$'s quantity choice in (5) were instead generalized to:

$$\max_{q_D} \lambda \Pi_D(q_D, q_R, w_D, w_R) + [1 - \lambda] \Pi(q_D, q_R, w_R), \quad \lambda \in (0, 1]. \quad (10)$$

Via the choice of $\lambda$, (10) reflects a range of possible priorities for $D$, reflecting that $D$ may care both about firmwide (VIP) profit and its own entity's profit. Solving the first-order conditions associated with (2) and (10) jointly yields decentralization quantities as a function of the wholesale price and transfer price:

$$q_i^d(w_D, w_R; \lambda) = \lambda q_i^d(w_D, w_R) + [1 - \lambda] q_i^c(w_R), \quad i = D, R. \quad (11)$$

Given the induced demand in (11), the VIP's input pricing problem is

$$\max_{w_D, w_R} \Pi(q_D^d(w_D, w_R; \lambda), q_R^d(w_D, w_R; \lambda), w_R).$$

Solving the first order condition yields $w_D^d(\lambda) = w_D^d/\lambda$ and $w_R^d(\lambda) = w_R^d$ which, when substituted into (11) and (1) confirms that $\lambda$ influences the preferred transfer price, but does not change the overall profit from decentralization.

Intuitively, the transfer price is tailored to achieve the desired level of retail competitiveness in $D$. If $D$'s priorities are closer to that of the VIP (i.e., $\lambda$ is lower), the VIP needs to increase the transfer price appreciably so as to keep the ideal level of
competitiveness in play ($\lambda$ times $w_D^d(\lambda)$ is held constant). Thus, the basic premise of the result continues to hold in this generalized case: the VIP hands the reigns over to a party whose priorities differ from its own, and such differential priorities provide an avenue to convince $R$ that it will not face excessive encroachment in the retail market.

A related question is whether decentralization itself (for any $\lambda$) is merely an imperfect substitute for competitive commitment. That is, a decision to decentralize serves as a means of precommitting the firm to a certain posture in the retail market; but, how close does this approach come to the outcome that would be achieved if the VIP could fully commit to its quantity in advance? Technically, this means that $R$ chooses retail quantity as in (2), but the VIP is able to pick $q_D$ and $w_R$ in advance. The first-order condition of (2) yields $q_R^*(q_D, w_R) = [\alpha - kq_D - \tau w_R] / 2$. Given this induced demand, the VIP chooses wholesale price and retail quantity by solving:

$$\text{Max}_{q_D, w_R} \Pi(q_D, q_R^*(q_D, w_R), w_R).$$  \hspace{1cm} (12)$$

Solving the first-order conditions of (12) yields $q_D = q_D^d$ and $w_R = w_R^d$. Substituting this into VIP profit confirms a somewhat remarkable result: decentralization replicates the firm's desired benchmark performance. The next proposition summarizes these results.

**Proposition 3.**

(i) Decentralization yields the same VIP profit for any $\lambda > 0$.

(ii) Decentralization achieves the VIP's optimal profit under quantity precommitment.

The proposition demonstrates two key elements. First, the salient feature of decentralization is that it entails providing decision rights to parties whose priorities differ from the VIP's, and thus the assumption of a complete focus on entity profit is merely a
convenient simplification. Second, the decentralized arrangement considered thus far achieves the VIP's desired outcome were it able to fully commit to its decisions in advance.\textsuperscript{3}

It is worth noting, however, that the optimal decentralized arrangement relies on activism by the VIP in terms of setting input prices. As the next section demonstrates, the results can continue to hold in circumstances wherein the VIP is also passive in pricing.

3.4. Negotiated Pricing

In the analysis thus far, we have considered the optimality of decentralization in an environment in which the VIP is able to determine transfer prices that govern related party trade and is privy to relevant information in doing so. Given such circumstances may not always be appropriate in practice, and the fact that decentralization of both production and pricing decisions is common, we next revisit the outcomes when decentralization entails handing control over input pricing to the subsidiaries.

To reflect pricing decisions made by the two subsidiaries, we employ the standard Nash bargaining solution, generalized to (possibly) asymmetric bargaining power (e.g., Myerson 1991). Besides allowing an axiomatic representation of outcomes when parties bargain over a decision, this approach is commonly employed because it also allows for a tractable characterization of equilibria without requiring an explicit representation of the specific bargaining process. In particular, denoting $U$'s profit by $\Pi_U(q_D, q_R, w_D, w_R) = w_Dq_D + w_R\tau q_R$, the chosen input prices are the outcome of a bargaining process that solves the generalized Nash product of the two subsidiaries' profits, taking into account the ensuing effects in the retail market:

$$\begin{align*}
\max_{w_D, w_R} \left[ \Pi_D(q_d^d(w_D, w_R), q_d^d(w_D, w_R), w_D, w_R) \right]^\beta \left[ \Pi_U(q_d^d(w_D, w_R), q_d^d(w_D, w_R), w_D, w_R) \right]^{1-\beta}.
\end{align*}$$

\textsuperscript{3} This finding is analogous to that in Vickers (1985), which demonstrates that strategic delegation can create an environment wherein the firm plays the role of a Stackelberg leader.
In (13), $\beta \in (0,1)$ reflects the relative influence (bargaining power) of $D$ in setting input prices (the limiting cases of $\beta = 0$ and $\beta = 1$ correspond to prices being unilaterally set by $U$ and $D$, respectively). The first-order conditions of (13) reveal the chosen input prices in the case of negotiated pricing, denoted $w^D_n(\beta)$ and $w^n_R(\beta)$ (the superscript reflects the *negotiated* outcome). As may be expected, $w^D_n(\beta)$ is decreasing in $\beta$, reflecting the fact that $D$ prefers a relatively low transfer price to boost its retail profitability, whereas $U$ prefers a relatively high transfer price to boost its profitability from sales to $D$. And, recall, decentralization is useful when a modest transfer price (i.e., one above but not too far above marginal cost) is in place so as to balance the VIPs priorities in the wholesale and retail markets. As such, so long as neither subsidiary holds too much influence in the determination of input prices, VIP profit under negotiated pricing, $\Pi^n(\beta)$, is preferred to centralization, despite the fact that input pricing is left to the devices of self-interested subsidiaries.

**Proposition 4.** There exist $\underline{\beta}$ and $\overline{\beta}$, $0 < \underline{\beta} < \overline{\beta} < 1$, such that decentralization with negotiated pricing yields higher VIP profit than centralization if and only if $\beta \in (\underline{\beta}, \overline{\beta})$.

A natural extension to this line of reasoning centers around the VIP’s preferred bargaining arrangement. In particular, in establishing organizational structure, the VIP may have a profound influence on the relative power of the two subsidiaries in setting prices, via provision of either formal or real authority. In this case, if the VIP could establish the relative influence and decision-making power of the divisions as part of its decentralized structure, what would be its preferred arrangement? The next proposition answers this question.

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4 In terms of formal authority, the VIP could, for example, set the particular form of bargaining (which itself determines who exerts the most influence), or set ground rules for the outcome if the two parties cannot come to agreement (similar in spirit to the status quo arrangement considered in Edlin and Reichelstein 1996). In terms of real authority, the VIP can "play favorites", demonstrate a willingness to rubber stamp appeals made by one party, etc.
PROPOSITION 5.

(i) The VIP's preferred assignment of bargaining rights is \( \beta = \beta^* = \frac{[2 - k]^2}{[6 - 4k - 3k^2 + 2k^3]} \).

(ii) Negotiated pricing with judicious assignment of bargaining rights replicates the VIP's preferred decentralized outcome, i.e., \( w_D^n(\beta^*) = w_D^d \), \( w_R^n(\beta^*) = w_R^d \), and \( \Pi^n(\beta^*) = \Pi^d \).

The first part of the proposition states the \( \beta \)-value that maximizes VIP profit (which, by Proposition 4 is sure to yield higher profit than centralization). The second part of the proposition underscores an intriguing property of this assignment of authority. That is, it not only reflects the VIP's preferred bargaining arrangement, but it also replicates the VIP's preferred decentralized outcome. This feature is particularly surprising because preferred pricing on two dimensions (\( w_D \) and \( w_R \)) is achieved by the negotiation process despite the process being identified by only one parameter (\( \beta \)). The reason this can be accomplished is that \( w_R^n(\beta) \) is free of \( \beta \). Intuitively, \( w_R^d \) is \( U \)'s preferred wholesale price were it given complete control over pricing. And, as \( U \)'s bargaining power is decreased, concessions to \( D \) are provided through lower \( w_D \), leaving \( w_R \) unchanged. In other words, the two subsidiaries are in agreement over not giving a break to the outside party rather exploiting their bargaining strengths to push \( w_D \) in the direction of their liking. Given this feature, a judicious choice of \( \beta \) can ensure \( w_D^n(\beta) = w_D^d \) and, thereby, replicate the desired outcome.

A notable feature of \( \beta^* \) as well as \( \underline{\beta} \) and \( \overline{\beta} \) is that they are free of both \( \alpha \) and \( \tau \). That is, the VIP can implement its preferred assignment of decision rights without knowing either the relative profitability of the retail market or the relative efficiency of the participants in the wholesale market. Thus, if the VIP is unaware of such information, it can nonetheless effectively utilize decentralization to achieve its desired purpose. This feature and results in Propositions 4 and 5 are depicted pictorially in Figure 2, which plots the net gain from decentralization for two \((\alpha, \tau)\)-pairs.
Figure 2. $\Pi^u(\beta) - \Pi^c$ as a function of $\beta$ for differing values of $\alpha$ and $\tau$.

The figure reiterates the following key points from this section: (i) firm profit under negotiated transfer pricing is higher than under centralization for $\beta \in (\underline{\beta}, \overline{\beta})$, (ii) firm profit under negotiated transfer pricing is maximized at $\beta = \beta^*$, in which case, firm profit is the same as that under decentralization, (i.e., $\Pi^u(\beta^*) = \Pi^d$), and (iii) the cutoff $\beta$-values, $\underline{\beta}$ and $\overline{\beta}$, and the optimal $\beta$-value, $\beta^*$, are unaffected by $(\alpha, \tau)$ values; thus, $\Pi^d$ can be implemented via negotiated pricing even when the VIP is uniformed of market demand and/or the relative efficiency of the firms.

3.5. Arm's Length Pricing Restrictions

In the paper's analysis thus far, we have presumed that the VIP ($U$ and $D$ in the case of negotiated pricing) can employ differential input prices to $D$ and $R$. In some circumstances, however, the use of distinct input prices for affiliated parties is not possible due to regulatory restrictions. Most familiar to accountants are the OECD guidelines applying the arm's length principle to transfer prices set by multinationals for tax purposes.\footnote{While several countries (including the US) permit firms to "de-couple" their transfer prices so that those used for tax purposes are different from those used for internal purposes, many firms are reluctant to have any transfer prices that deviate significantly from market price, fearing regulatory backlash (Durst 2002; Ernst and Young 2003; Johnson 2006).} Further, "parity" requirements, forcing equal pricing to related and unrelated parties, are
often imposed even when taxation is not an issue.\textsuperscript{6} While such restrictions have no effect on centralized decision making (the VIP simply ignores the price in making production decisions), they can have effects on decentralized outcomes.

In particular, returning to the baseline analysis under decentralization with arm's length (parity) restrictions, the VIP's chosen wholesale price solves (7) with the restriction \( w_D = w_R = w \). The first-order condition with respect to \( w \) of this problem yields:

\[
w = w^* = \psi w_D^d + [1 - \psi] w_R^d,
\]

where \( \psi = \frac{[2 - k^2][2 - k\tau]}{4[1 - k\tau] + 2\tau[k^3 + 4\tau] - k^2[2 + 3\tau^2]} \). (14)

Intuitively, when restrained in its pricing, the VIP opts for a common price which is a weighted average of the separate prices it would like to charge to the respective parties. The added constraint on pricing reduces the attractiveness of decentralization but does not necessarily eliminate its benefits. In particular, consider the effect of differential efficiency between \( D \) and \( R \). If \( D \) and \( R \) are equally efficient (i.e., \( \tau = 1 \)), the VIP would like to provide related-party discounts (i.e., \( w_D^d < w_R^d \)) so as to best balance profits in its two markets. As \( \tau \) increases, the preferred arrangement entails greater concessions to \( R \), making the unfettered \( w_D^d \) and \( w_R^d \) closer. In this case, the imposition of a requirement that \( w_D^d = w_R^d \) is not particularly costly. This intuition is borne out in a comparison of \( \Pi(q_D^d(w^*, w^*), q_R^d(w^*, w^*), w^*) \) and \( \Pi^c \), as confirmed in the next proposition.

**PROPOSITION 6.** In the presence of arm's length pricing restrictions, VIP profit is higher under decentralization than under centralization if and only if \( \tau > \overline{\tau} \).

The proposition demonstrates that as long as the preferred arrangement under unfettered pricing does not entail too large a related-party discount, the imposition of arm's

\textsuperscript{6} Such parity requirements fall under the broad umbrella of uniform pricing regulations (e.g., the Robinson-Patman Act). Explicit codification of parity requirements is particularly common in industries where the input in question entails access to capacity, such as telecommunications and electric utilities. The heated debate over "net neutrality" has also brought this issue to the forefront in the context of broadband content pricing. For more on the imposition of parity requirements, see Ismail (2004), Sappington and Weisman (2005), and Reitzes and Woroch (2007).
length pricing restrictions does not derail the fundamental benefits of decentralization. Such a preference is reflected succinctly in an input efficiency cutoff.

Besides demonstrating that the basic tensions of the paper's results can remain intact under parity requirements, this section also speaks to the issues of observability and commitment that permeate the strategic transfer pricing literature (Caillaud and Rey 1994; Goex 2000; Narayanan and Smith 2000). As in other settings of strategic transfer pricing, observability of and precommitment to related party prices are critical in this setting for strategic benefits of transfer pricing to arise. However, when arm's length restrictions are in play, commitment and observability are ensured. That is, regulatory restrictions on preferential pricing mean that when \( R \) learns its own price, it invariably learns \( D \)'s input price as well. And, since \( D \)'s input price cannot be adjusted without concomitant adjustments in \( R \)'s price, this also entails a credible commitment. In other words, arm's length pricing restrictions can serve to reinforce commitments to chosen related-party prices which, in turn, can reinforce the benefits to decentralization.

**COROLLARY.** In the absence of observability and commitment, arm's length pricing restrictions serve to restore the optimality of decentralization if and only if \( \tau > \overline{\tau} \).

The fact that regulatory restrictions serve to reinforce commitment to a particular transfer pricing arrangement to the benefit is akin to the result in Narayanan and Smith (2000) who show that tax considerations can provide an ancillary benefit of making strategically beneficial transfer prices ex post optimal. Note that the regulatory benefit identified herein necessitates that the item transferred internally also be sold externally. Thus, the ability of common parity regulations to substitute for observability and commitment further differentiates the results here from those in the extant strategic transfer pricing literature.

As a final aside, we stress that similar benefits can arise even when the restrictions do not entail strict adherence to arm's length pricing. That is, if regulatory restrictions
instead impose an implicit cap on intracompany discounts (e.g., Smith 2002) decentralization can still prove beneficial. The source of benefits to arm's length restrictions is that they serve to credibly convey to external parties that the related party price is above marginal cost; as such, the internal price does not have to be identical to the external price to be useful.

4. Conclusion

Conflicting priorities among related parties and the ensuing distortions that arise in pricing and production have garnered much attention in managerial accounting. The standard view posits that the accountant's job is to design transfer pricing rules so as to minimize such distortions. This paper demonstrates that such a view is incomplete when the scope of the firm's priorities extends to both wholesale and retail markets. In such a case, a modicum of conflict (manifest in transfer prices with markups above marginal cost) can actually better coordinate the parties' behavior.

Centralized decision making engenders excessive retail encroachment, which in turn translates into lower wholesale profit. With transfer pricing-induced distortions, however, the firm is able to commit to less aggressive encroachment on its wholesale customer's retail market. By effectively ceding retail market share, the firm expands wholesale profitability. The paper's results demonstrate that decentralization with judicious choice of transfer prices (or careful dissemination of decision rights) best balances profitability in the wholesale and retail markets. Given the prevalence of circumstances wherein an input supplier also has a retail arm, the results shed some light on the efficacy of organizational forms and accounting policies which are routinely observed.
APPENDIX

Proof of Lemma 1. Substituting the quantities from (3) into (4) yields the following wholesale pricing problem for the VIP in the centralized setting:

$$\max_{w_R} \frac{\alpha^2[2 - k]^2 + \alpha[8 - 4k^2 + k^3]\tau w_R - [8 - 3k^2]\tau^2 w_R^2}{[4 - k^2]^2}.$$  \hspace{1cm} (A1)

The first-order condition of (A1) with respect to $w_R$ yields:

$$\frac{\tau\alpha[8 - 4k^2 + k^3] - 2[8 - 3k^2]\tau w_R}{[4 - k^2]^2} = 0.$$  

Solving the above linear equation for $w_R$ yields $w_R^c$ in Lemma 1. Substituting $w_R^c$ into (3) gives $q_D^c$ and $q_R^c$. Finally, using $w_R^c$ in (A1) yields $\Pi^c$, the VIP's profit under centralization.

Proof of Lemma 2. Substituting the quantities from (6) into (7) yields the following pricing problem for the VIP in the decentralized setting:

$$\max_{w_D, w_R} \frac{1}{[4 - k^2]^2} \left( \alpha^2[2 - k]^2 - 2[2 - k^2][w_D - k\tau w_R]w_D - [8 - 3k^2]\tau^2 w_R^2 + \alpha[2 - k][2\tau w_R(2 + k) - k^2(w_D + \tau w_R)] \right).$$  \hspace{1cm} (A2)

The first-order conditions with respect to $w_D$ and $w_R$, respectively, are:

$$\frac{2[2 - k^2][w_D - k\tau w_R] - \alpha[2 - k]k^2}{[4 - k^2]^2} = 0,$$

and

$$\frac{\tau\alpha[8 - 4k^2 + k^3] + 2kw_D[2 - k^2] - 2\tau w_R[8 - 3k^2]}{[4 - k^2]^2} = 0.$$  

Jointly solving the above two equations for $w_D$ and $w_R$ yields $w_D^d$ and $w_R^d$ in Lemma 2. Substituting these input prices into (6) gives $q_D^d$ and $q_R^d$. Finally, using $w_D^d$ and $w_R^d$ in (A2) yields $\Pi^d$, the VIP's profit under decentralization.

Proof of Proposition 1.

(i) The proof follow from the expression for $w_D^d$ in Lemma 2, and noting $k \in (0,1)$.

(ii) Using $q_D^d$ from Lemma 2 and $q_R^c$ from Lemma 1,
\[ q_D - q_R = \frac{\alpha[2 - k]}{2(2 - k^2)} - \frac{\alpha[2 - k][4 + k]}{2(8 - 3k^2)} = -\frac{\alpha k[1 - k][4 - k^2]}{2[16 - 14k^2 + 3k^4]} < 0. \]

(iii) Using \( q_R^d \) from Lemma 2 and \( q_R^c \) from Lemma 1,
\[ q_R^d - q_R^c = \frac{\alpha[1 - k]}{2(2 - k^2)} - \frac{2\alpha[1 - k]}{8 - 3k^2} = \frac{\alpha k^2[1 - k]}{2[16 - 14k^2 + 3k^4]} > 0. \]

(iv) Using \( w_R^d \) from Lemma 2 and \( w_R^c \) from Lemma 1,
\[ w_R^d - w_R^c = \frac{\alpha}{2\tau} - \frac{\alpha[8 - 4k^2 + k^3]}{2\tau[8 - 3k^2]} = \frac{\alpha k^2[1 - k]}{2\tau[8 - 3k^2]} > 0. \]

Proof of Proposition 2. Using \( \Pi^d \) from Lemma 2 and \( \Pi^c \) from Lemma 1,
\[ \Pi^d - \Pi^c = \frac{\alpha^2[3 - 2k]}{4(2 - k^2)} - \frac{\alpha^2[6 - k][2 - k]}{4[8 - 3k^2]} = \frac{\alpha^2k^2[1 - k]^2}{4[16 - 14k^2 + 3k^4]} > 0. \]

Proof of Proposition 3.

(i) The first-order conditions of (2) and (10) correspond to the following equations:
\[ \alpha - 2q_D - kq_R - \lambda w_D = 0 \text{ and } \alpha - 2q_R - kq_D - \tau w_R = 0. \]

Solving these equations for the two quantities yields:
\[ q_D^d(w_D, w_R; \lambda) = \frac{\alpha[2 - k] - 2\lambda w_D + k\tau w_R}{4 - k^2} \text{ and } q_R^d(w_D, w_R; \lambda) = \frac{\alpha[2 - k] - 2\tau w_R + k\lambda w_D}{4 - k^2}. \]

Substituting these quantities into \( \Pi(q_D, q_R, w_R) \) yields the following pricing problem for the VIP:
\[ \max_{w_D, w_R} \frac{1}{[4 - k^2]^2} \left( \alpha^2[2 - k]^2 - 2[2 - k^2][\lambda w_D - k\tau w_R]\lambda w_D - [8 - 3k^2]\tau^2 w_R^2 + \right) \]
\[ \alpha[2 - k][2\tau w_R(2 + k) - k^2(\lambda w_D + \tau w_R)] \]. \hfill (A3) \]

Notice (A3) is the same as (A2) except that \( w_D \) is replaced by \( \lambda w_D \). Hence, as expected, the first-order conditions of (A3) yield input prices of \( w_D^d(\lambda) = w_D^d/\lambda \) and \( w_R^d(\lambda) = w_R^d \). Substituting these prices back into (A3), and simplifying, shows (A3) equals \( \frac{\alpha^2[3 - 2k]}{4[2 - k^2]} \). From Lemma 2, this is \( \Pi^d \).
(ii) The VIP's problem in (12) is equivalent to:

$$\max_{q_D, w_R} \frac{1}{2} \left( \alpha[(2-k)q_D + \tau w_R] - \tau^2 w_R^2 - [2-k^2] q_D^2 \right).$$  \hspace{1cm} (A4)

The first-order conditions of (A4) correspond to the following equations:

$$\alpha - \frac{\alpha k}{2} - [2-k^2] q_D = 0 \text{ and } \frac{\tau}{2} [\alpha - 2 \tau w_R] = 0.$$

Solving these equations yields $q_D = \frac{\alpha [2-k]}{2[2-k^2]}$ and $w_R = \frac{\alpha}{2 \tau}$ which, from Lemma 2, are the same as $q_D^d$ and $w_R^d$, respectively. Substituting these into (A4) yields VIP profit of $\frac{\alpha^2 [3-2k]}{4[2-k^2]}$. From Lemma 2, this is $\Pi^d$. \hfill \square

**Proof of Proposition 4.** When $D$ makes production decisions for the VIP, the retail quantities as a function of input prices are as in (6). Using these quantities, $D$'s profit is:

$$\Pi_D(q_D^d(w_D, w_R), q_R^d(w_D, w_R), w_D, w_R) = \left[ \frac{\alpha(2-k) - 2w_D + k\tau w_R}{4-k^2} \right]^2.$$

Similarly, using (6), $U$'s profit is:

$$\Pi_U(q_D^d(w_D, w_R), q_R^d(w_D, w_R), w_D, w_R) = w_D q_D^d(w_D, w_R) + w_R \tau q_R^d(w_D, w_R)$$

$$= \frac{\alpha[2-k][w_D + \tau w_R] - 2[w_D^2 - k w_D \tau w_R + \tau^2 w_R^2]}{4-k^2}.$$

Using the above divisional profit expressions, under negotiated pricing, the input prices solve:

$$\max_{w_D, w_R} \left[ \Pi_D(q_D^d(w_D, w_R), q_R^d(w_D, w_R), w_D, w_R) \right]^{\beta} \left[ \Pi_U(q_D^d(w_D, w_R), q_R^d(w_D, w_R), w_D, w_R) \right]^{1-\beta}.$$

Solving the two first-order conditions of the above yields:

$$w_R^n(\beta) = \frac{\alpha}{2 \tau} \text{ and } w_D^n(\beta) = \frac{\alpha[6 - \beta(2-k) - k - A(\beta)]}{8}, \text{ where } A(\beta) = \sqrt{[2-k][2 + \beta^2(2-k) - k + 2\beta(6+k)].}$$

Using these input prices, the VIP's profit under negotiated pricing, denoted $\Pi^n(\beta)$, is calculated as follows:
\[\Pi^n(\beta) = \Pi(q^d_D(w^n_D(\beta),w^n_R(\beta)),q^d_R(w^n_D(\beta),w^n_R(\beta)),w^n_R(\beta)) = \]
\[
\frac{\alpha^2 \left( 28 + 2k - 6k^2 - k^3 - 2\beta k[4 - 2k - k^2] - \beta^2[4 - 2k - 2k^2 + k^3] + A(\beta)[6 - k^2 - \beta(2 - k^2)] \right)}{16(2 - k)(2 + k)^2}. \tag{A5}
\]

The VIP's profit is higher under decentralization and negotiated pricing than under centralization if and only if \(\Pi^n(\beta) - \Pi^c > 0\). The expression for \(\Pi^n(\beta)\) is in (A5) and the expression for \(\Pi^c\) is presented in Lemma 2. Substituting these expressions, it is easy to verify that \(\Pi^n(\beta) - \Pi^c\) is a concave function of \(\beta\) with two roots, denoted \(\beta_1\) and \(\beta_2\), \(0 < \beta_1 < \beta_2 < 1\), listed below:

\[
\beta_1 = \frac{F(k) - kG(k)\sqrt{2(8 - 3k^2)}}{H(k)} \quad \text{and} \quad \beta_2 = \frac{F(k) + kG(k)\sqrt{2(8 - 3k^2)}}{H(k)},
\]

where

\[
F(k) = 768 - 1280k + 512k^2 + 224k^3 - 260k^4 + 112k^5 - 23k^6 - 6k^7 + 3k^8,
\]

\[
G(k) = 112 - 232k + 130k^2 + 17k^3 - 37k^4 + 11k^5 - k^6, \quad \text{and}
\]

\[
H(k) = 1152 - 1536k - 384k^2 + 1248k^3 - 314k^4 - 204k^5 + 103k^6 - 18k^7 + 3k^8.
\]

Given the concavity of \(\Pi^n(\beta) - \Pi^c\), it follows that negotiated pricing yields higher VIP profit than centralization if and only if \(\beta \in (\beta_1, \beta_2)\).

\[\square\]

**Proof of Proposition 5.**

(i) Under negotiated pricing, the VIP's preferred assignment solves \(\max_{\beta} \Pi^n(\beta)\), where \(\Pi^n(\beta)\) is in (A5). The first-order condition of this problem yields:

\[
\beta = \beta^* = \frac{[2 - k]^2}{6 - 4k - 3k^2 + 2k^3}.
\]

(ii) With \(\beta = \beta^*\), the VIP's profit is \(\Pi^n(\beta^*)\). Substituting \(\beta = \beta^*\) in (A5) yields:

\[
\Pi^n(\beta^*) = \frac{\alpha^2[3 - 2k]}{4[2 - k^2]}.
\]

From Lemma 2, the above profit expression is equal to \(\Pi^d\).

\[\square\]

**Proof of Proposition 6.** Under decentralization and arm's length pricing, the VIP's problem is as in (A2) with \(w_D = w_R = w\), the single price. The first-order condition with respect to \(w\) of this problem yields:
\[ w = w^* = \frac{\alpha(2 - k)[2 \tau(2 + k) - k^2(1 + \tau)]}{2[4(1 - k\tau) + 2\tau(k^3 + 4\tau) - k^2(2 + 3\tau^2)]}. \]

Substituting this input price into (A2) yields VIP profit of:

\[ \frac{\alpha^2[2 - k][2(1 + 3\tau^2) - k(1 + \tau)^2]}{4[4(1 - k\tau) + 2\tau(k^3 + 4\tau) - k^2(2 + 3\tau^2)]}. \]

Using the above profit expression and \( \Pi^c \) from Lemma 1, the VIP prefers decentralization with arm's length pricing to centralization if and only if:

\[-[8 + 4k - 6k^2 - k^3] + 2\tau k[4 - 2k - 3k^2 + k^3] > 0.\]

Solving for \( \tau \) in the above equation, \( \tau = \left[8 + 4k - 6k^2 - k^3\right]/\left[2k(4 - 2k - 3k^2 + k^3)\right], \)
completes the proof.

**Proof of the Corollary.** Without regulatory restrictions, observability, or commitment, the VIP's chosen \( w_D \) cannot influence \( R \)'s quantity response function. Thus, in choosing \( w_D \), the VIP takes \( q_R \) as given, thereby focusing only on the effect of \( w_D \) on \( q_D \). Clearly, this entails \( w_D = 0 \) and, from (6), this entails equivalent solutions under centralization and decentralization.

With an arm's length pricing restriction, the commitment to an external price guarantees the related-party price. Thus, VIP profit under decentralization in this case is as in the proof of Proposition 6. Comparing decentralization and centralization as in Proposition 6 then yields the corollary.
REFERENCES


